المجال الكهربي

Ch. 1: Electric Fields

Electricity & Magnetism (Starts from CH. 22 in TEXTBOOK)

Phys_1118

Note: The following slides are modified shared slides which made by the faculty members of Physics Dept

Ch. 1: Electric Fields

This chapter covers the following subjects:

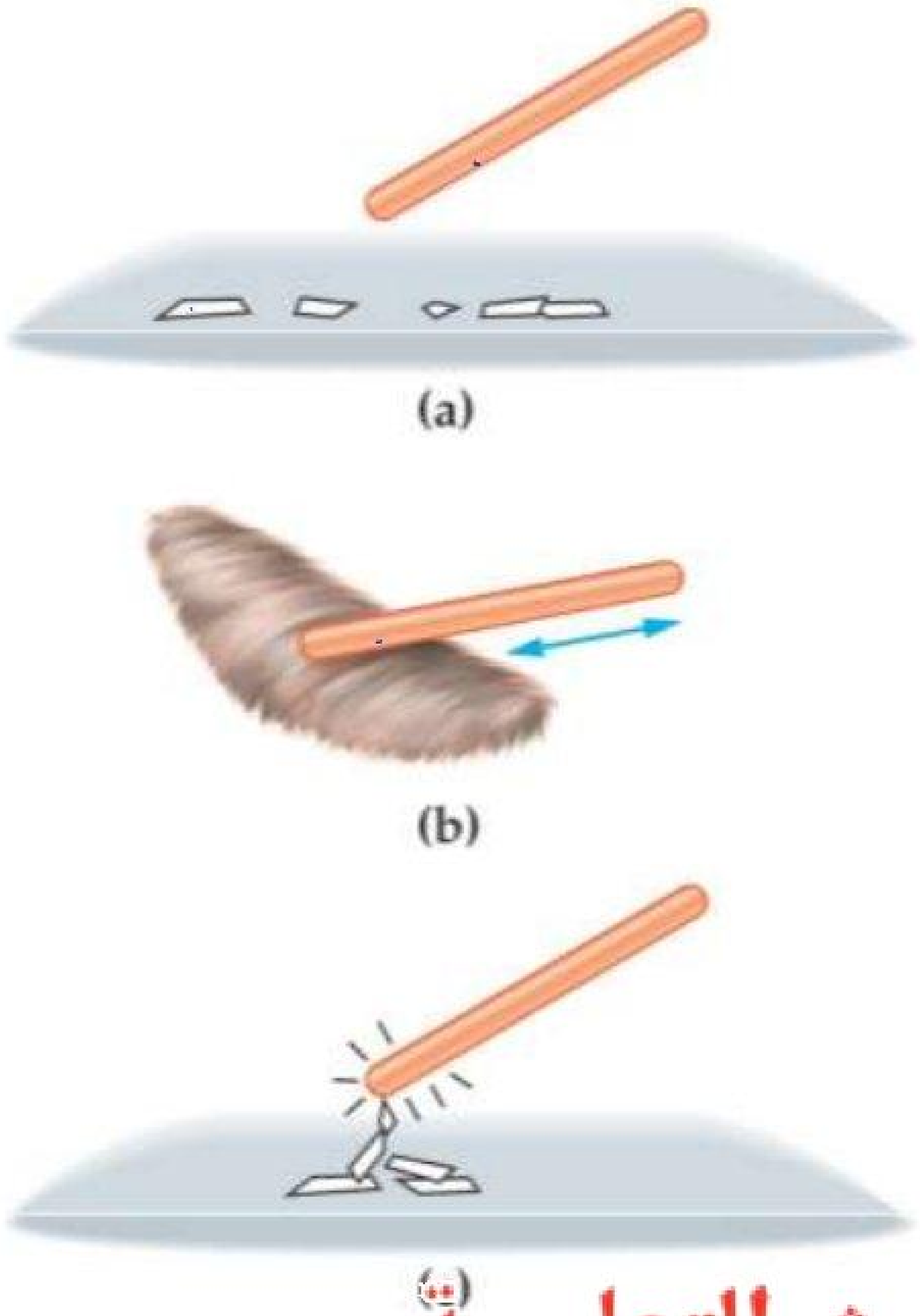
- *Properties of Electric Charges*
- *Charging Objects By Induction*
- *Coulomb's Law*
- *Electric Field Created by One Charge and Group of Charges*
- *Electric Field of a Continuous Charge Distribution*
- *Electric Field Lines*
- *Motion of Charged Particles in a Uniform Electric Field.*

Properties of Electric Charges:

خصائص الشحنات الكهربية

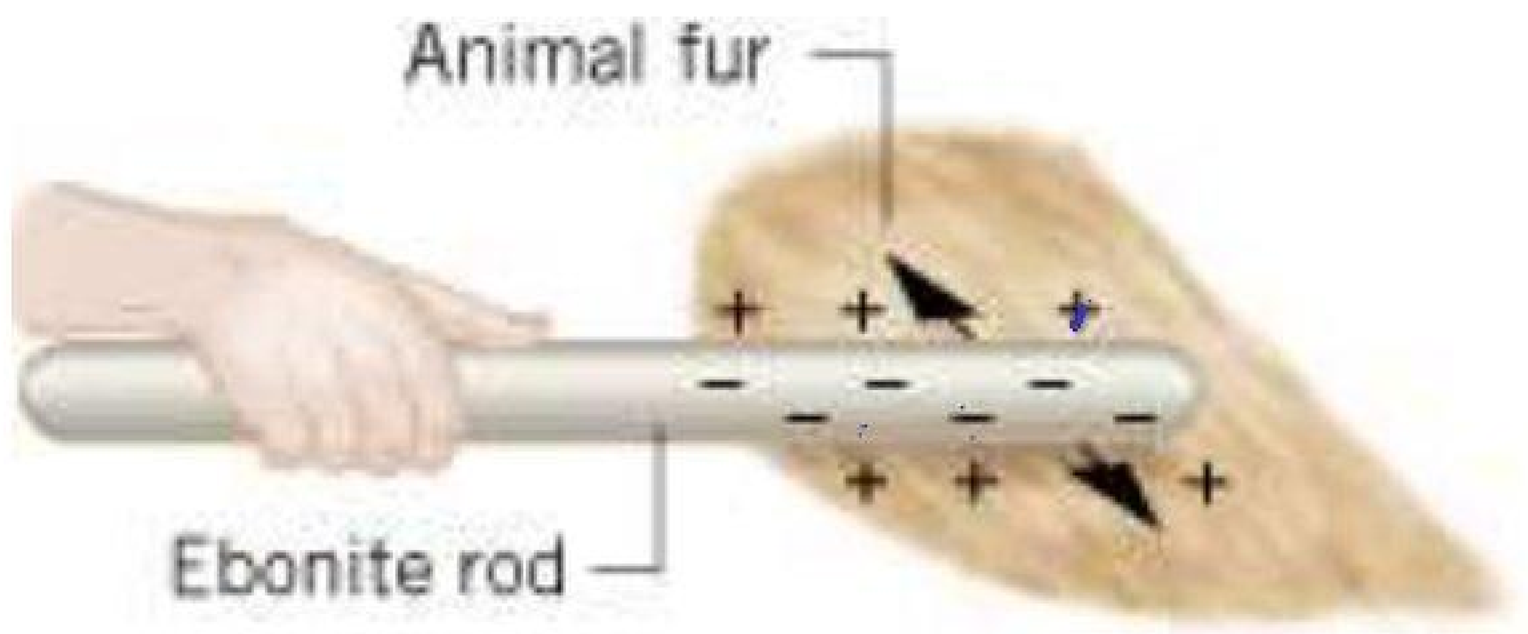
The effects of electric charge were first observed as static electricity:

After being rubbed on a piece of fur, a hard rubber rod acquires a charge and can attract small objects.



- It is possible to transfer electric charge from one object to another. Usually electrons are transferred, and the body that gains electrons acquires an excess of negative charge. The body that loses electrons has an excess of positive charge.

مبدأ بقاء الشحنة :
الشحنة الكهربية لا تفنى
ولا تستحدث من العدم



- During rubbing process, **the net electric charge of an entire isolated system remains constant (<u>the electric charge is always conserved</u>).**

→ This is referred to as the **law of conservation of electric charge**.

عمود مطاط + فرو ← العمود اكتسب شحنة (-) ، الفرو اكتسب شحنة (+)

- Rubbing a rubber rod with fur transfers negative charge to the rubber
  - Fur has excess positive charge due to the loss of negative charge

عمود زجاج + حرير ← العمود اكتسب شحنة (+) ، الحرير اكتسب شحنة (-)

- Rubbing a glass rod with silk transfers negative charge to the silk
  - Glass rod has excess positive charge

- Opposite charges attract, like charges repel

- Then, there are two kinds of electric charges: positive (+) and negative (-)
- Charges with the same sign repel, and changes with opposite sign attract.

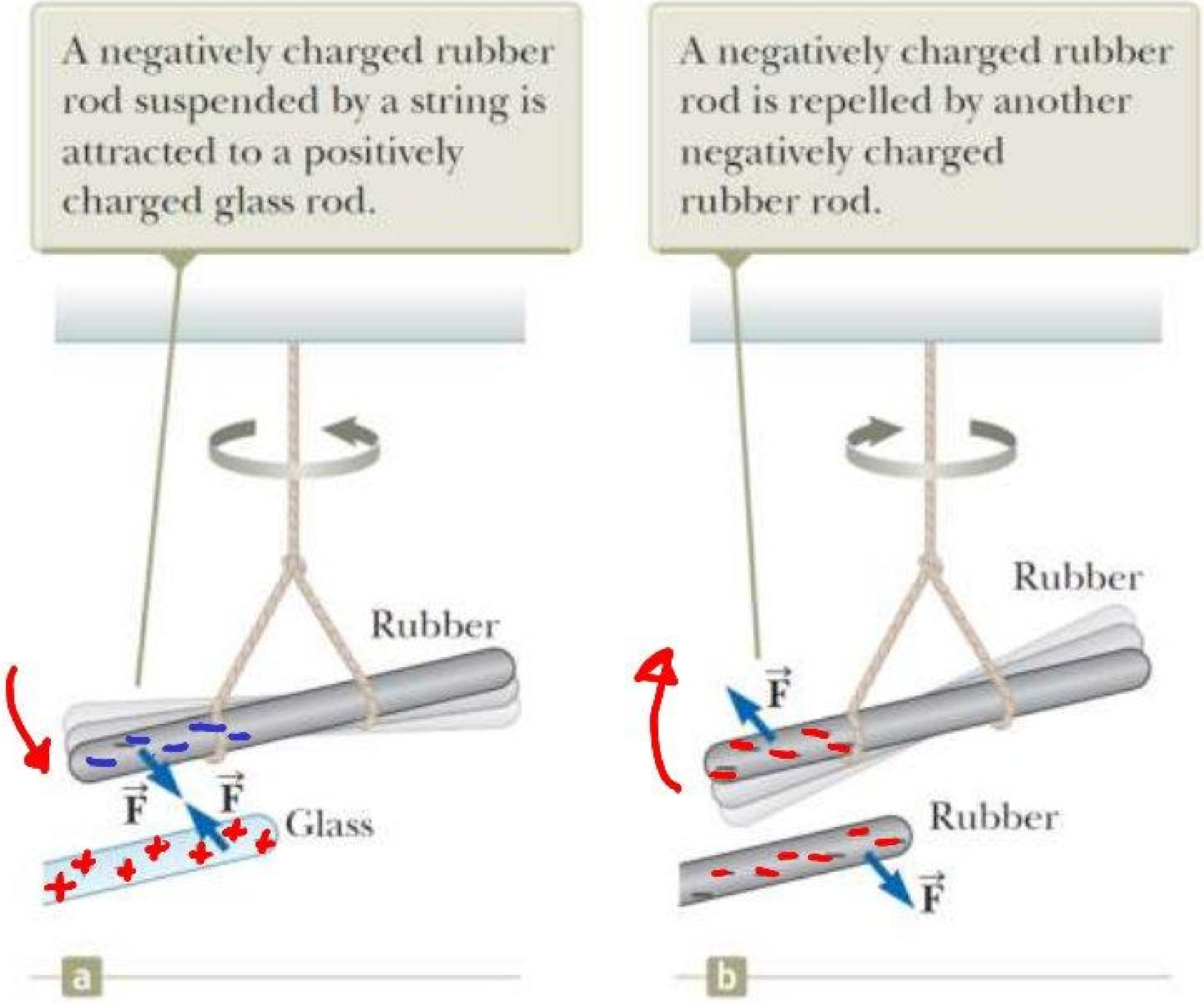
الشحنات من نفس تتنافر والشحنات عكس النوع تجاذب

- Charge is **quantized**
  - An electric charge (q) is always a multiple of a fundamental unit of charge, symbolized by $\mathbf{e} = \mathbf{1.6 \times 10^{-19}\ C}$ such that:

$$q = Ne$$

where N is an integer $(\pm e, \pm 2e, \pm 3e, \ldots)$.

  - **$1C = 6.24 \times 10^{18}$** electrons or protons.



Experiments show that:

- Negatively charged rubber rod is attracted to positively charged glass rod
- Negatively charged rubber rod is repelled by another negatively charged rubber rod

Conductors vs Insulators

- **<u>Conductors</u>:**

Electrical conductor is a material that conducts electrical charges. In other words, electrical conductors are materials in which some of the electrons are free electrons that are not bound to atoms and can move relatively freely through the material,

Example: Iron, silver, cuppor…

- **<u>Insulators</u>:**

Insulator is a material that conducts electric charges poorly. They are materials in which all electrons are bound to atoms and cannot move freely through the material. *Example: Wood, glass, rubber…*

Particles	Charge (C)	Mass (kg)
Electron (e)	-1.6022×10^{-19} C	$m_e = 9.1095 \times 10^{-31}$ kg
Proton (p)	$+1.6022 \times 10^{-19}$ C	$m_p = 1.6726 \times 10^{-27}$ kg
Neutron (n)	0	$m_n = 1.6749 \times 10^{-27}$ kg

المواد

" عوازل " insulator

← عندها عدد محدود من الالكترونات الحره

خشب - بلاستيك -

" موصلات " conductor

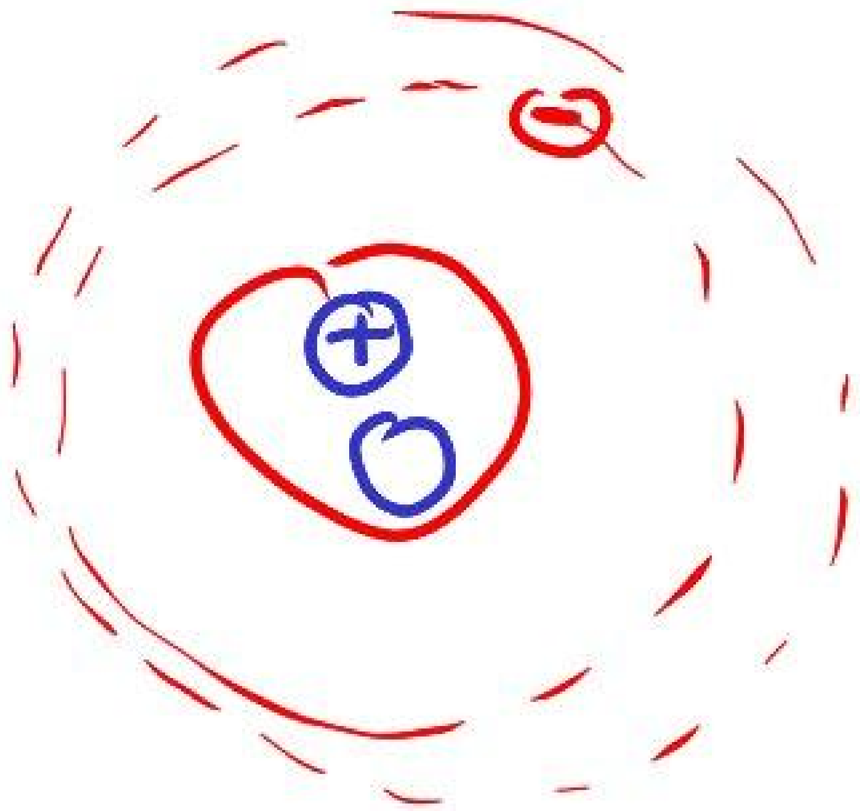
← عندها عدد كبير جدا من الالكترونات الحره

" النحاس - الالومنيوم - ...

المواد مكونه من ذرات ← e سالبه

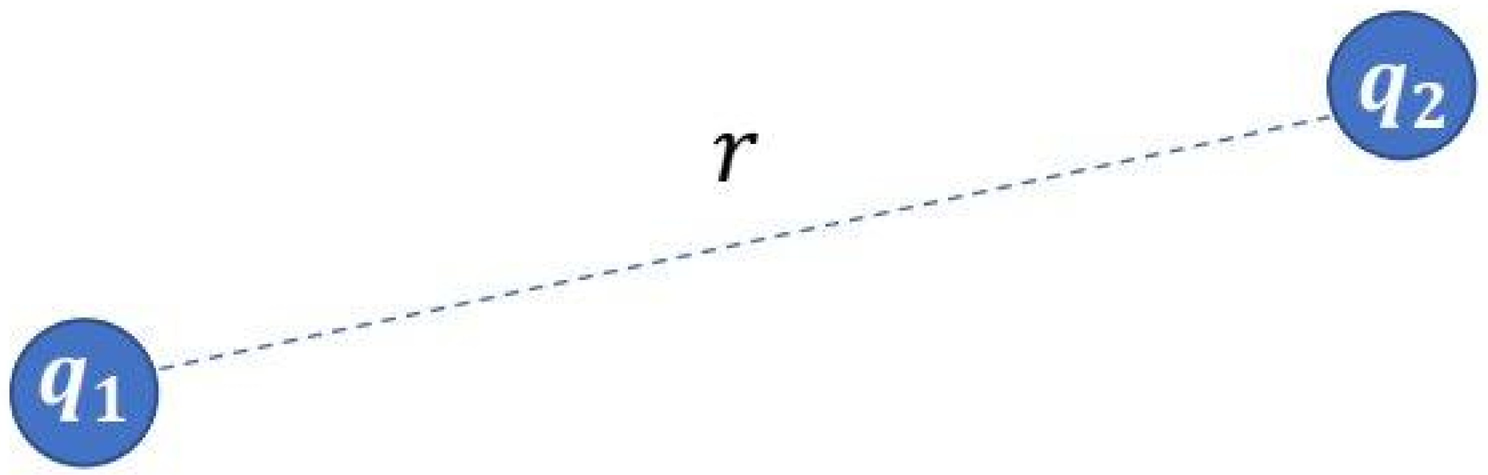
← P موجبه

← N متعادله



2- Coulomb's Law

قانون كولوم



- **The electric force between two charges is:**

$$\boxed{F_e = k_e \frac{|q_1||q_2|}{r^2}}$$

where k_e or simply k is called the **Coulomb constant,** $k_e = 8.9875 \times 10^9\ \text{N.m}^2/\text{C}^2$

This Coulomb constant is also written in the form:

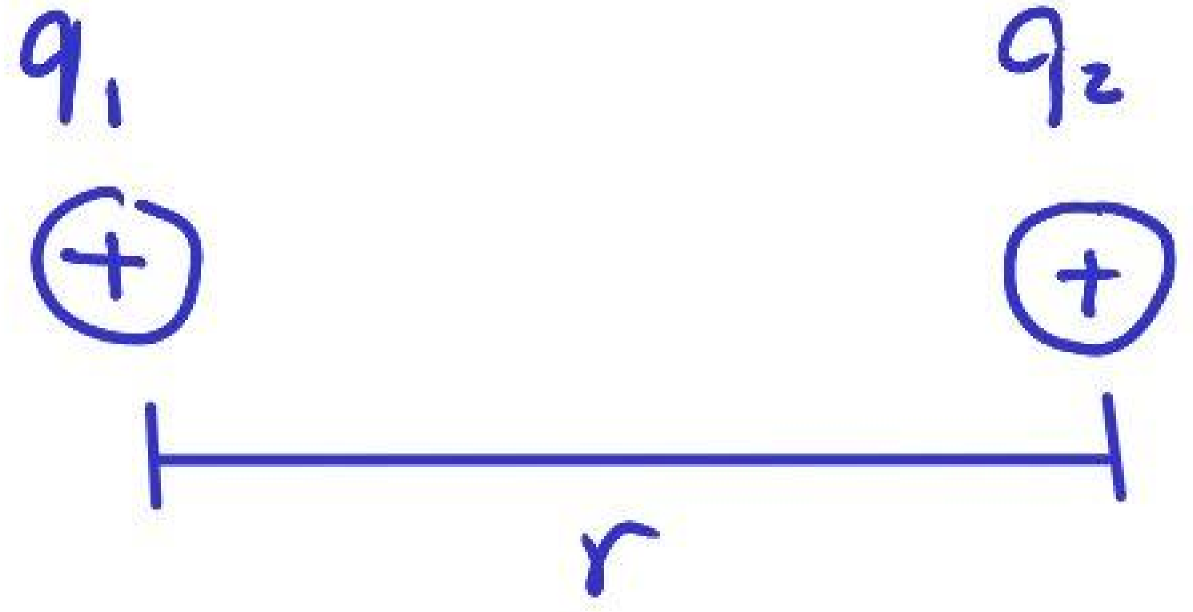
$$k_e = \frac{1}{4\pi\epsilon_0}$$

where ε_o the constant is known as the **permittivity of free space** and has the value: $\epsilon_0 = 8.854\,2 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$

- The SI unit of charge is the Coulomb (C)
  - $e = 1.6 \times 10^{-19}$ C

قانون كولوم « حساب فقط مقدار القوة »

$$\boxed{F_e = \frac{k_e |q_1||q_2|}{r^2}}$$



① $F_e \rightarrow$ القوة الكهربية بين الشحنات

② $q_1 , q_2 \rightarrow$ قيم الشحنات الكهربية بالموجب

③ $r \rightarrow$ المسافة الفاصلة بين الشحنتين

(4) $k_e \rightarrow$ ثابت كولوم

$$\boxed{k_e = \frac{1}{4\pi\epsilon_0} = 9 * 10^9}$$

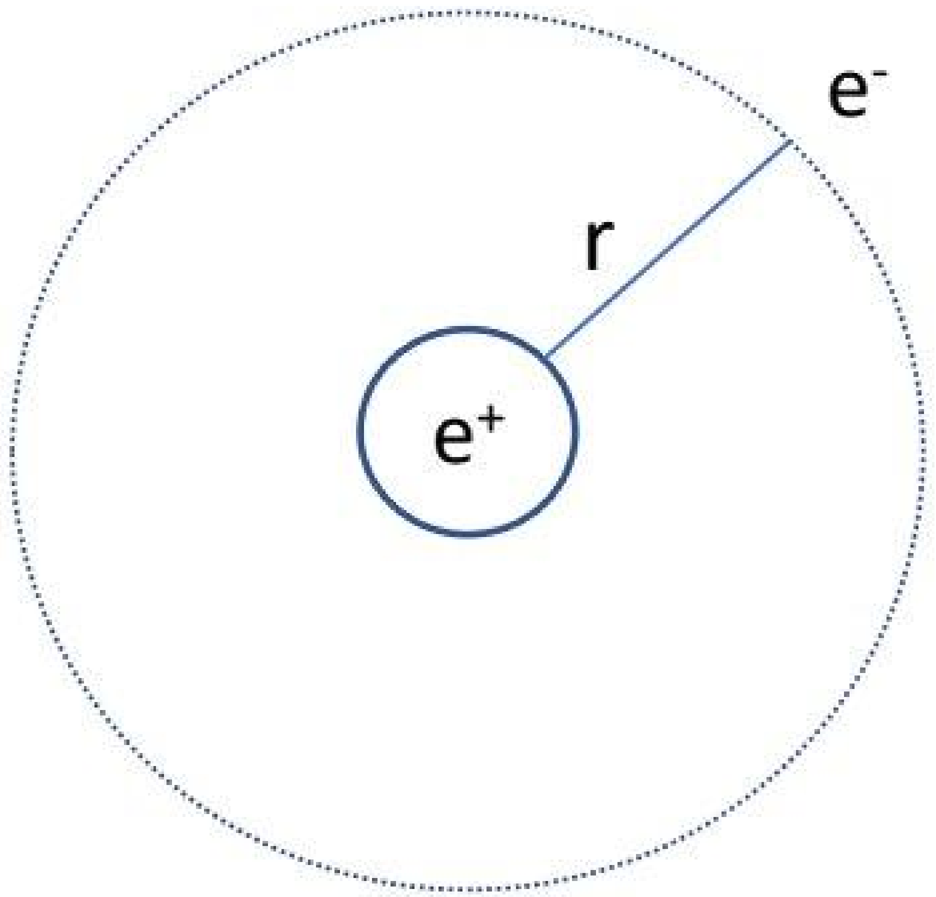
$\epsilon_0 \rightarrow$ السماحية الكهربية للفراغ

$$\epsilon_0 = 8.854 * 10^{-12}$$

Example 1:

- The electron and proton of a hydrogen atom are separated by a distance of 5.3×10^{-11} m. Find the electric force?

$$\boxed{\boldsymbol{F_e = k_e \frac{|q_1||q_2|}{r^2}}}$$



$$= \boldsymbol{k_e} \frac{|e|\,|-e|}{r^2} = (9\text{x}10^9)\ \frac{(1.6\text{x}10^{-19})^2}{(5.3\text{x}10^{-11})^2}$$

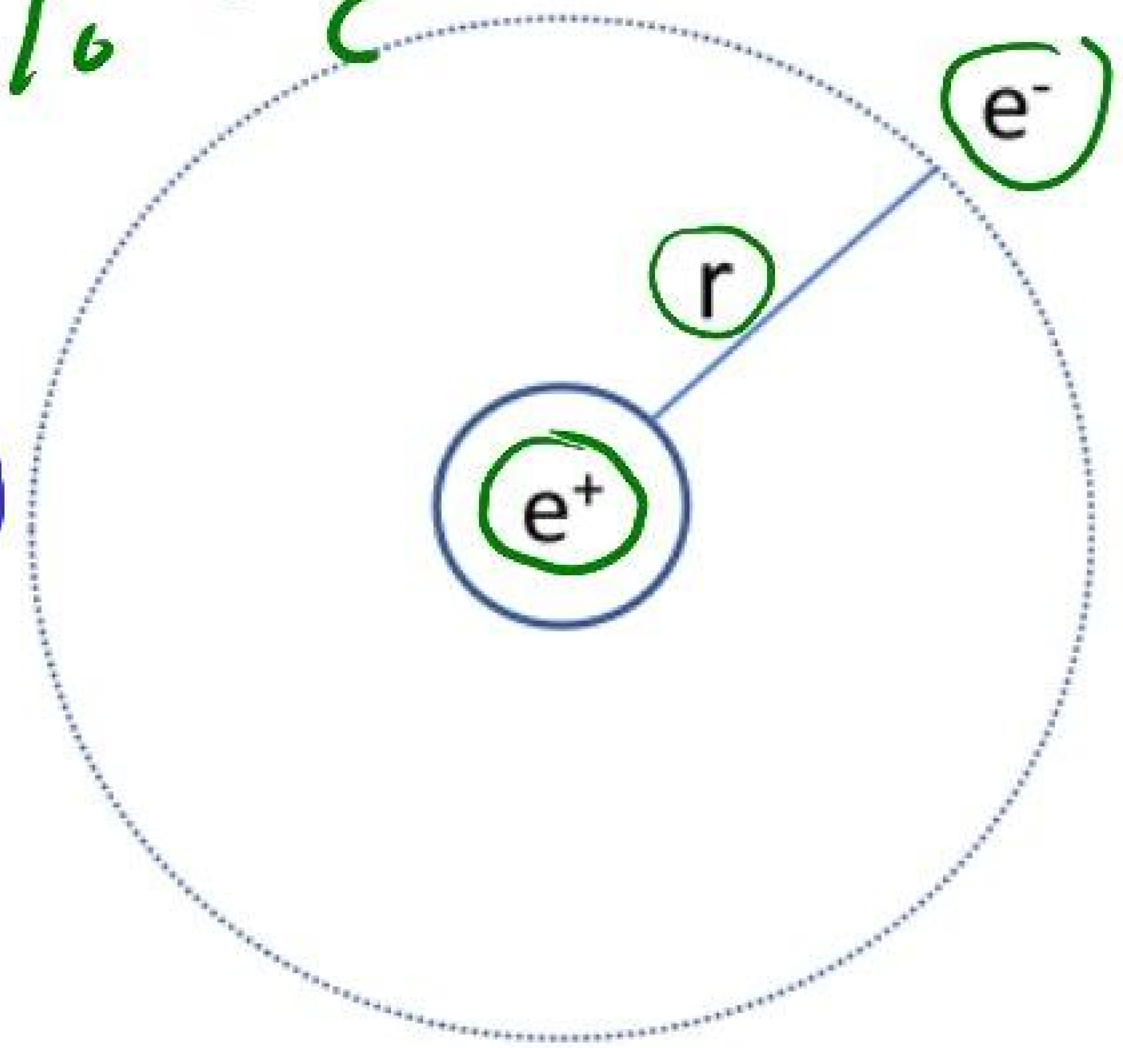
$$= 8.2\text{x}10^{-8}\text{N}$$

Example 1:

- The electron and proton of a hydrogen atom are separated by a distance of 5.3×10^{-11} m. Find the electric force?

$$\boxed{r = 5.3 * 10^{-11} \ m}$$

$$|q_e| = |q_p| = 1.6 * 10^{-19} \ C$$



$$F_e = \frac{k_e \, q_1 \, q_2}{r^2} = \frac{9 * 10^9 * 1.6 * 10^{-19} * 1.6 * 10^{-19}}{(5.3 * 10^{-11})^2}$$

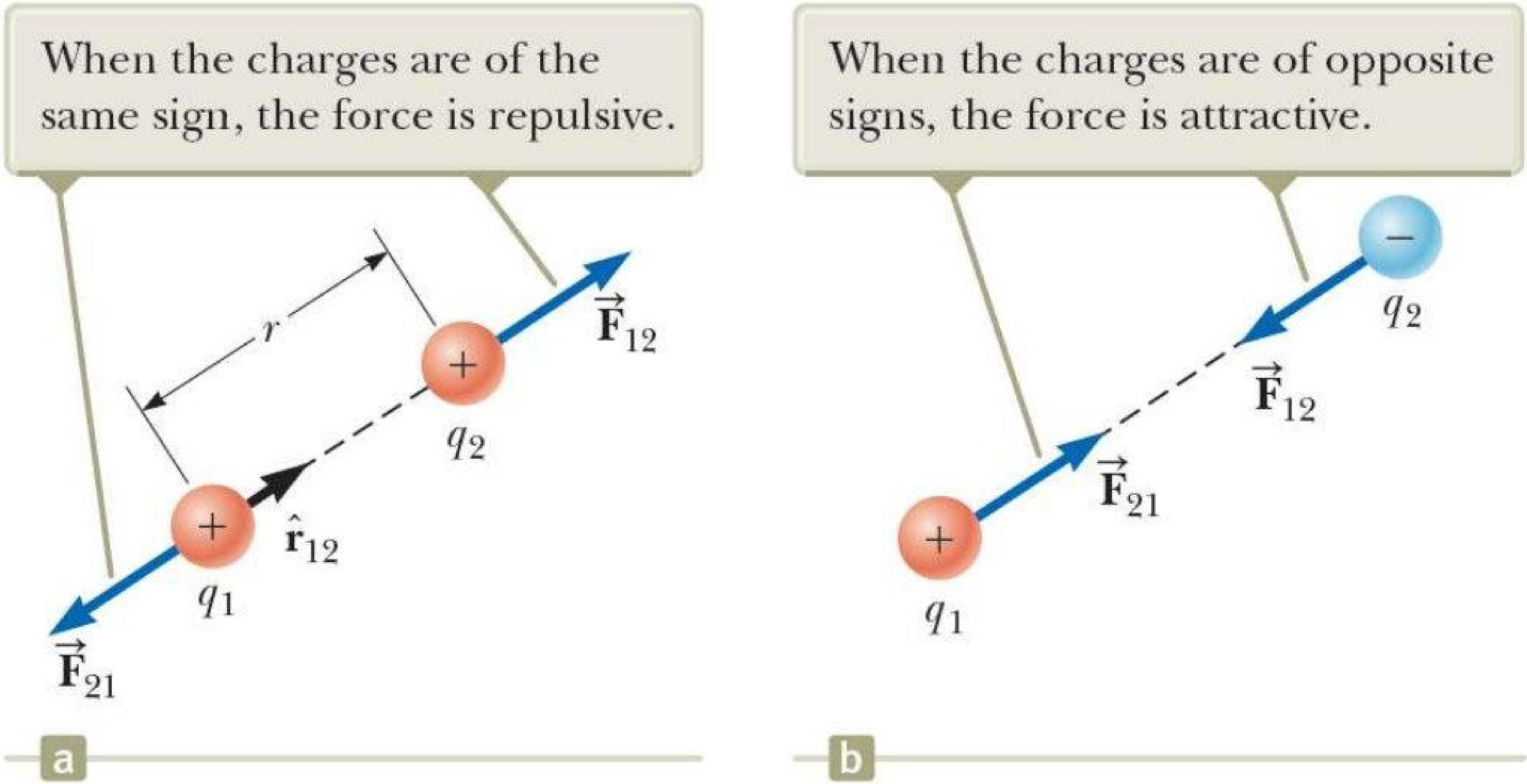
$$F_e = 8.2 * 10^{-8} \ N$$

- Remember! Force is a vector quantity $\vec{\boldsymbol{F}}$

→ Vector form of Coulomb's law:

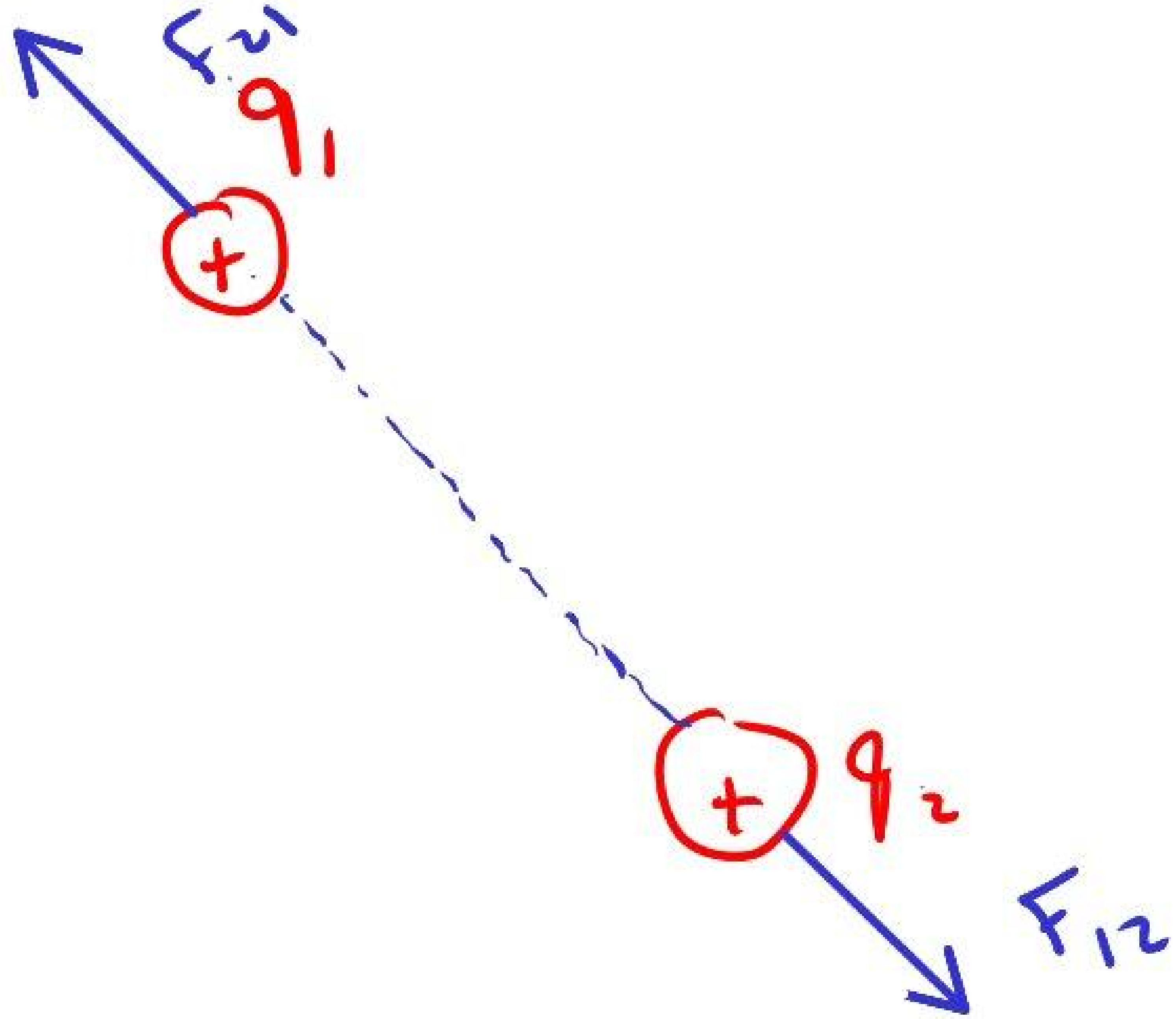
$$\boxed{\vec{\boldsymbol{F}}_e = k_e \frac{q_1 q_2}{r^2} \hat{\boldsymbol{r}}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector directed from q_1 toward q_2

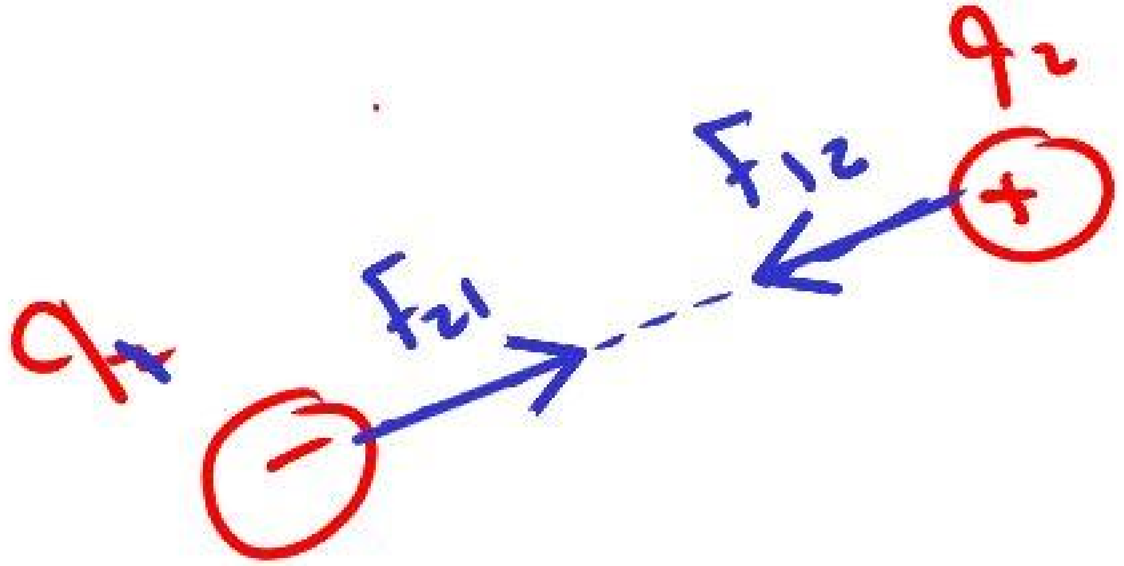


$$\boxed{\vec{\boldsymbol{F}}_{12} = -\vec{\boldsymbol{F}}_{21}}$$

اكاديمية الغيث التعليمية

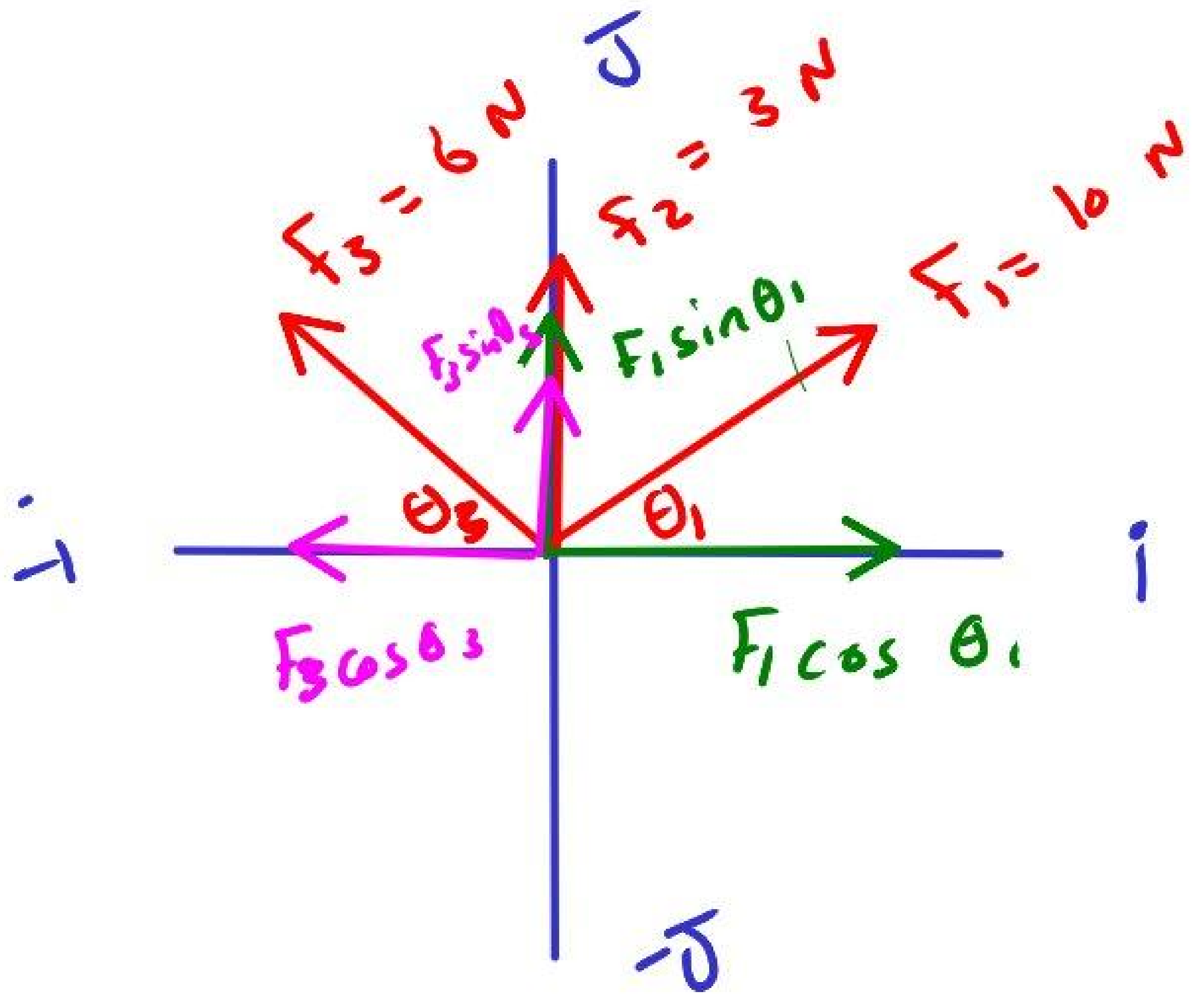


$$|F_{12}| = |F_{21}|$$



$$|F_{12}| = |F_{21}|$$

$$\boxed{F_T = F_x\, i + F_y\, j}$$



$$F_x = F_1 \cos\theta_1\, i - F_3 \cos\theta_3\, i$$

$$F_y = F_1 \sin\theta_1\, j + F_2\, j + F_3 \sin\theta_3\, j$$

- The resultant force on any one charge equals the vector sum of the forces exerted by the other individual charges that are present
  - Remember to add the forces *as vectors*

- The **resultant force** on q_1 is the vector sum of all the forces exerted on it by other charges:

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{41}$$

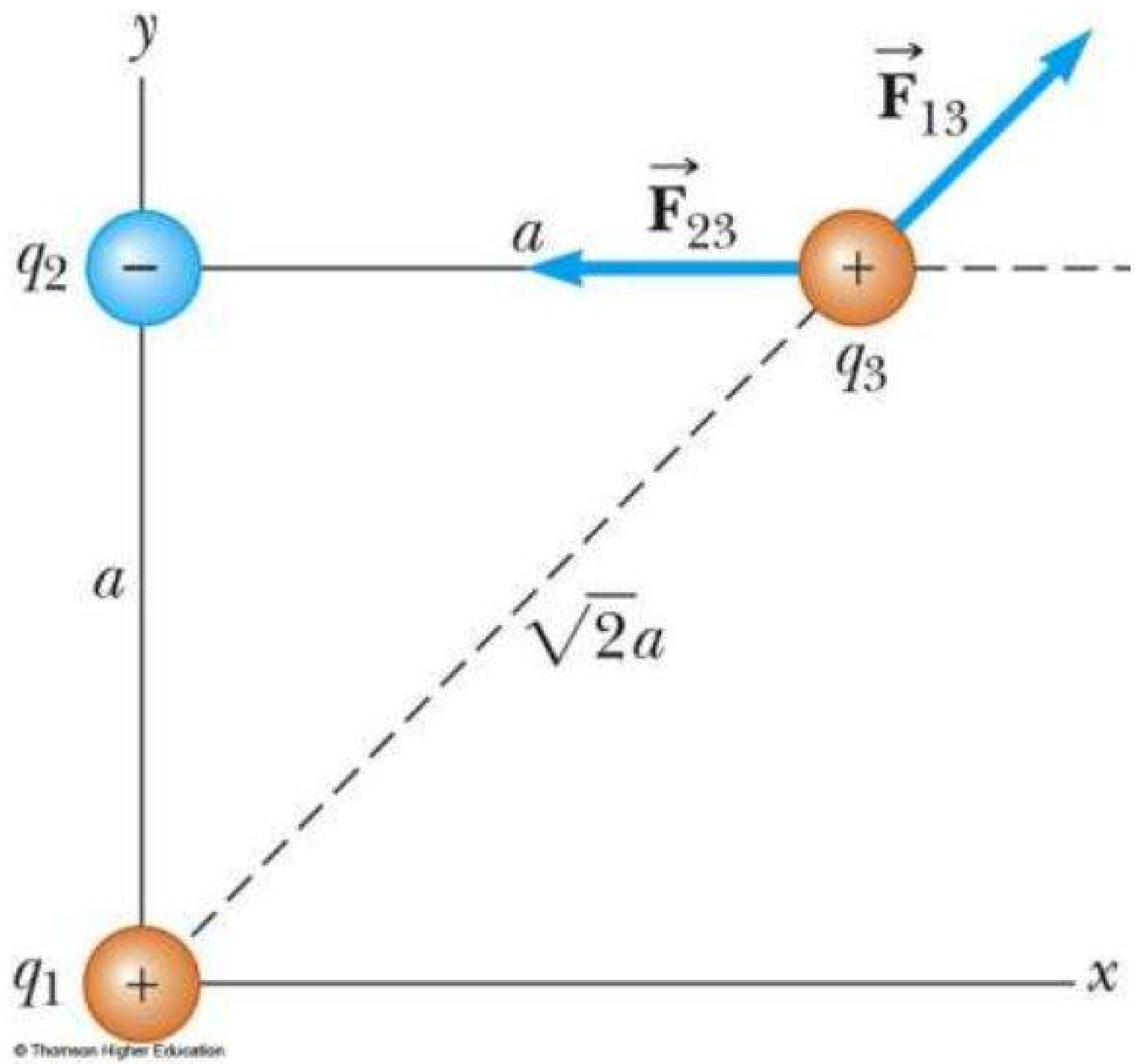
Example 3:

- Consider **three point charges** located at the corners of a right triangle as shown in Figure, where $q_1 = q_3 = 5\ \mu C$, $q_2 = -2\ \mu C$, *and* $a = 0.1\ m$.

Find the resultant force exerted on q_3.

Solution:

- The force exerted by q_1 on q_3 is $\vec{F}_{13}$
- The force exerted by q_2 on q_3 is $\vec{F}_{23}$
- The *resultant force* exerted on q_3 is the vector sum of $\vec{F}_{13}$ and $\vec{F}_{23}$



- Consider **three point charges** located at the corners of a right triangle as shown in Figure, where $q_1 = q_3 = 5\ \mu C$, $q_2 = -2\ \mu C$, *and* $a = 0.1\ m$.

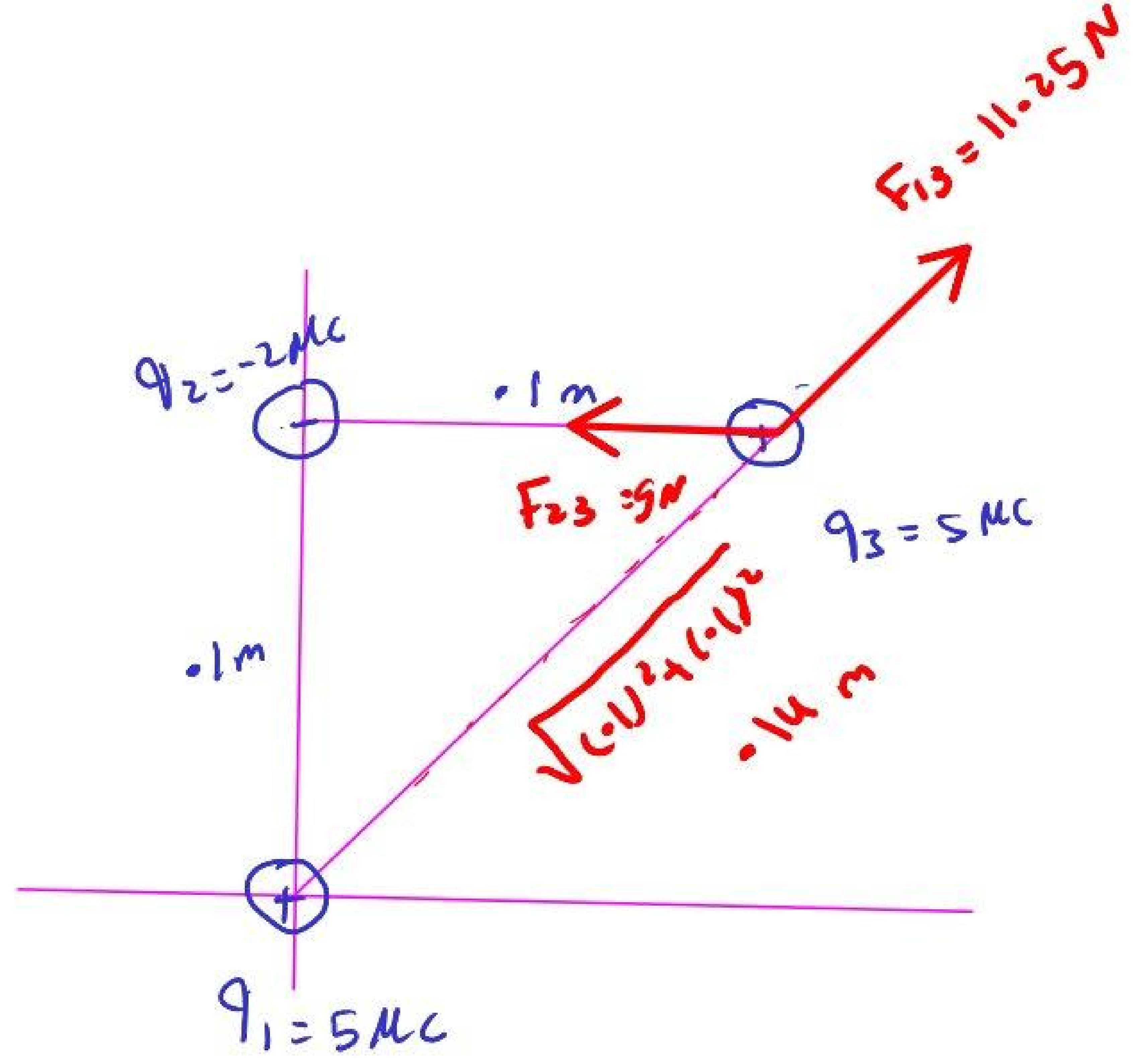
Find the resultant force exerted on q_3.

$$F_{13} = \frac{k_e\ q_1\ q_3}{r_{13}^2} = \frac{9*10^9 * (5*10^{-6})^2}{(.14)^2}$$

$$\boxed{F_{13} = 11.25\ N}$$

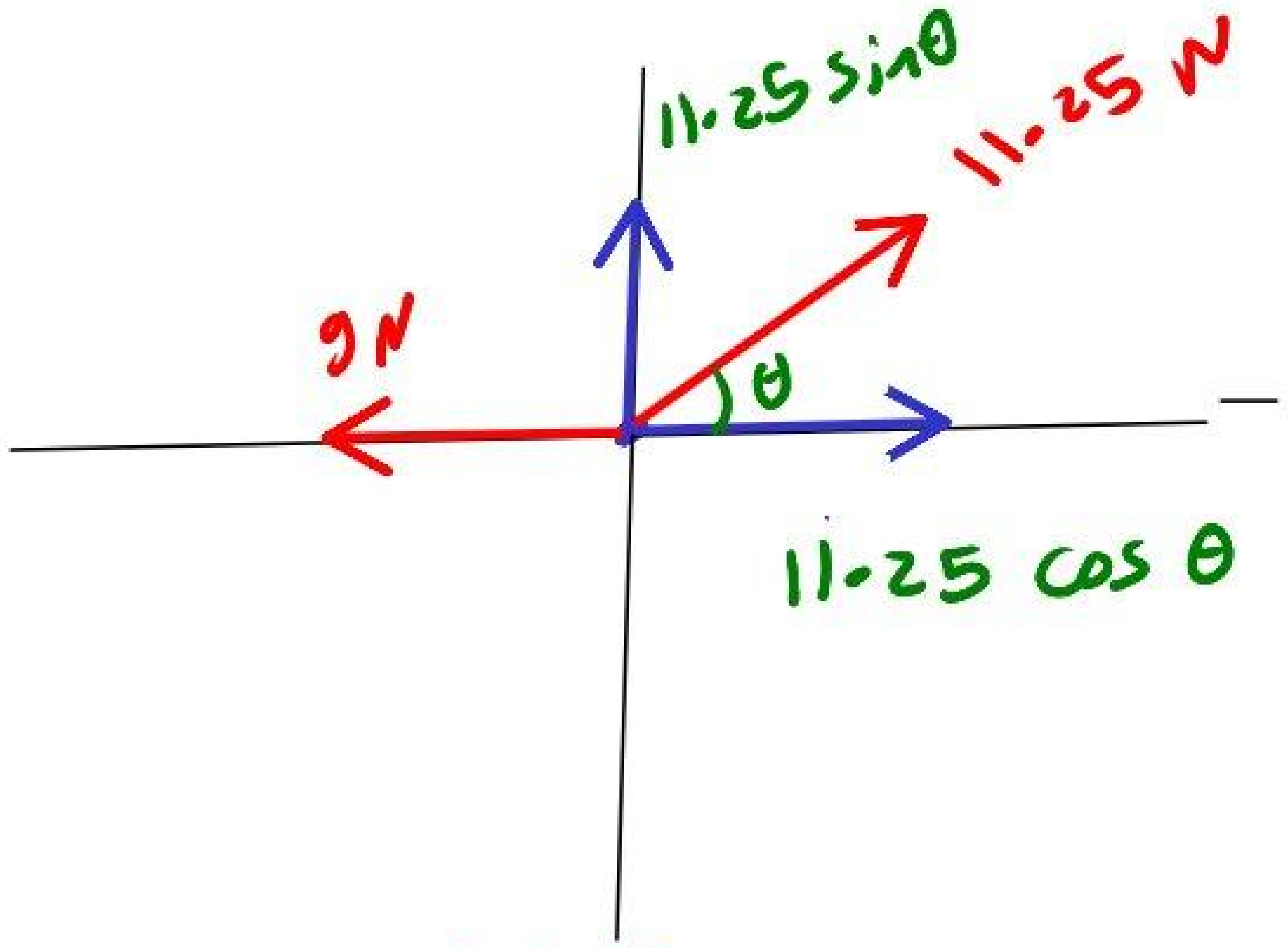
$$F_{23} = \frac{k_e\ q_2\ q_3}{r_{23}^2} = \frac{9*10^9 * (2*10^{-6})(5*10^{-6})}{(.1)^2}$$

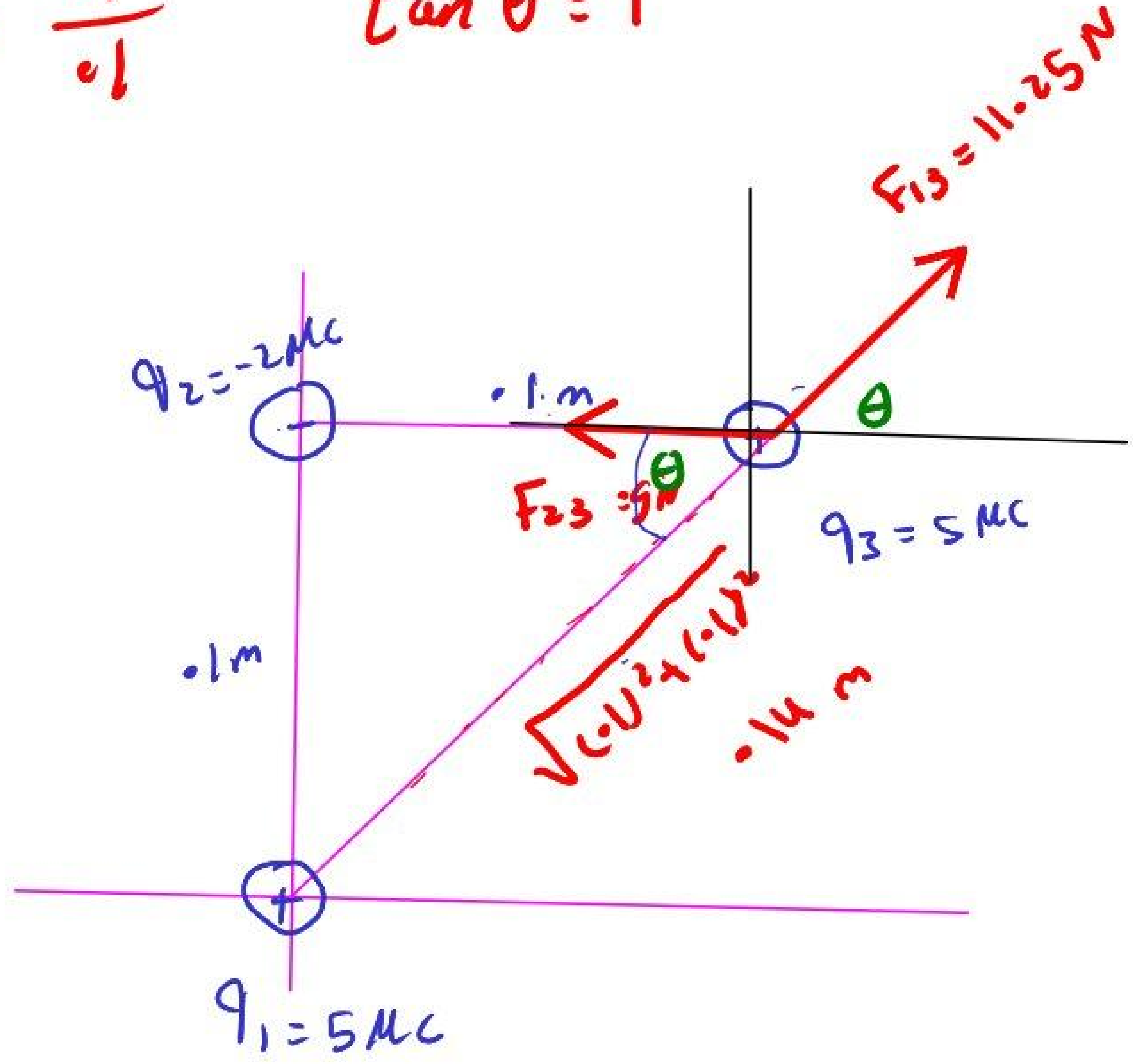
$$\boxed{F_{23} = 9\ N}$$



$\theta = \tan^{-1} 1 \longrightarrow \theta = 45$

$\tan\theta = \frac{.1}{.1}$ $\quad \tan\theta = 1$





cos 45

$$F_x = \left(11.25 * \frac{.1}{.14} - 9\right) i = -1.04\, i$$

sin 45

$$F_y = 11.25 * \frac{.1}{.14} J = 7.95\, J$$

$$\boxed{F_t = -1.04\, i + 7.95\, J}$$

$$F_t = \sqrt{(F_x)^2 + (F_y)^2}$$

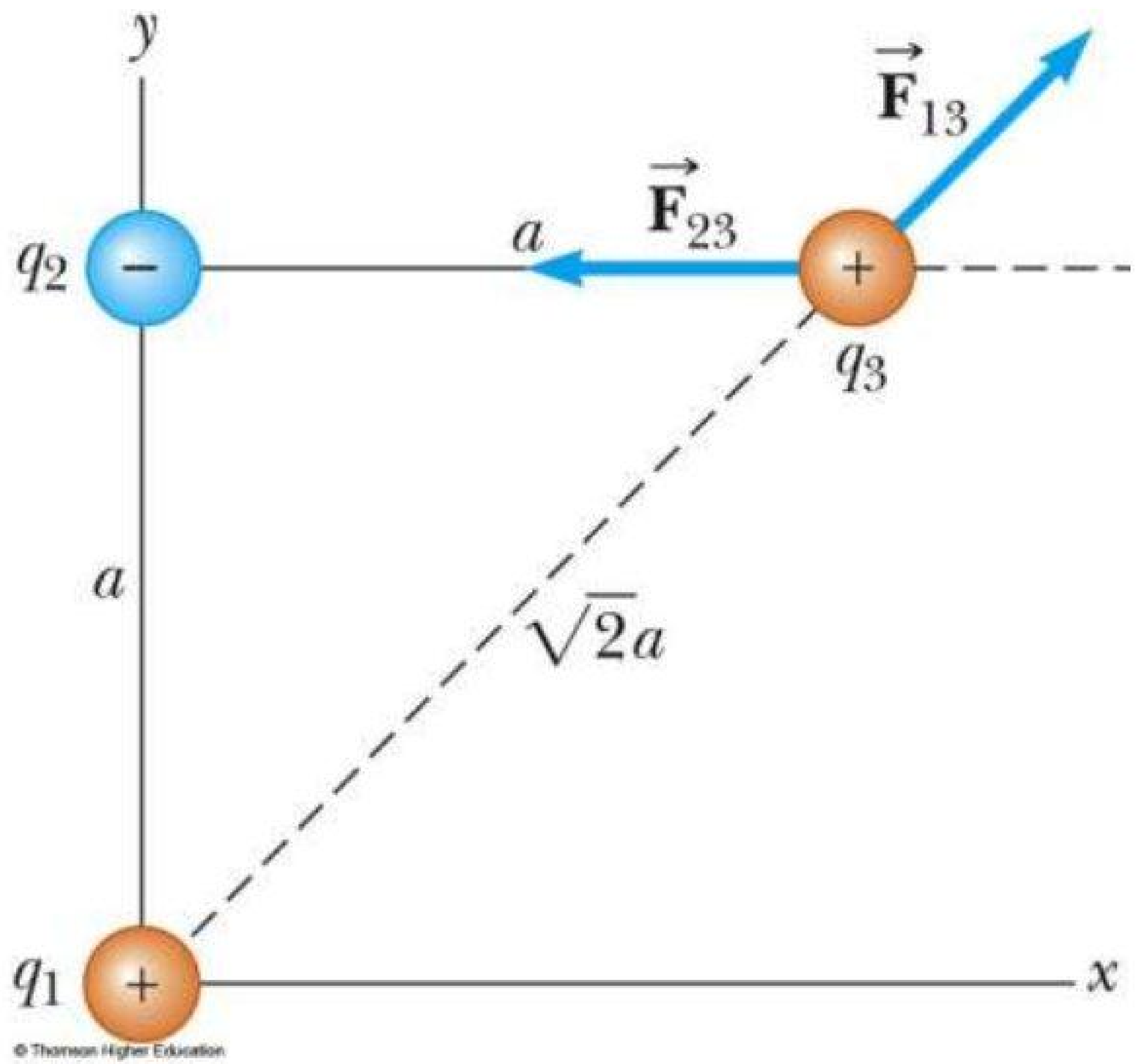
$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Solution:

Given that $q_1 = q_3 = 5\ \mu C$, $q_2 = -2\ \mu C$, and $a = 0.1$ m, the resultant Force exerted on q_3??

- $\vec{F}_{13}$: repulsive
- $\vec{F}_{23}$: attractive

$$F_{13} = k_e \frac{|q_1||q_3|}{r^2} \quad \text{(Coulomb's law)}$$



We can know from Figure that: $a^2 + a^2 = b^2$

$\rightarrow b^2 = 2\,a^2 \rightarrow b = \sqrt{2}\,a$

$$F_{13} = \frac{(9\times10^9)(5\times10^{-6})(5\times10^{-6})}{2(0.1)^2} = 11\ \text{N}$$

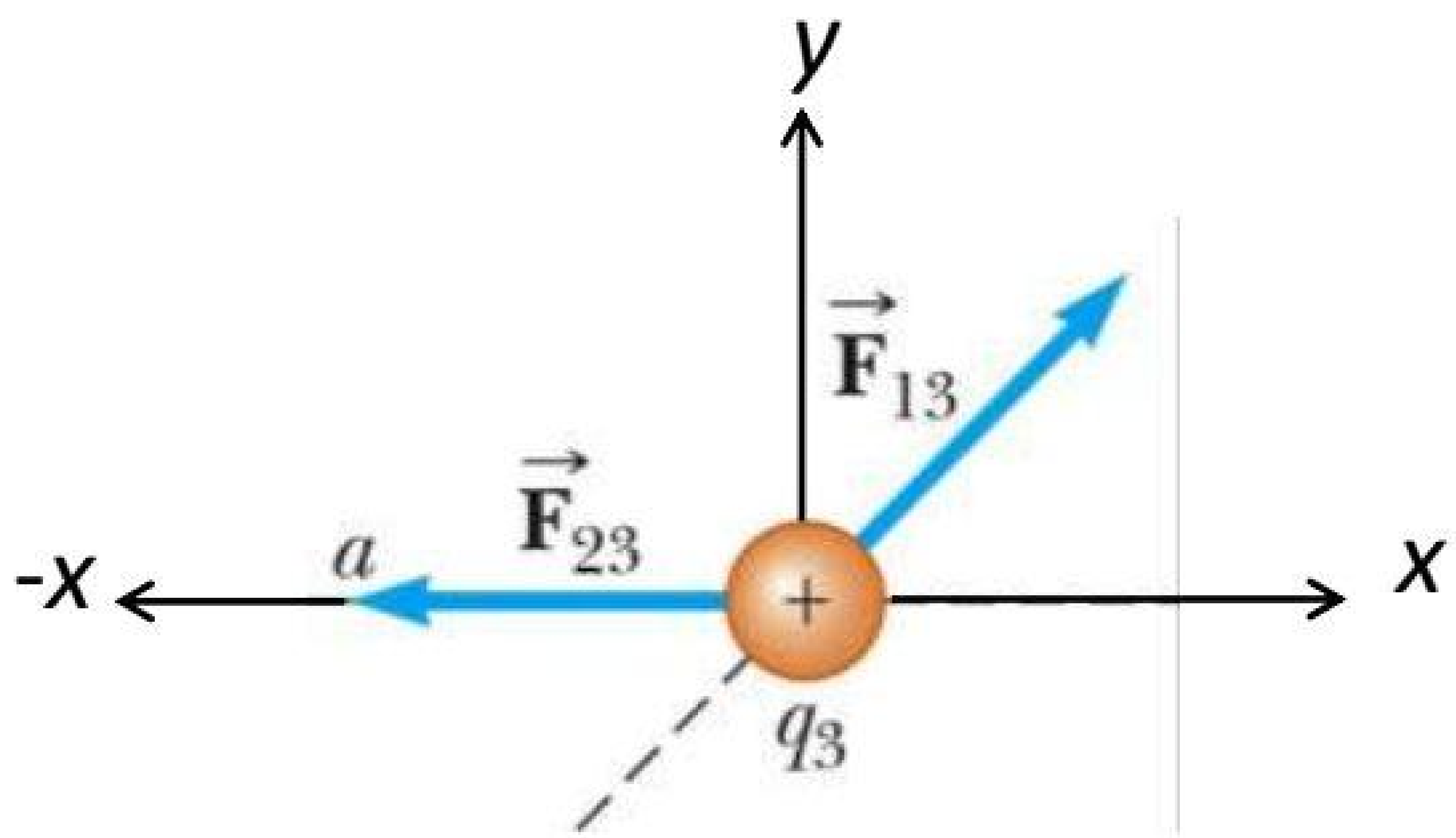
$$F_{23} = k_e \frac{|q_2||q_3|}{a^2} = 9\ \text{N}$$

- The resultant force $\overrightarrow{F_3}$:

$$F_x = F_{13}\cos 45^{\circ} - F_{23} = 11\left(\frac{\sqrt{2}}{2}\right) - 9 = -1.1\ N$$

$$F_y = F_{13}\sin 45^{\circ} = 7.9\ N$$

$$\boxed{\overrightarrow{F_3} = (-1.1\,\hat{\imath} + 7.9\,\hat{\jmath})\ \text{N}}$$



If you want to calculate the magnitude:

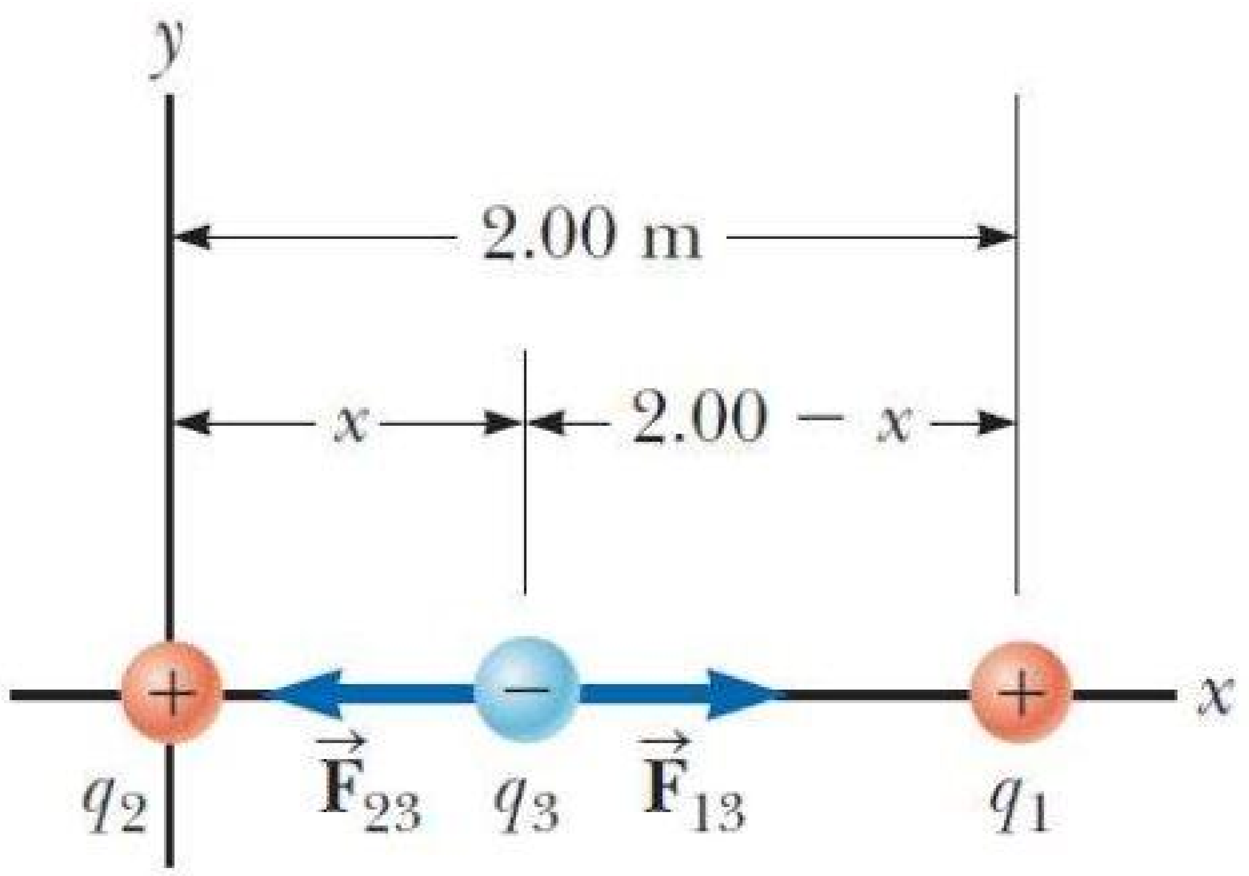
$$|\vec{F}_3| = \sqrt{F_x{}^2 + F_y{}^2} = \ldots..$$

And the direction of $\vec{F}_3$:

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \ldots.$$

Example 23.3 Where Is the Net Force Zero? AM

Three point charges lie along the x axis as shown in Figure 23.8. The positive charge $q_1 = 15.0\ \mu C$ is at $x = 2.00$ m, the positive charge $q_2 = 6.00\ \mu C$ is at the origin, and the net force acting on q_3 is zero. What is the x coordinate of q_3? $(x = ?)$



SOLUTION

The net force acting on $q_3 = 0$

$$\overrightarrow{\boldsymbol{F_{23}}} + \overrightarrow{\boldsymbol{F_{13}}} = \mathbf{0}$$

$$-F_{23} + F_{13} = 0$$

$$F_{23} = F_{13}$$

$$k_e \frac{|q_2||q_3|}{x^2} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2}$$

$$(2.00 - x)^2|q_2| = x^2|q_1|$$

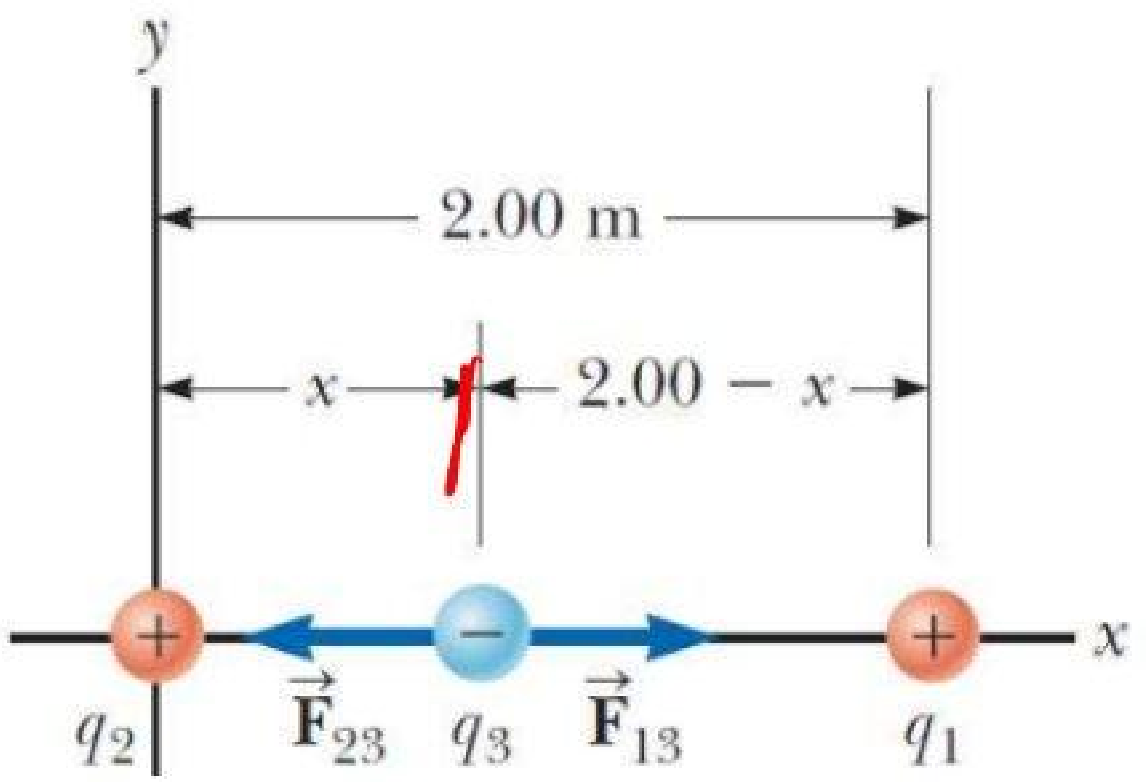
$$\rightarrow \quad (2.00 - x)\sqrt{|q_2|} = \pm x\sqrt{|q_1|}$$

$$x = \frac{2.00\sqrt{|q_2|}}{\sqrt{|q_2|} \pm \sqrt{|q_1|}}$$

$$x = \frac{2.00\sqrt{6.00 \times 10^{-6}\ \text{C}}}{\sqrt{6.00 \times 10^{-6}\ \text{C}} + \sqrt{15.0 \times 10^{-6}\ \text{C}}} = \boxed{0.775\ \text{m}}$$

Example 23.3 Where Is the Net Force Zero? AM

Three point charges lie along the x axis as shown in Figure 23.8. The positive charge $q_1 = 15.0\ \mu C$ is at $x = 2.00$ m, the positive charge $q_2 = 6.00\ \mu C$ is at the origin, and the net force acting on q_3 is zero. What is the x coordinate of q_3? ($x = ?$)



SOLUTION

$$\sum F_x = 0$$

$$F_{13} - F_{23} = 0$$

$$F_{13} = F_{23}$$

$$\frac{\cancel{k_e} q_1 \cancel{q_3}}{(2-x)^2} = \frac{\cancel{k_e} q_2 \cancel{q_3}}{(x)^2}$$

$$\frac{q_1}{(2-x)^2} = \frac{q_2}{x^2}$$

$$\frac{15 * \cancel{10^{-6}}}{(2-x)^2} = \frac{6 * \cancel{10^{-6}}}{x^2}$$

$$15x^2 = 6(2-x)^2$$

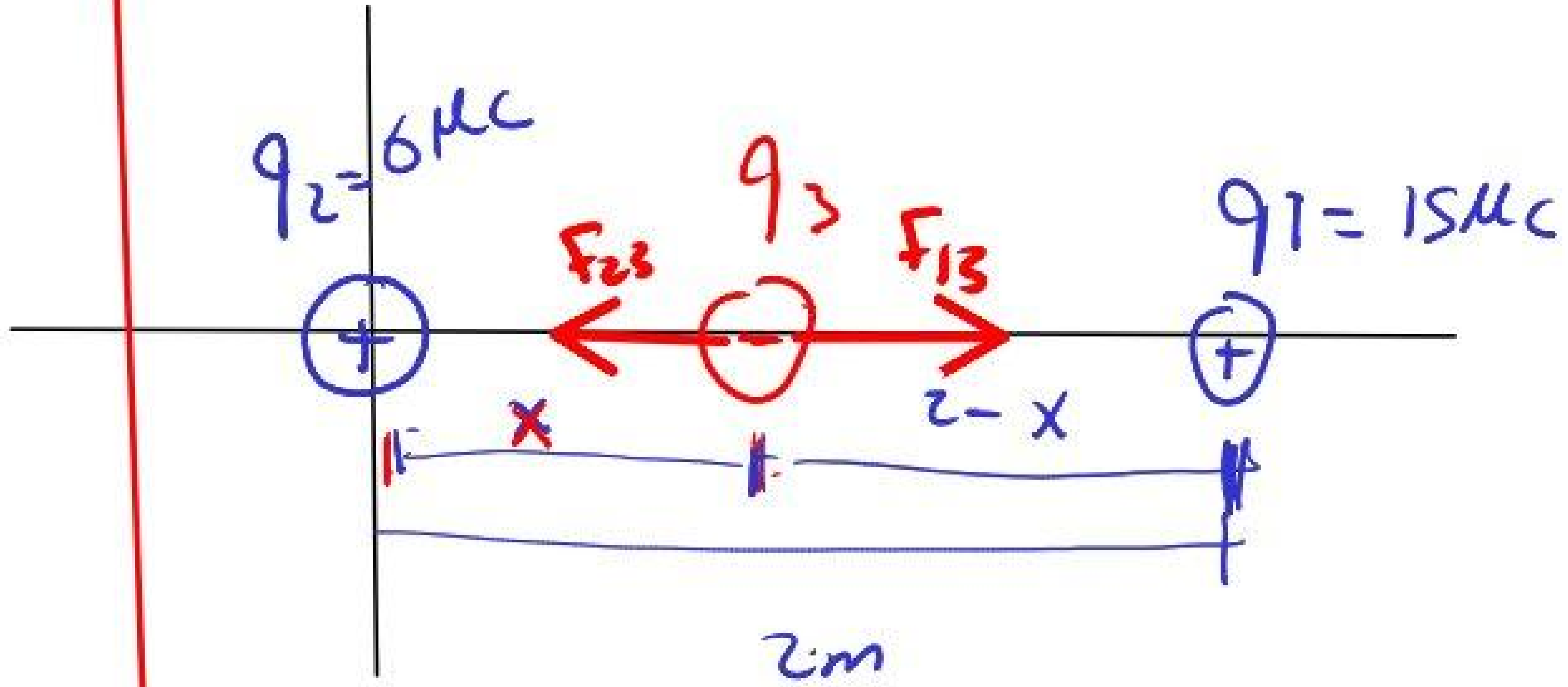
$$15x^2 = 6(4 - 4x + x^2)$$

$$15x^2 = 24 - 24x + 6x^2$$

$$9x^2 + 24x - 24 = 0$$

$$\boxed{x = .774\ m \rightarrow accepted}$$

$x = -3.44\ m \rightarrow$ rejected



For you to practice:

- *Solve Example **22.4** in the Textbook!*

3- The Electric Field:

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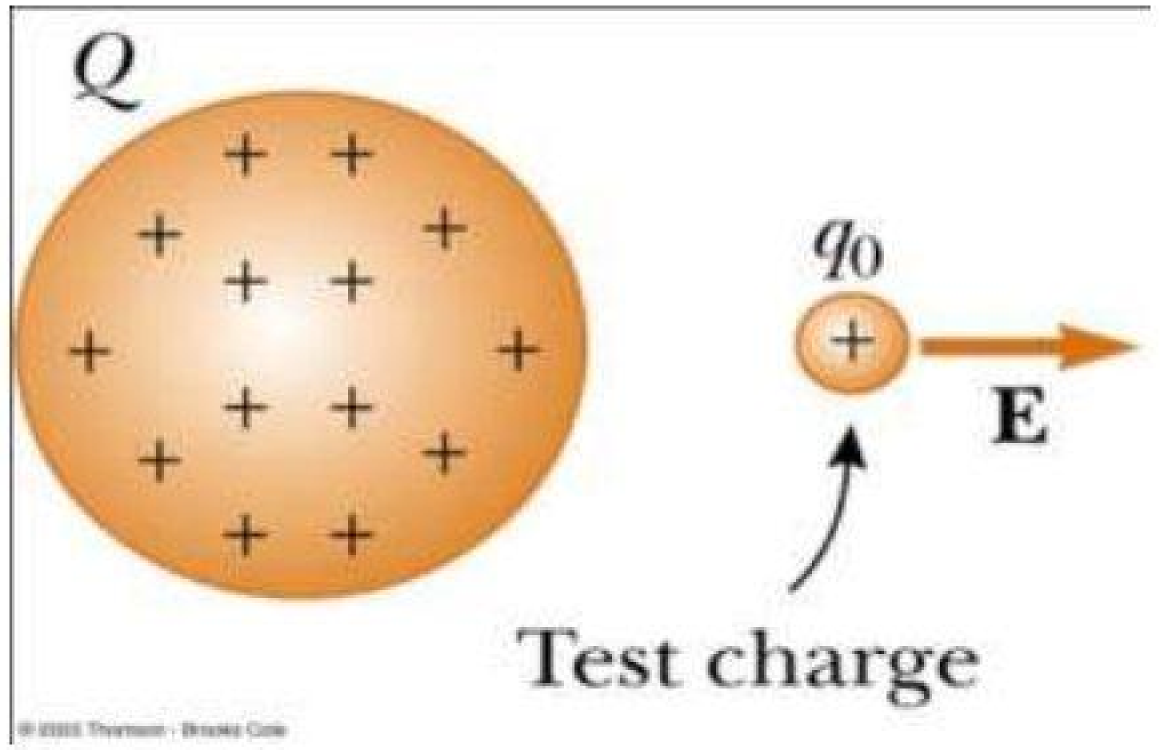
- The electric field $\vec{E}$ at some point in space is defined as the electric force $\vec{F}_e$ that acts on a small positive test charge placed at that point divided by the magnitude of the test charge:

$$\boxed{\vec{E} = \frac{\vec{F}_e}{q_0}} \rightarrow \text{In general, } \boxed{\vec{F}_e = q\vec{E}}$$

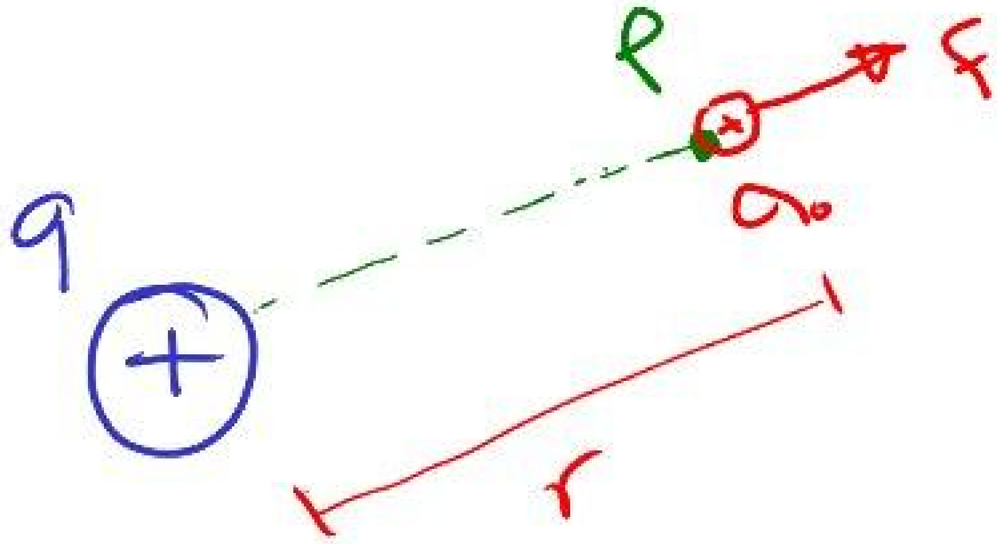


- SI Unit of the electric field: N/C

- **Note:** $\vec{F}_e = q\vec{E}$ This is valid for a point charge only.
  - One of zero size
  - For larger objects, the field may vary over the size of the object

electric field (E)

عند وجود شحنة كهربية بيكون مجال كهربي حول الشحنة



① $$\boxed{E_p = \frac{F}{q_0}}$$

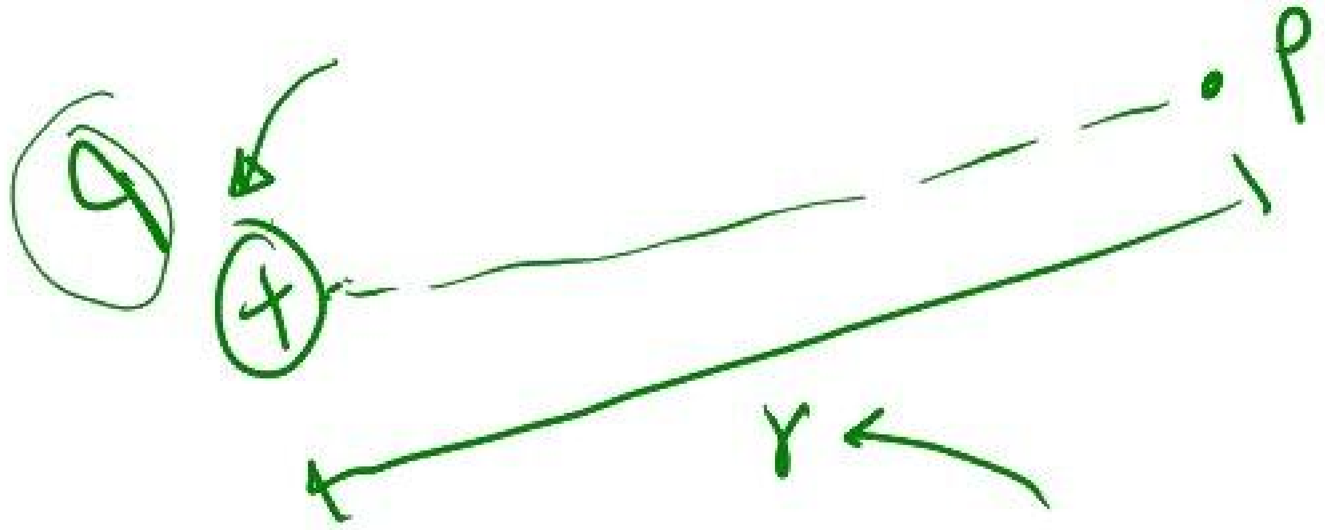
$F \rightarrow$ القوة على test charge

$q_0 \rightarrow$ قيمة الـ test charge

② $$E = \frac{k_e\, q\, \cancel{q_0} / r^2}{\cancel{q_0}}$$

$k_e \rightarrow$ ثابت كولوم $= 9 * 10^9$

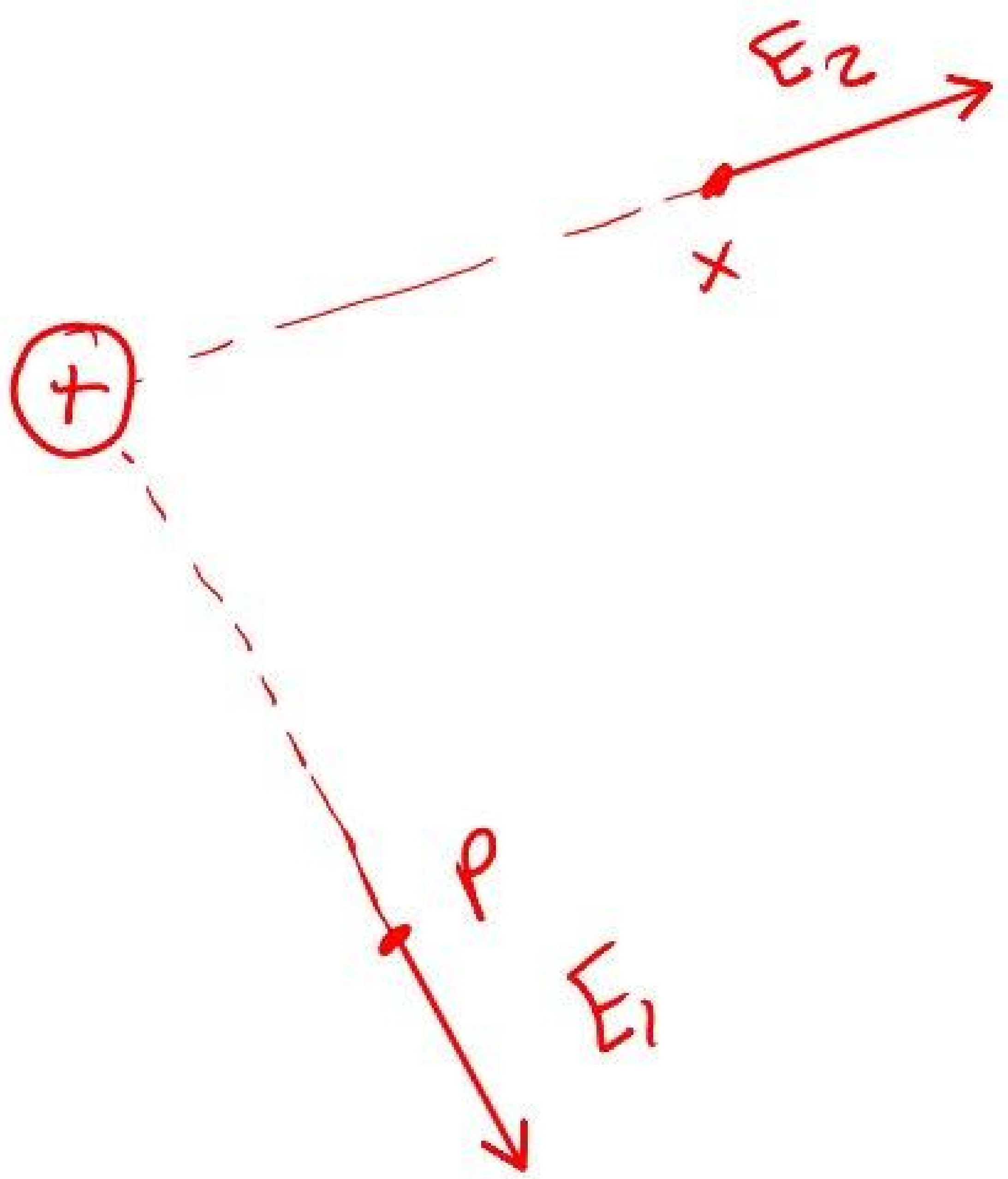
$$\boxed{E = \frac{k_e\, q}{r^2}}$$

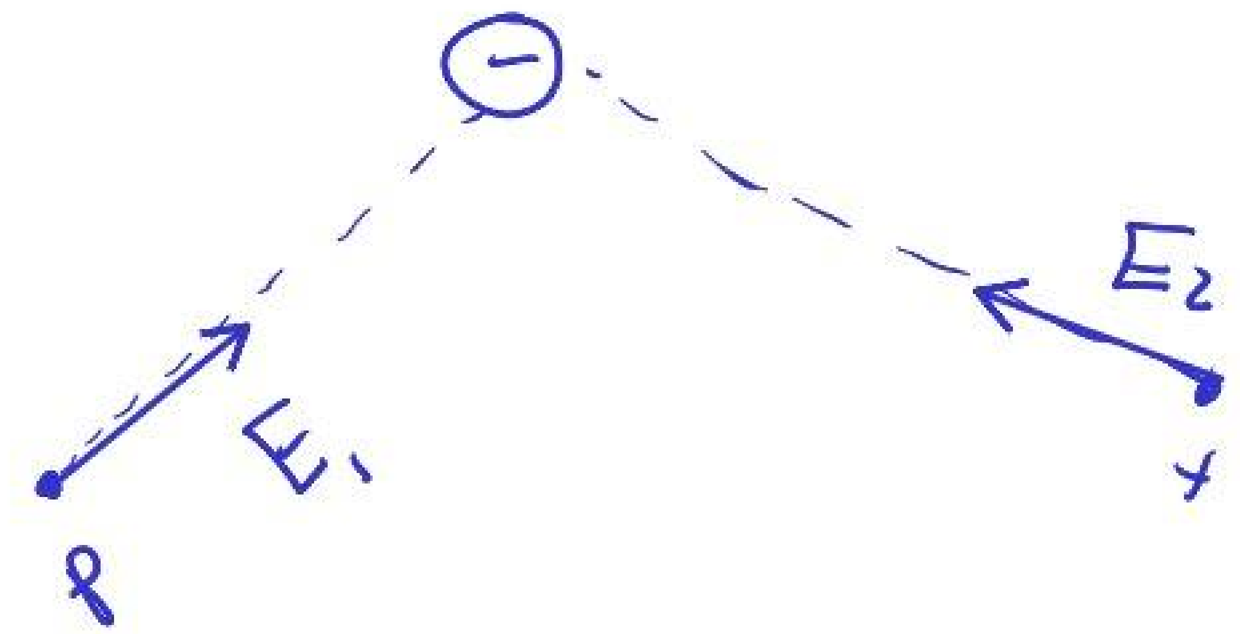


$q \rightarrow$ الشحنة المسببة للمجال

$r \rightarrow$ المسافة بين النقطة P والشحنة

اتجاه المجال





لو عندي أكثر من شحنة بنحسب المجال نتيجة كل شحنة على حده كمتجه

وبعدين نجمع المتجهات

- Remember <u>Coulomb's law</u>, between the source charge (q)and test charge (q_0), can be expressed as:

$$\boxed{\vec{\boldsymbol{F}}_e = k_e \frac{q q_0}{r^2} \hat{\boldsymbol{r}}}$$

- Then, the electric field due to the charge q can be written as:

$$\vec{\boldsymbol{E}} = \frac{\vec{F}_e}{q_0} \rightarrow \boxed{\vec{\boldsymbol{E}} = k_e \frac{q}{r^2} \hat{\boldsymbol{r}}}$$

So that, $\boxed{E = k_e \frac{|q|}{r^2}}$, and the SI unit is N/C.

- $\vec{\mathbf{E}}$ is the electric field produced by some charge or a group of charges, separate from the **test charge** (q_0)

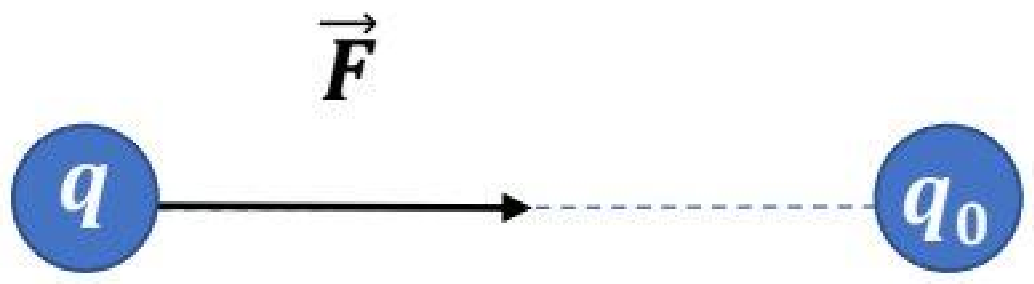
- The existence of an electric field is a property of the **source charge** (q)
  - The presence of the test charge is not necessary for the field to exist

- The test charge (q_0) serves as a detector of the electric field.

Example 4:

- $q_0 = 18 \times 10^{-8} C$
- If the force F = $36 \times 10^{-8} N$

What is the electric field?

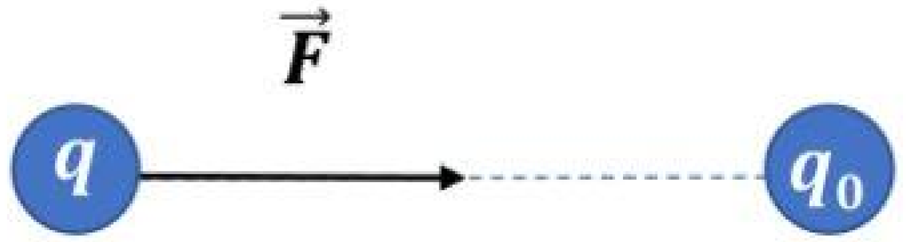


$$E = \frac{F}{|q_0|} = \frac{36 \times 10^{-8} N}{18 \times 10^{-8} C} = 2\ \text{N/C}$$

Example 4:

- $q_0 = 18 \times 10^{-8} C$
- If the force F = 36 × $10^{-8} N$

What is the electric field?



$q_0 = 18 * 10^{-8}$ C

$F = 36 * 10^{-8}$ N

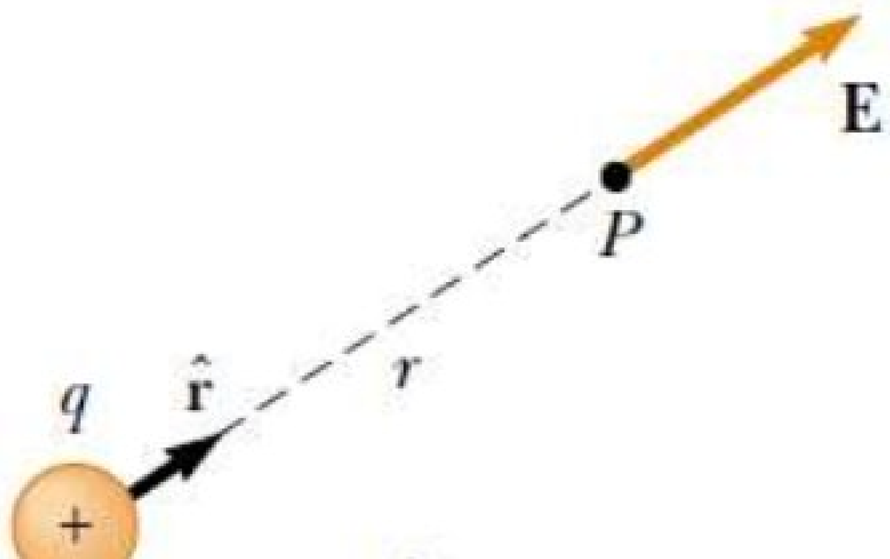
$$\boxed{E = \frac{F}{q_0} = \frac{36 * 10^{-8}}{18 * 10^{-8}} = 2 \ N/C}$$

About Electric Field Direction:

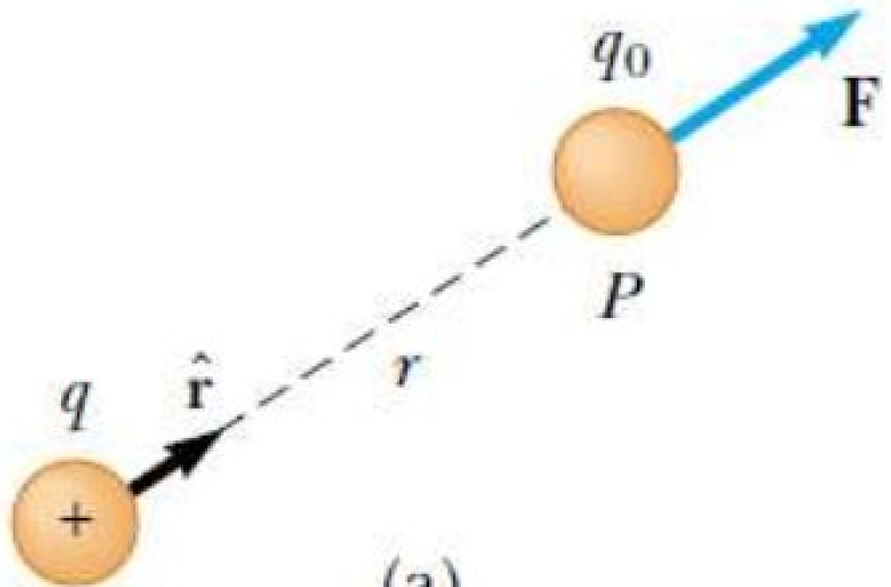
- Direction of $\vec{E}$: is the direction of the force a positive test charge q_0 experiences when placed in the field.

- q is positive
- the electric field at P points <u>radially outward</u> from q.
- the force is in the same direction as the field.

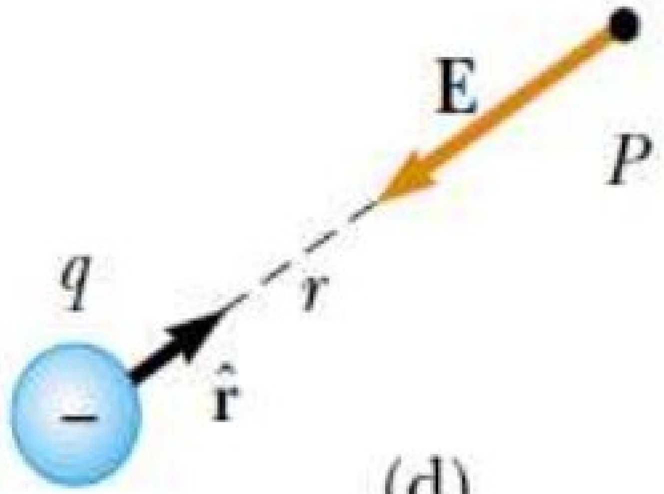
- q is negative
- the electric field at P points <u>radially inward</u> toward q.
- the force and the field are in opposite directions.



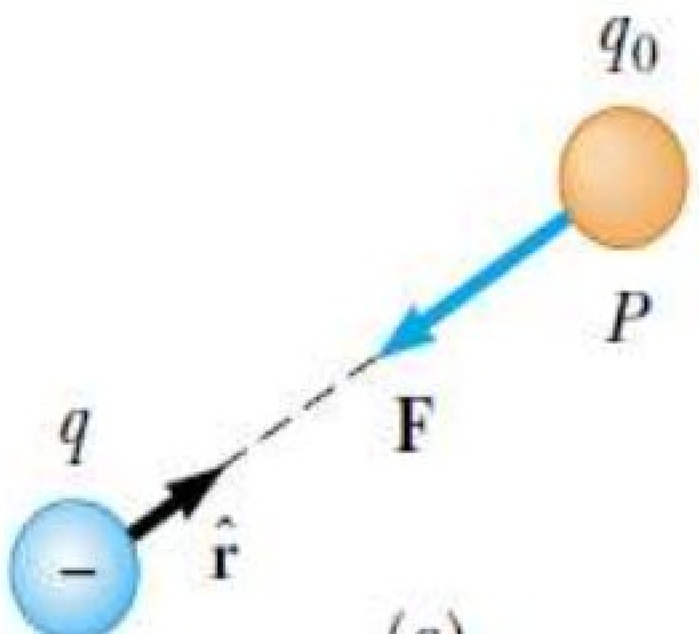
(b)



(a)



(d)

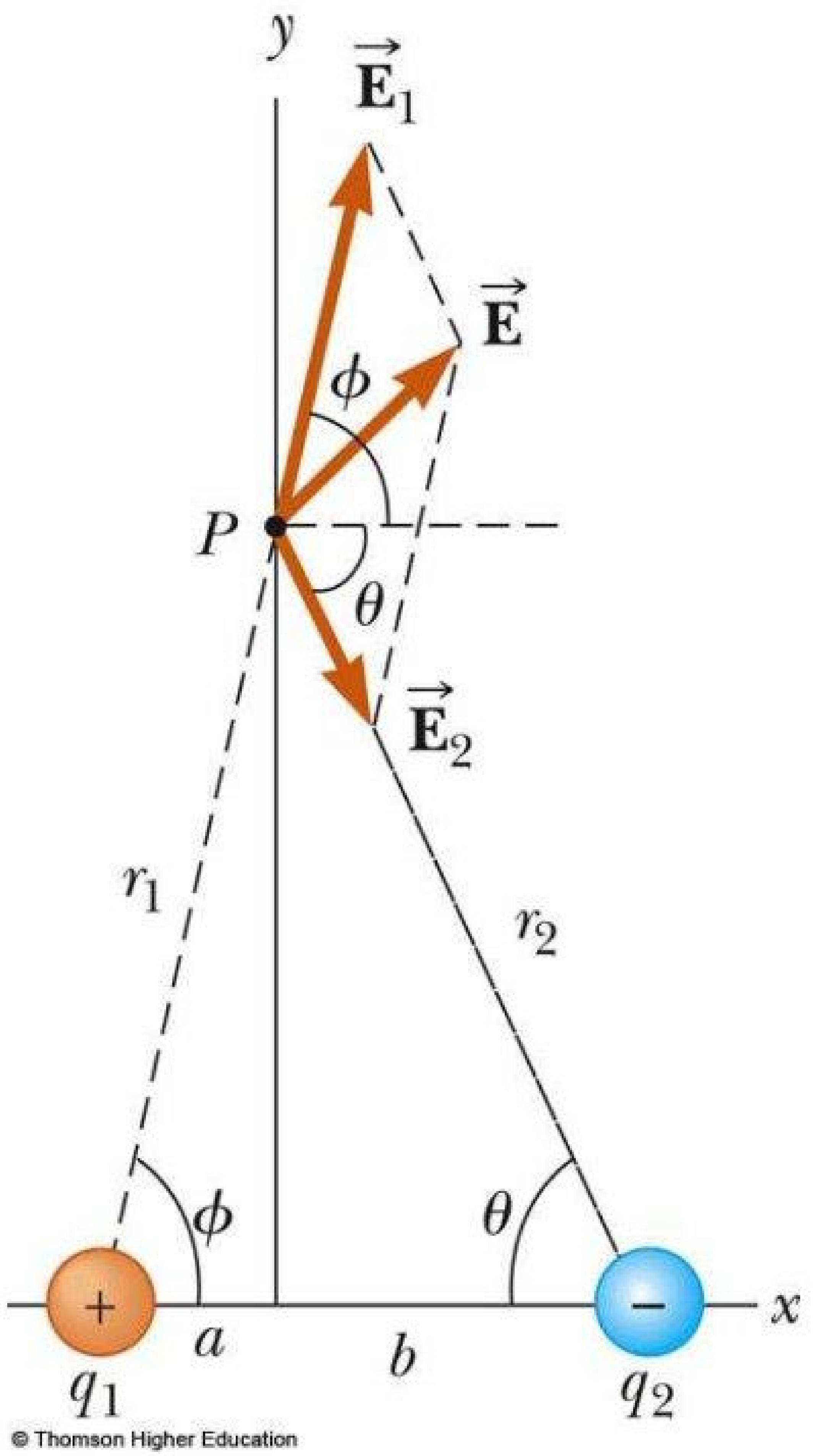


(c)

Superposition principle

- Find the electric field due to q_1, $\vec{\mathbf{E}}_1$
- Find the electric field due to q_2, $\vec{\mathbf{E}}_2$
- $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$
  - Remember, the electric fields are added as vectors
  - The direction of the individual fields is the direction of the force on a positive test charge
- **In general, the electric field due to a group of point charges:**

$$\boxed{\vec{E} = k_e \sum_i \frac{q_i}{r_i^2}\,\hat{r}_i}$$

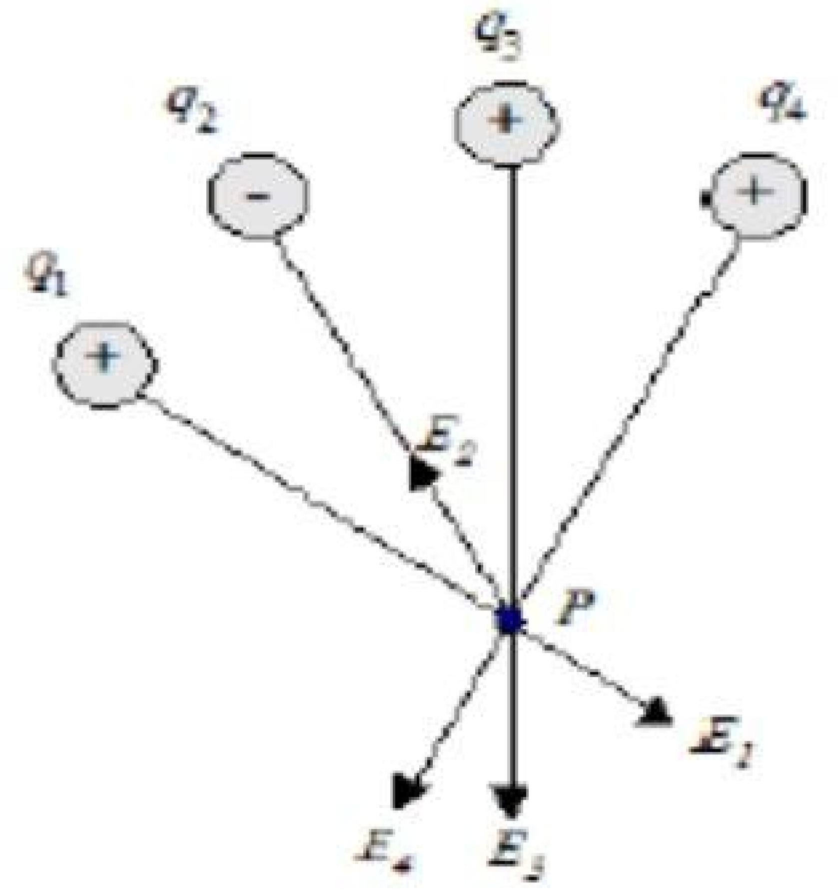


▪ Electric field due to a group of point charges

▪ At any point P, the total electric field due to a group of source charges equals the **<u>vector sum</u>** of the electric fields of all the charges.

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

where r_i is the distance from the *ith* source charge q_i to the point P and $\hat{r}_i$ is a unit vector directed from q_i toward P.



Example 23.6 Electric Field Due to Two Charges

Charges q_1 and q_2 are located on the x axis, at distances a and b, respectively, from the origin as shown in Figure 23.12.

(A) Find the components of the net electric field at the point P, which is at position $(0, y)$.

SOLUTION

Conceptualize Compare this example with Example 23.2. There, we add vector forces to find the net force on a charged particle. Here, we add electric field vectors to find the net electric field at a point in space. If a charged particle were placed at P, we could use the particle in a field model to find the electric force on the particle.

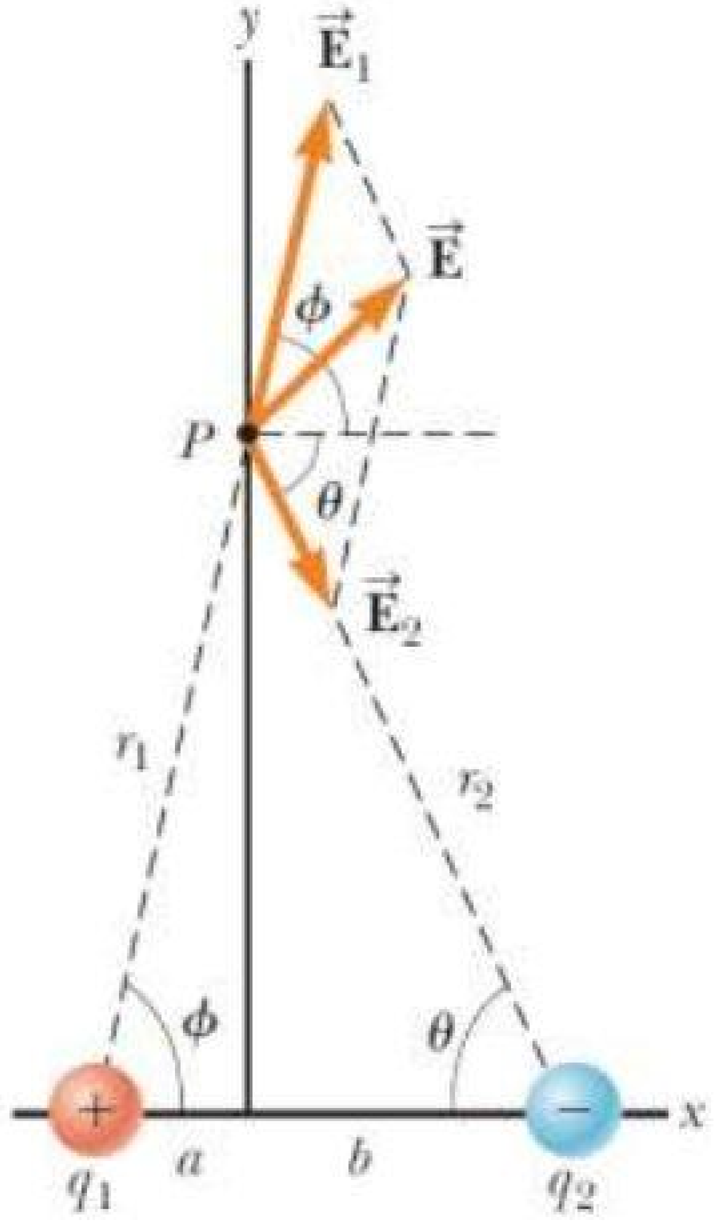


Figure 23.12 (Example 23.6) The total electric field $\vec{\mathbf{E}}$ at P equals the vector sum $\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$, where $\vec{\mathbf{E}}_1$ is the field due to the positive charge q_1 and $\vec{\mathbf{E}}_2$ is the field due to the negative charge q_2.

Categorize We have two source charges and wish to find the resultant electric field, so we categorize this example as one in which we can use the superposition principle represented by Equation 23.10.

Analyze Find the magnitude of the electric field at P due to charge q_1:

$$E_1 = k_e \frac{|q_1|}{r_1^2} = k_e \frac{|q_1|}{a^2 + y^2}$$

Find the magnitude of the electric field at P due to charge q_2:

$$E_2 = k_e \frac{|q_2|}{r_2^2} = k_e \frac{|q_2|}{b^2 + y^2}$$

Write the electric field vectors for each charge in unit-vector form:

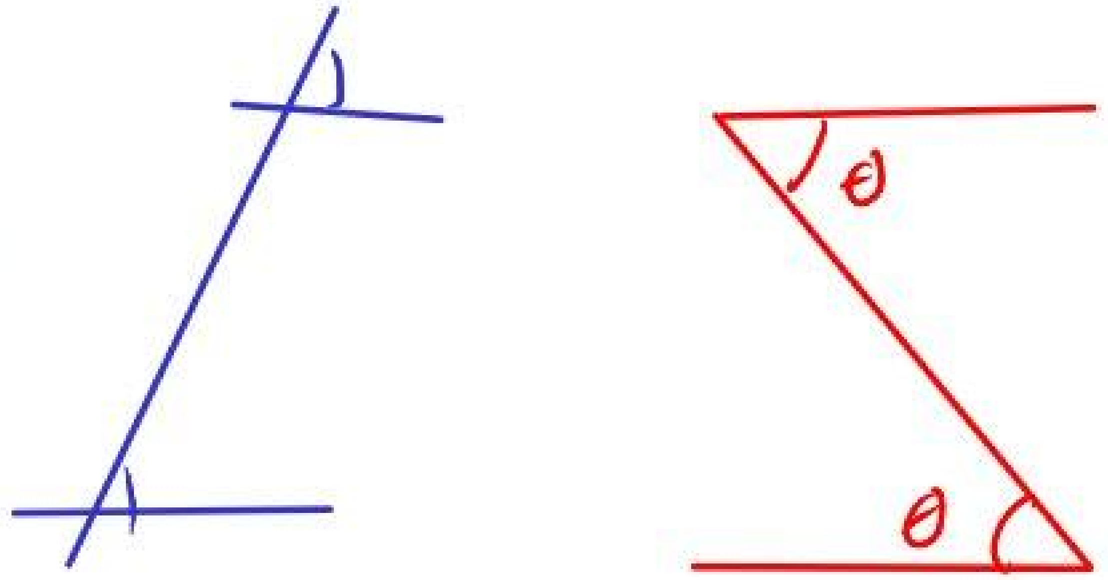
$$\vec{\mathbf{E}}_1 = k_e \frac{|q_1|}{a^2 + y^2} \cos\phi\,\hat{\mathbf{i}} + k_e \frac{|q_1|}{a^2 + y^2} \sin\phi\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}_2 = k_e \frac{|q_2|}{b^2 + y^2} \cos\theta\,\hat{\mathbf{i}} - k_e \frac{|q_2|}{b^2 + y^2} \sin\theta\,\hat{\mathbf{j}}$$

Example 23.6 Electric Field Due to Two Charges

Charges q_1 and q_2 are located on the x axis, at distances a and b, respectively, from the origin as shown in Figure 23.12.

(A) Find the components of the net electric field at the point P, which is at position $(0, y)$.

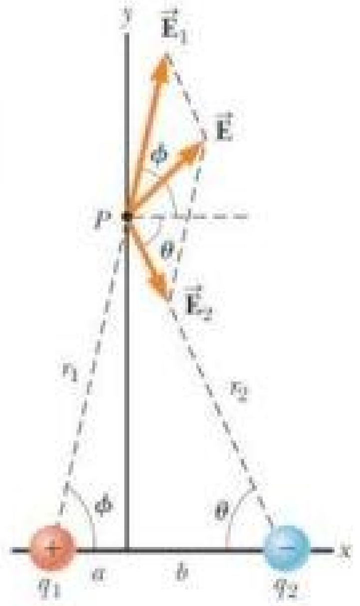


$$\boxed{E_1 = \frac{k_e q_1}{r_1^2} = \frac{k_e q_1}{a^2+y^2}}$$

$$r_1 = \sqrt{a^2+y^2}$$

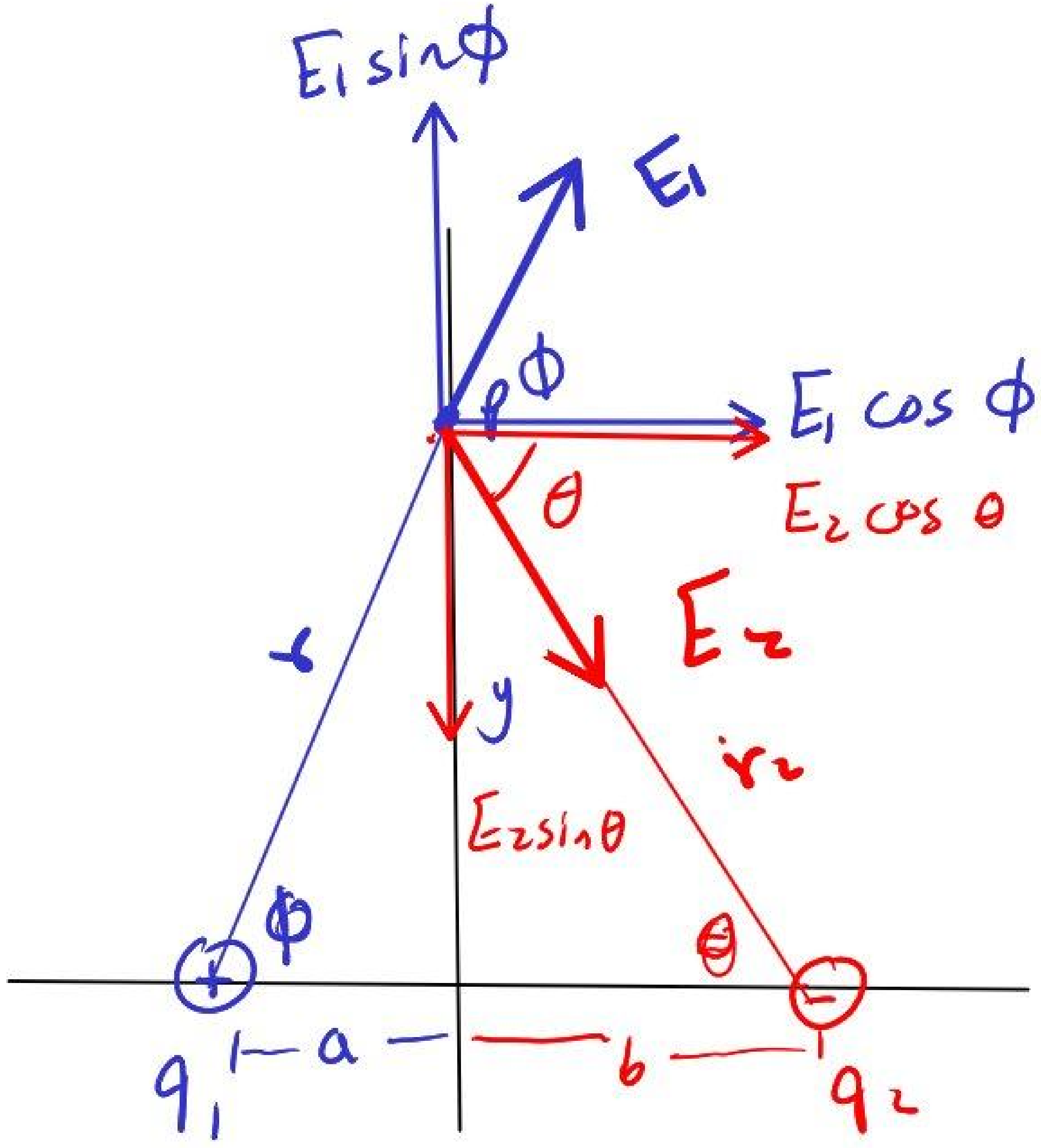
$$\boxed{E_2 = \frac{k_e q_2}{r_2^2} = \frac{k_e q_2}{b^2+y^2}}$$

$$r_2 = \sqrt{b^2+y^2}$$



$$\boxed{E_x = (E_1 \cos\phi + E_2 \cos\theta)\,i = \frac{k_e q_1}{a^2+y^2}\cos\phi + \frac{k_e q_2}{b^2+y^2}\cos\theta}$$

$$\boxed{E_y = (E_1 \sin\phi - E_2 \sin\theta)\,j = \frac{k_e q_1}{a^2+y^2}\cos\phi - \frac{k_e q_2}{b^2+y^2}\cos\theta}$$



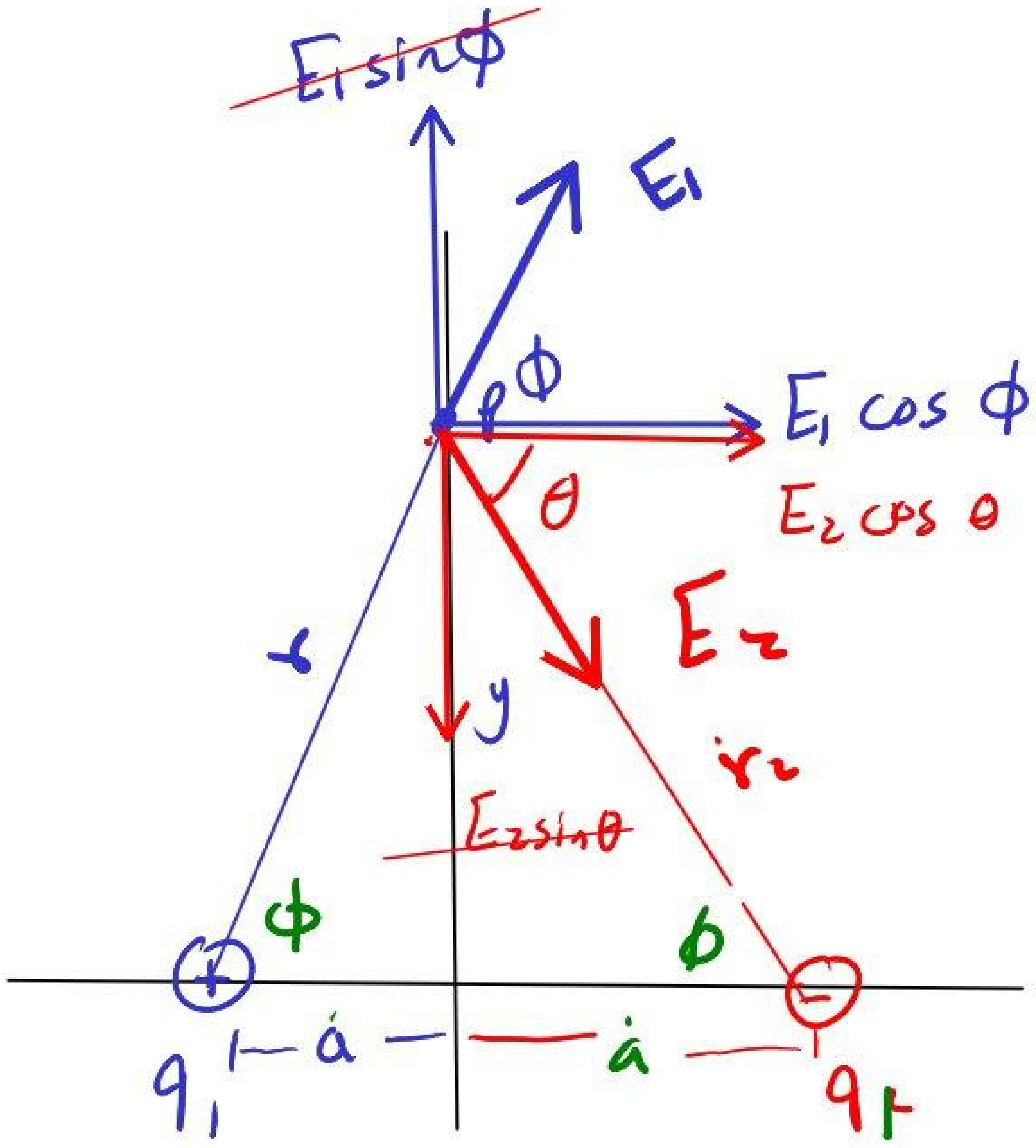
(B) Evaluate the electric field at point P in the special case that $|q_1| = |q_2|$ and $a = b$.

$$E_x = (E_1\cos\phi + E_2\cos\theta)\,i = \frac{k_e q_1}{a^2+y^2}\cos\phi + \frac{k_e q_2}{b^2+y^2}\cos\theta$$

$$E_y = (E_1\sin\phi - E_2\sin\theta)\,j = \frac{k_e q_1}{a^2+y^2}\cos\phi - \frac{k_e q_2}{b^2+y^2}\cos\theta$$

$$E_x = \frac{2k_e q_1}{a^2+y^2}\cos\phi \quad N/C$$

$$E_y = 0 \quad N/C$$



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Write the components of the net electric field vector:

$$(1)\quad E_x = E_{1x} + E_{2x} = \boxed{k_e \frac{|q_1|}{a^2 + y^2}\cos\phi + k_e \frac{|q_2|}{b^2 + y^2}\cos\theta}$$

$$(2)\quad E_y = E_{1y} + E_{2y} = \boxed{k_e \frac{|q_1|}{a^2 + y^2}\sin\phi - k_e \frac{|q_2|}{b^2 + y^2}\sin\theta}$$

(B) Evaluate the electric field at point P in the special case that $|q_1| = |q_2|$ and $a = b$.

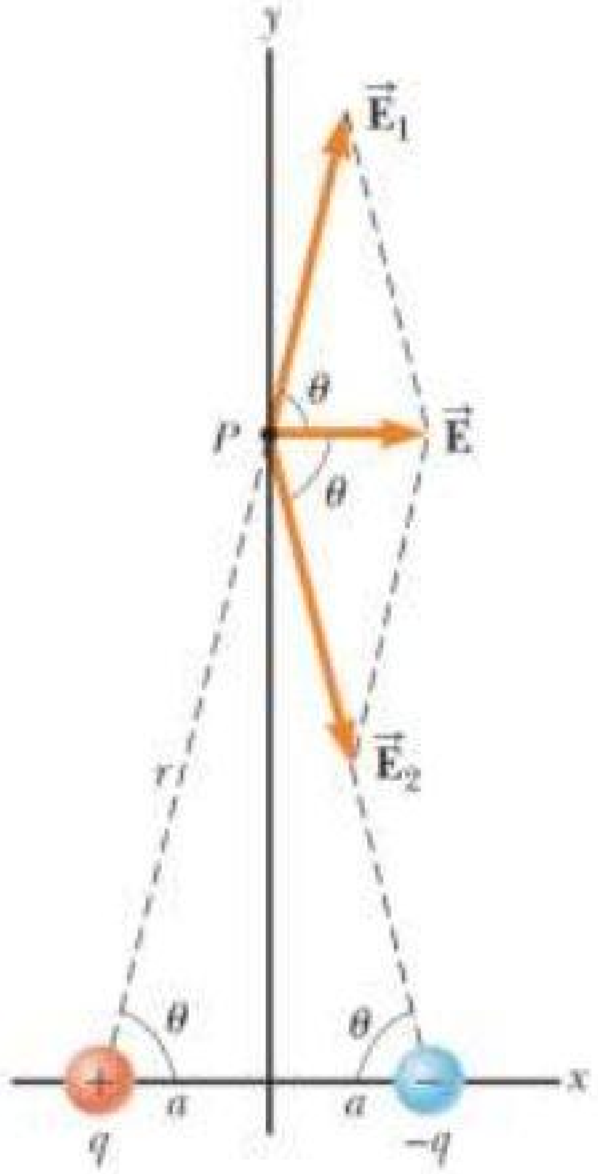


Figure 23.13 (Example 23.6) When the charges in Figure 23.12 are of equal magnitude and equidistant from the origin, the situation becomes symmetric as shown here.

SOLUTION

Conceptualize Figure 23.13 shows the situation in this special case. Notice the symmetry in the situation and that the charge distribution is now an electric dipole.

Categorize Because Figure 23.13 is a special case of the general case shown in Figure 23.12, we can categorize this example as one in which we can take the result of part (A) and substitute the appropriate values of the variables.

Analyze Based on the symmetry in Figure 23.13, evaluate Equations (1) and (2) from part (A) with $a = b$, $|q_1| = |q_2| = q$, and $\phi = \theta$:

$$(3)\quad E_x = k_e \frac{q}{a^2 + y^2}\cos\theta + k_e \frac{q}{a^2 + y^2}\cos\theta = 2k_e \frac{q}{a^2 + y^2}\cos\theta$$

$$E_y = k_e \frac{q}{a^2 + y^2}\sin\theta - k_e \frac{q}{a^2 + y^2}\sin\theta = 0$$

From the geometry in Figure 23.13, evaluate $\cos\theta$:

$$(4)\quad \cos\theta = \frac{a}{r} = \frac{a}{(a^2 + y^2)^{1/2}}$$

Substitute Equation (4) into Equation (3):

$$E_x = 2k_e \frac{q}{a^2 + y^2}\left[\frac{a}{(a^2 + y^2)^{1/2}}\right] = \boxed{k_e \frac{2aq}{(a^2 + y^2)^{3/2}}}$$

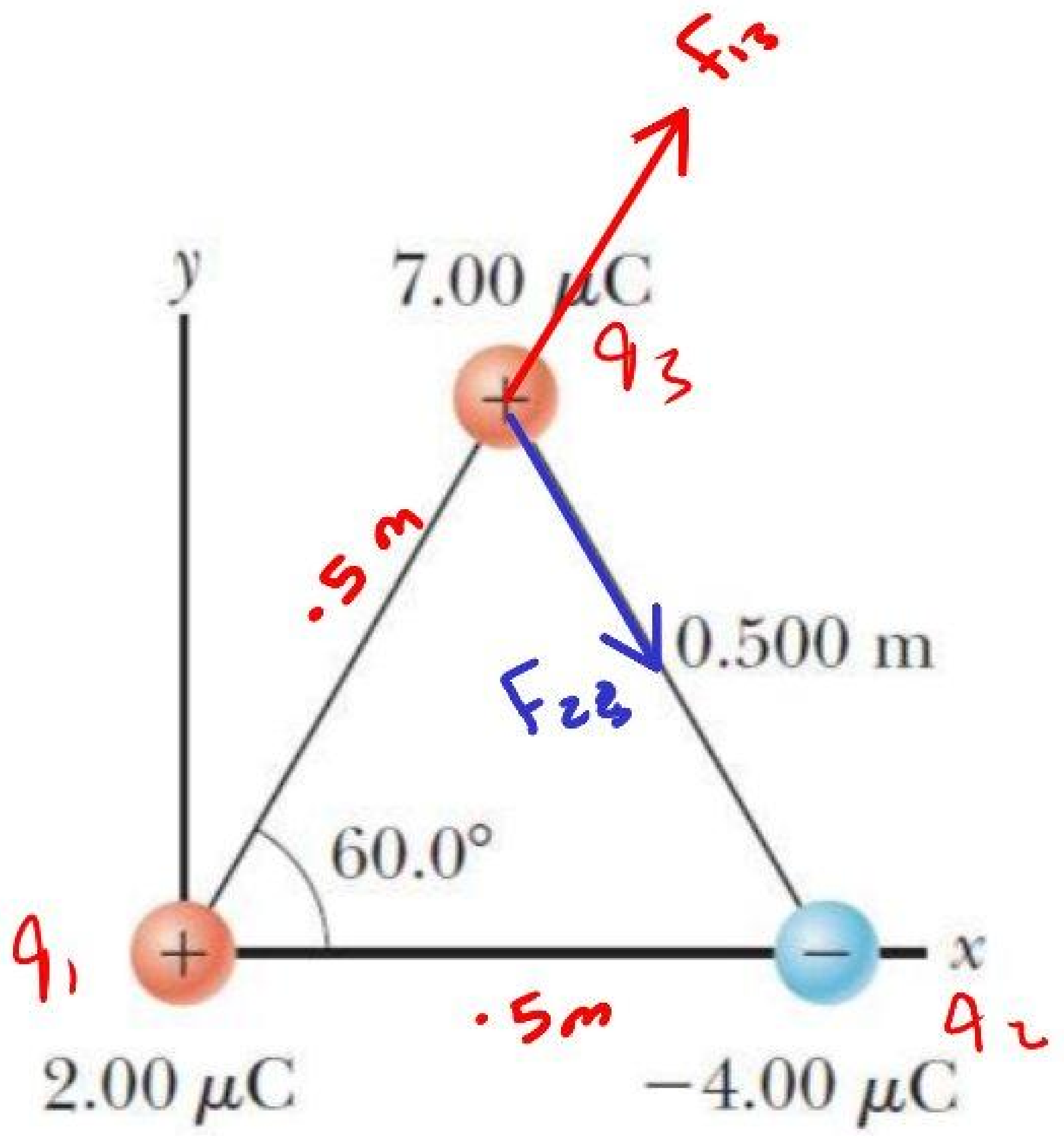
15. **M** Three charged particles are located at the corners of an equilateral triangle as shown in Figure P23.15. Calculate the total electric force on the $7.00\text{-}\mu C$ charge.

$$F_{13} = \frac{k_e q_1 q_3}{r_{13}^2} = \frac{9*10^9 * (2*10^{-6})(7*10^{-6})}{(.5)^2}$$

$$\boxed{F_{13} = .504 \ N}$$

$$F_{23} = \frac{k_e q_2 q_3}{r_{23}^2} = \frac{9*10^9 * 4*10^{-6} * 7*10^{-6}}{(.5)^2}$$

$$\boxed{F_{23} = 1.008 \ N}$$



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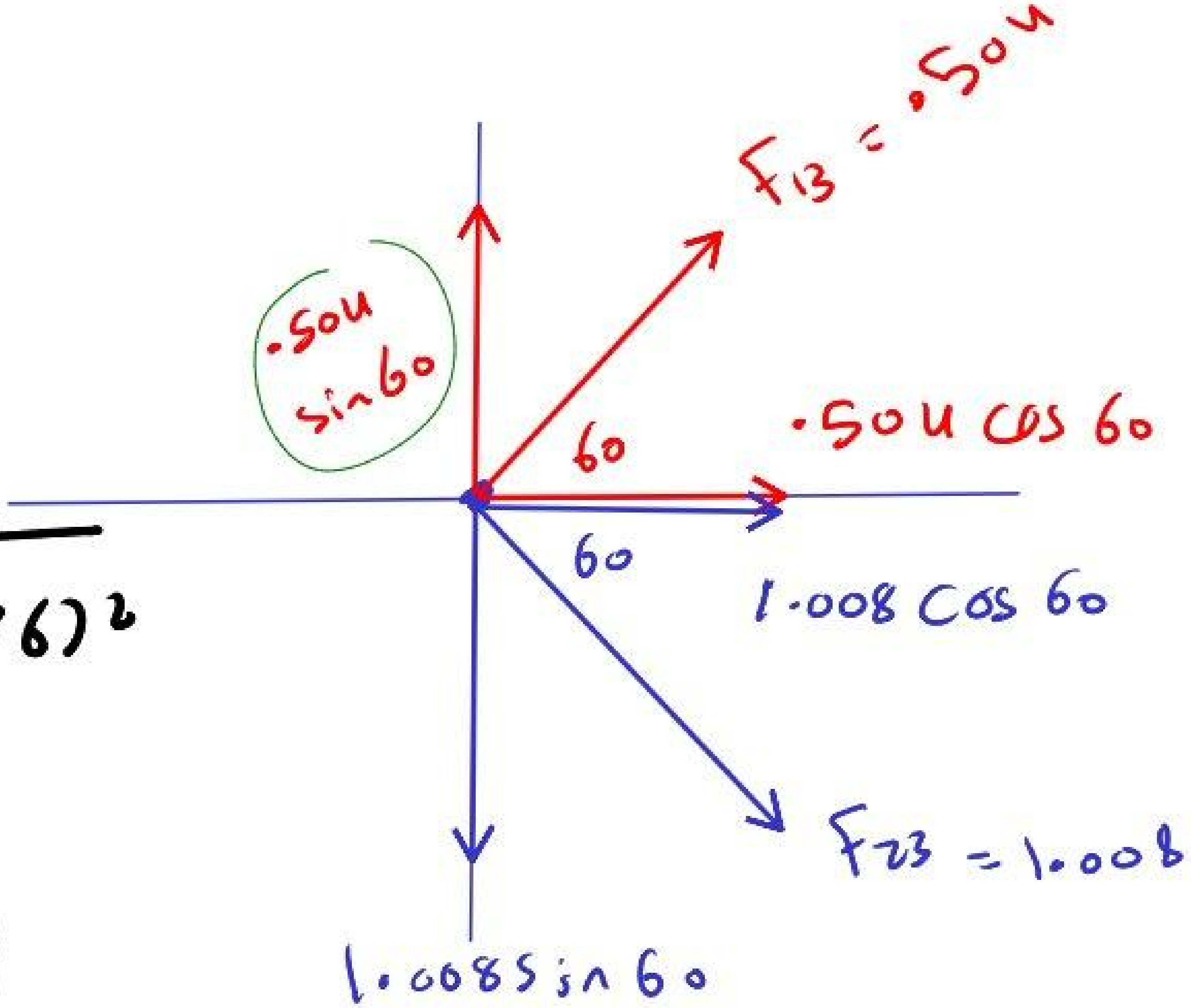
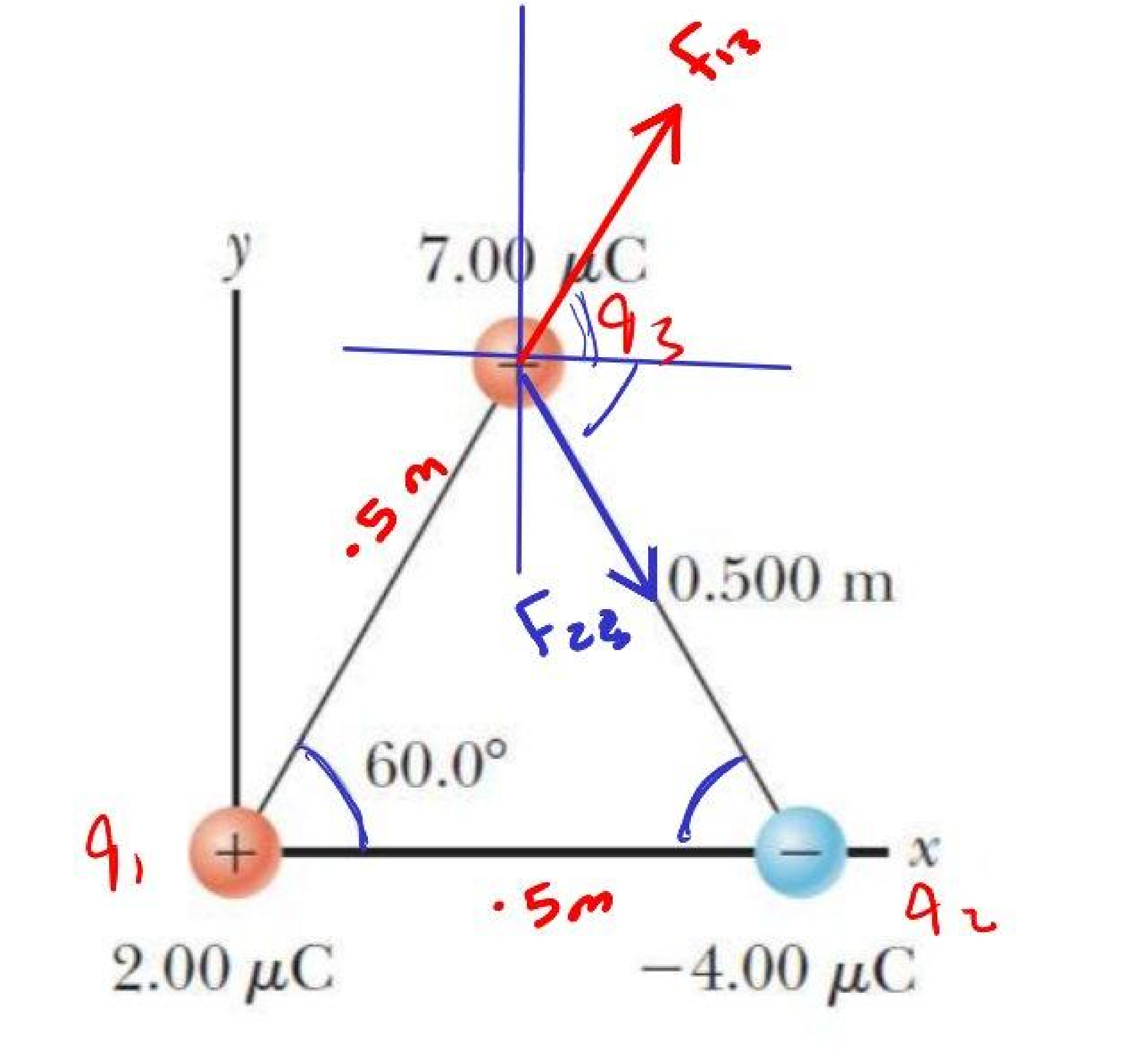
$$F = \sqrt{(.756)^2 + (-.436)^2}$$

$$\theta = \tan^{-1}\left(\frac{.436}{.756}\right)$$





$$F_x = .504 \cos 60 + 1.008 \cos 60 = .756 \text{ N}$$

$$F_y = .504 \sin 60 - 1.008 \sin 60 = -.436 \text{ N}$$

$$\boxed{F_t = .756\,i - .436\,j \quad \text{N}}$$

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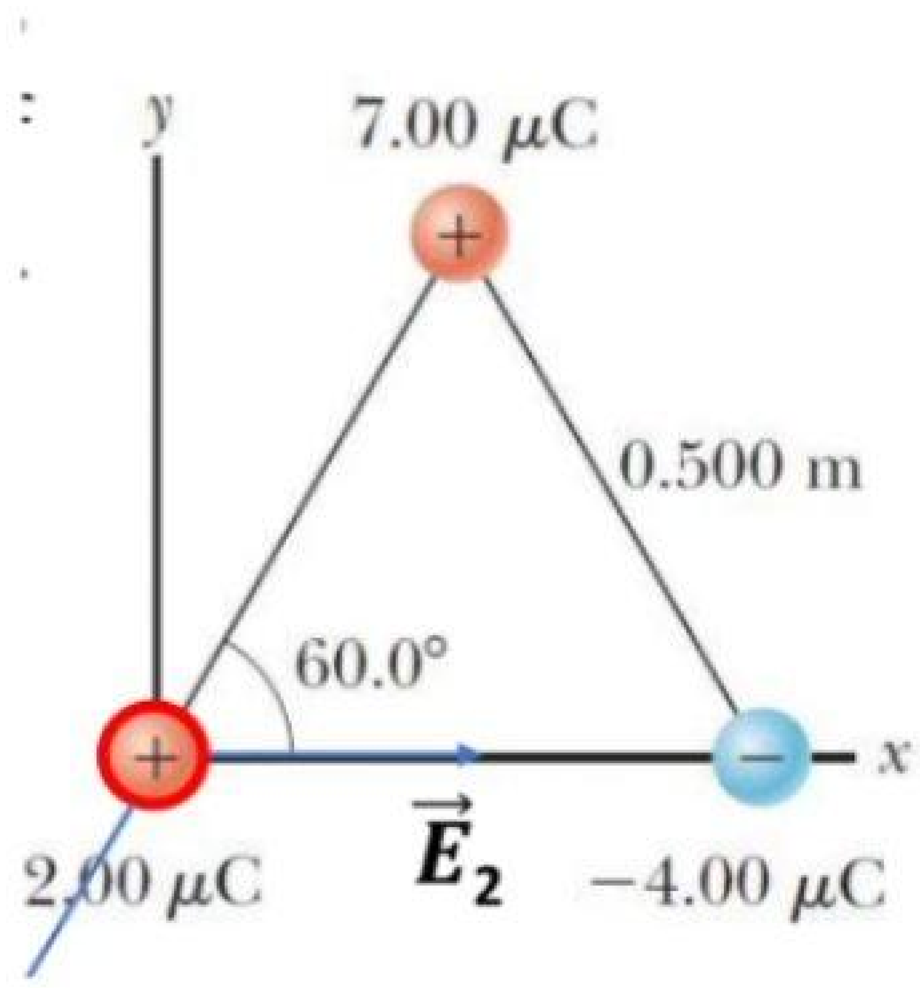
***Problem2:** Three charges are at the corners of an equilateral triangle. **A)** Calculate the electric field at the position of the $2\mu C$ charge due to the $7\ \mu C$ and $-4\ \mu C$ charges. **B)** Determine the electric force on the $2\mu C$ charge.

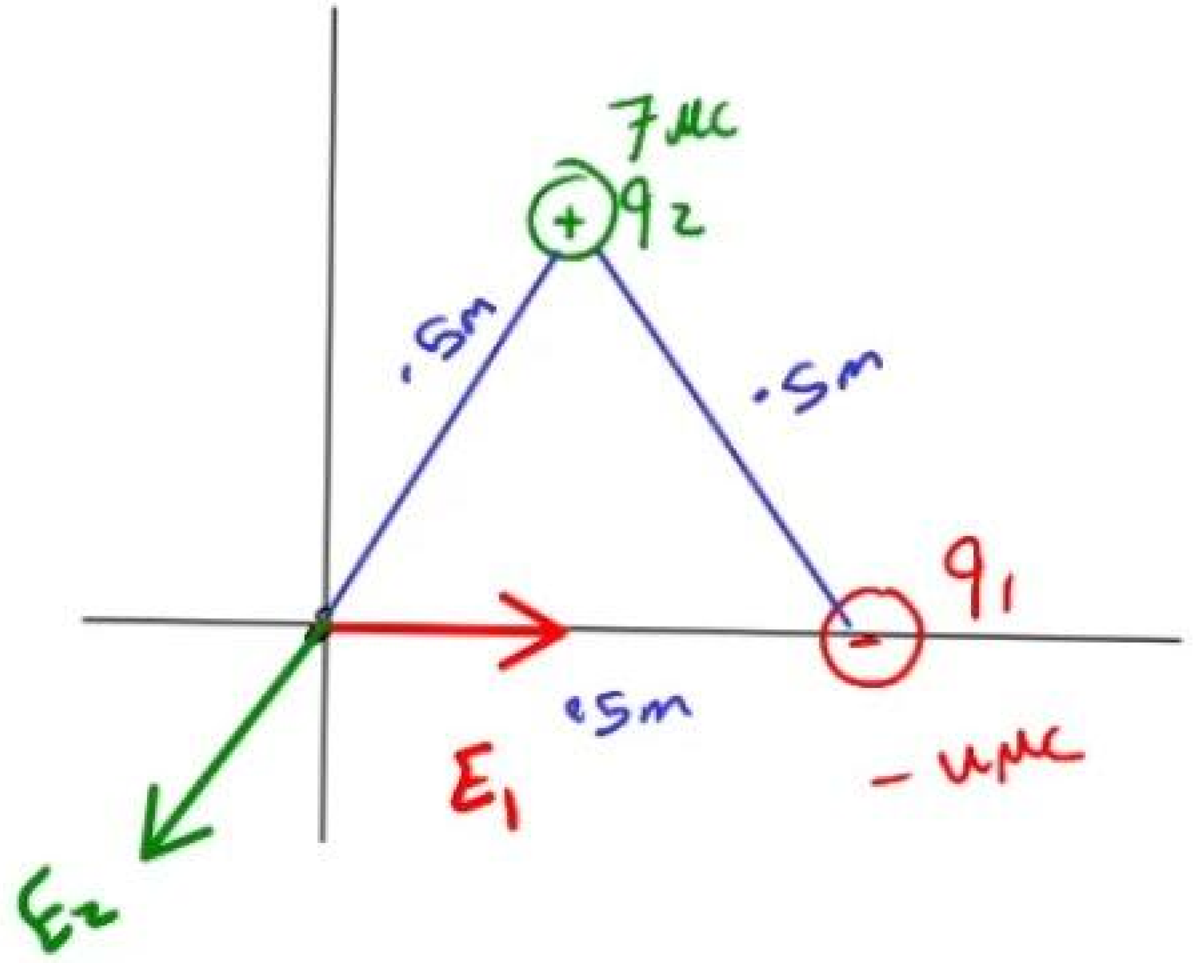


$$E_1 = \frac{k q_1}{r_1^2} = \frac{9*10^9 * (4*10^{-6})}{(.5)^2}$$

$$\boxed{E_1 = 144000\ N/C}$$

$$E_2 = \frac{k_e q_2}{r_2^2} = \frac{9*10^9 * 7*10^{-6}}{(.5)^2}$$

$$\boxed{E_2 = 252000\ N/C}$$



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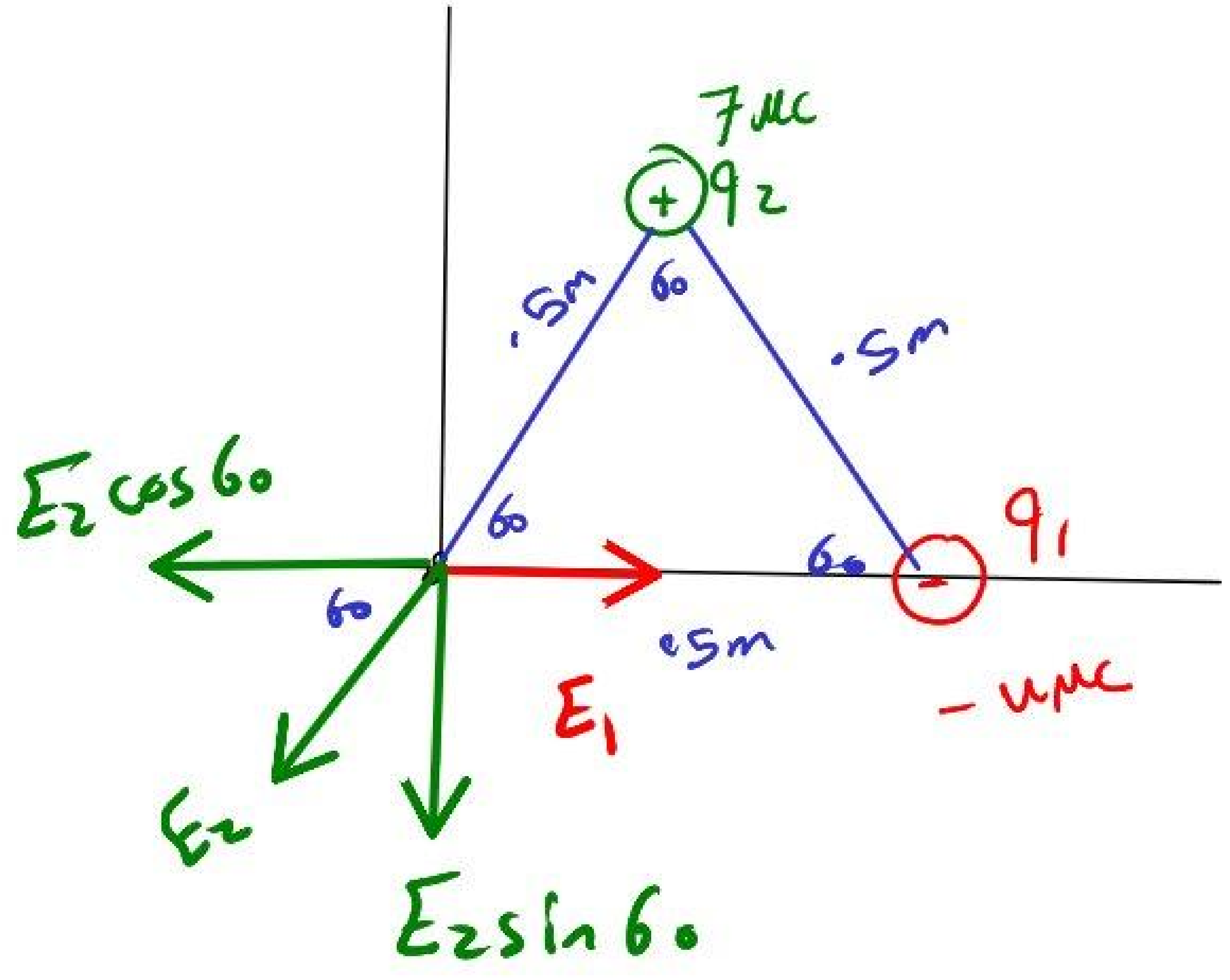
$$\boxed{E_1 = 144000\ N/C}$$

$$\boxed{E_2 = 252000\ N/C}$$



$$E_x = 144000 - 252000 \cos 60$$

$$E_x = 18000\ N/C$$

$$E_y = -E_2 \sin 60 = -218238.4\ N/C$$

$$\boxed{E_t = 18000\,i - 218238.4\,J}$$

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(b) F on $q_3 = 2\ \mu c$

$$F = q_3 E = 2\times10^{-6}\,[18000\,i - 218238.4\,J]$$

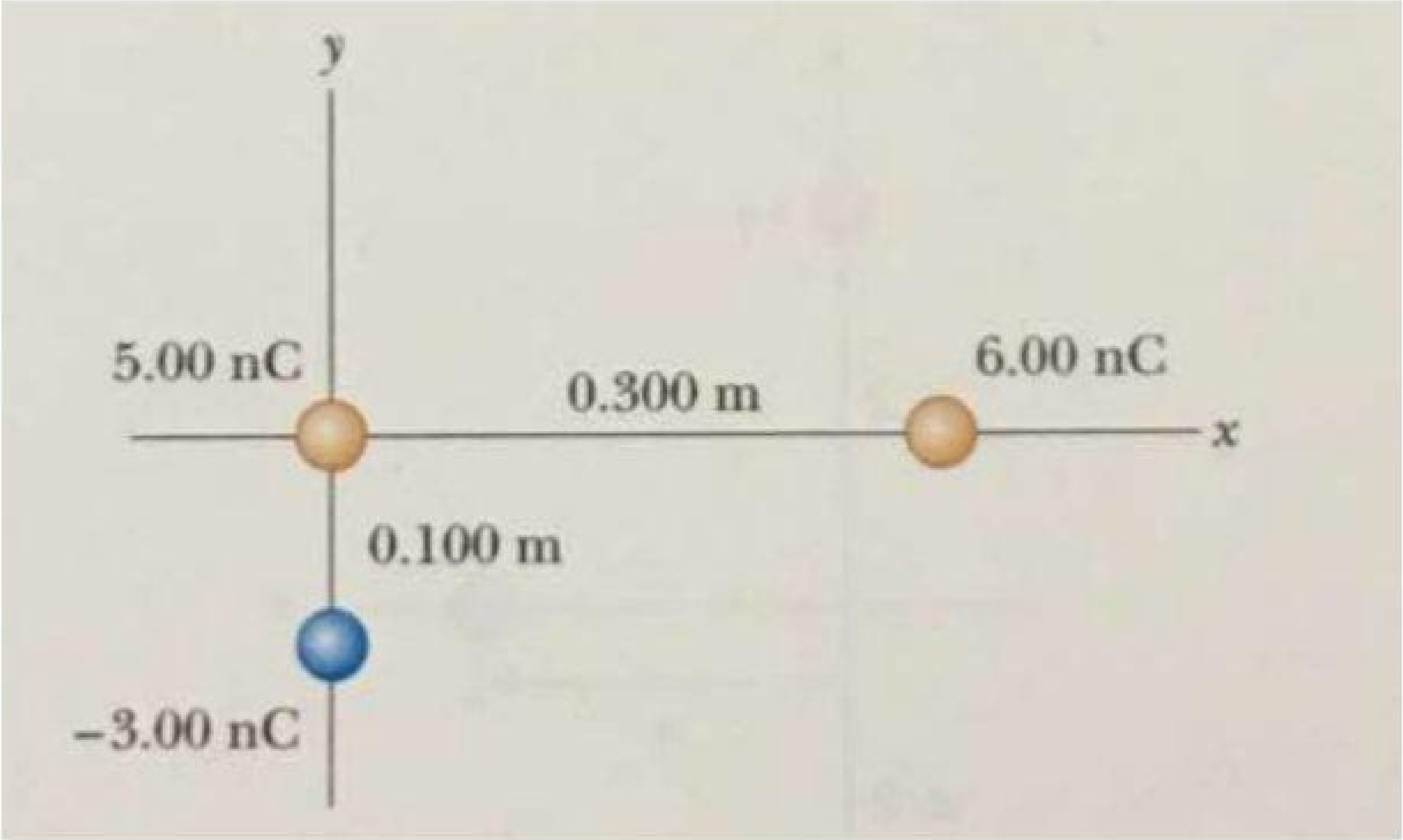
$$\boxed{F_t = .036\,i - .436\,J}\ N$$

*Problem3:

- Three point charges are arranged as shown in Figure.

a) Find **the electric field vector** that the *6 nC* and *-3 nC* charges together create at the origin.

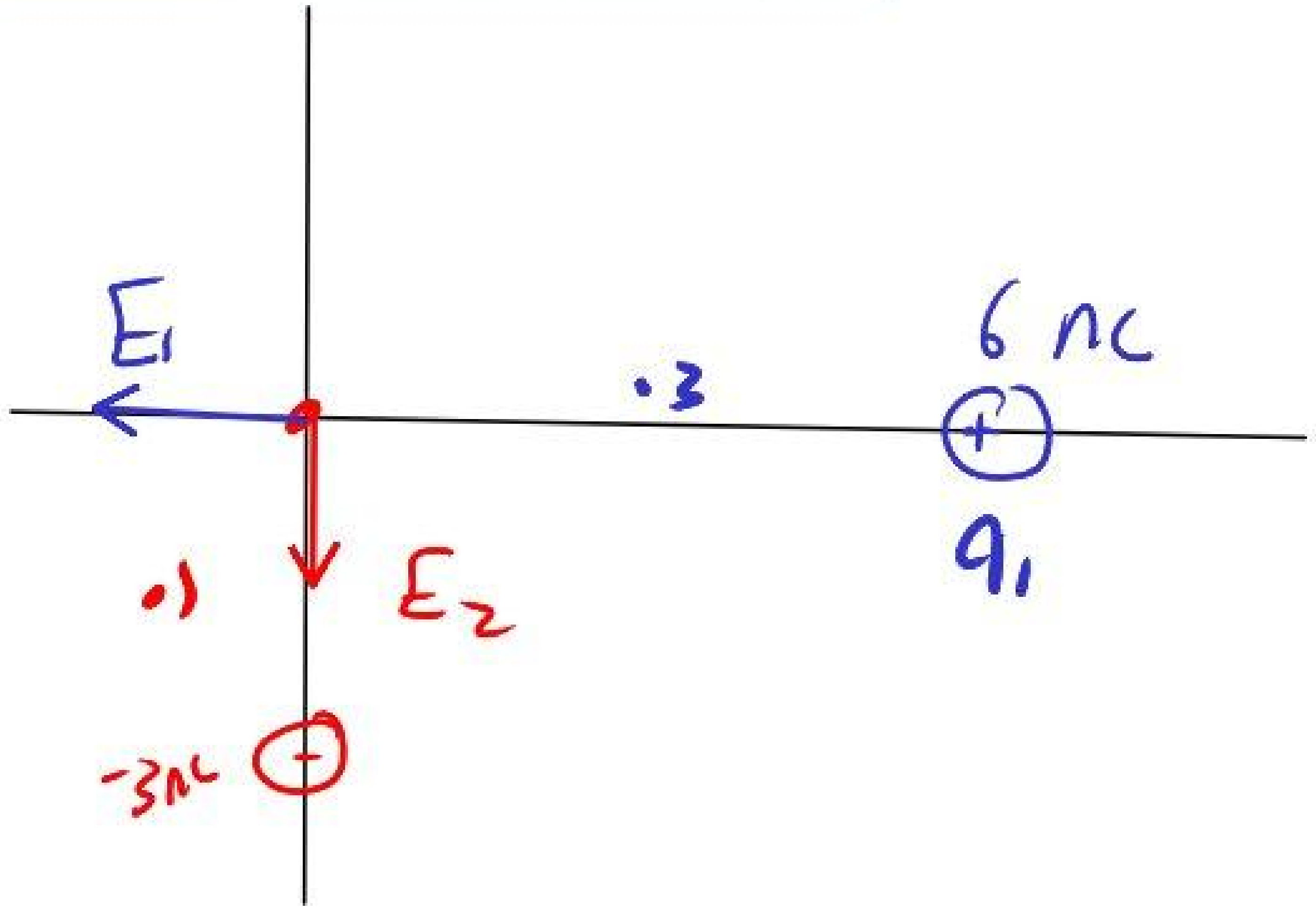
b) Find **the electric force vector** on the *5 nC* charge.



$$E_1 = \frac{k q_1}{r_1^2} = \frac{9\times10^{9} \times 6\times10^{-9}}{(.3)^2}$$

$$E_1 = 600 \ N/C$$

$$E_2 = \frac{k q_2}{r_2^2} = \frac{9\times10^{9} \times 3\times10^{-9}}{(.1)^2} = 2700 \ N/C$$



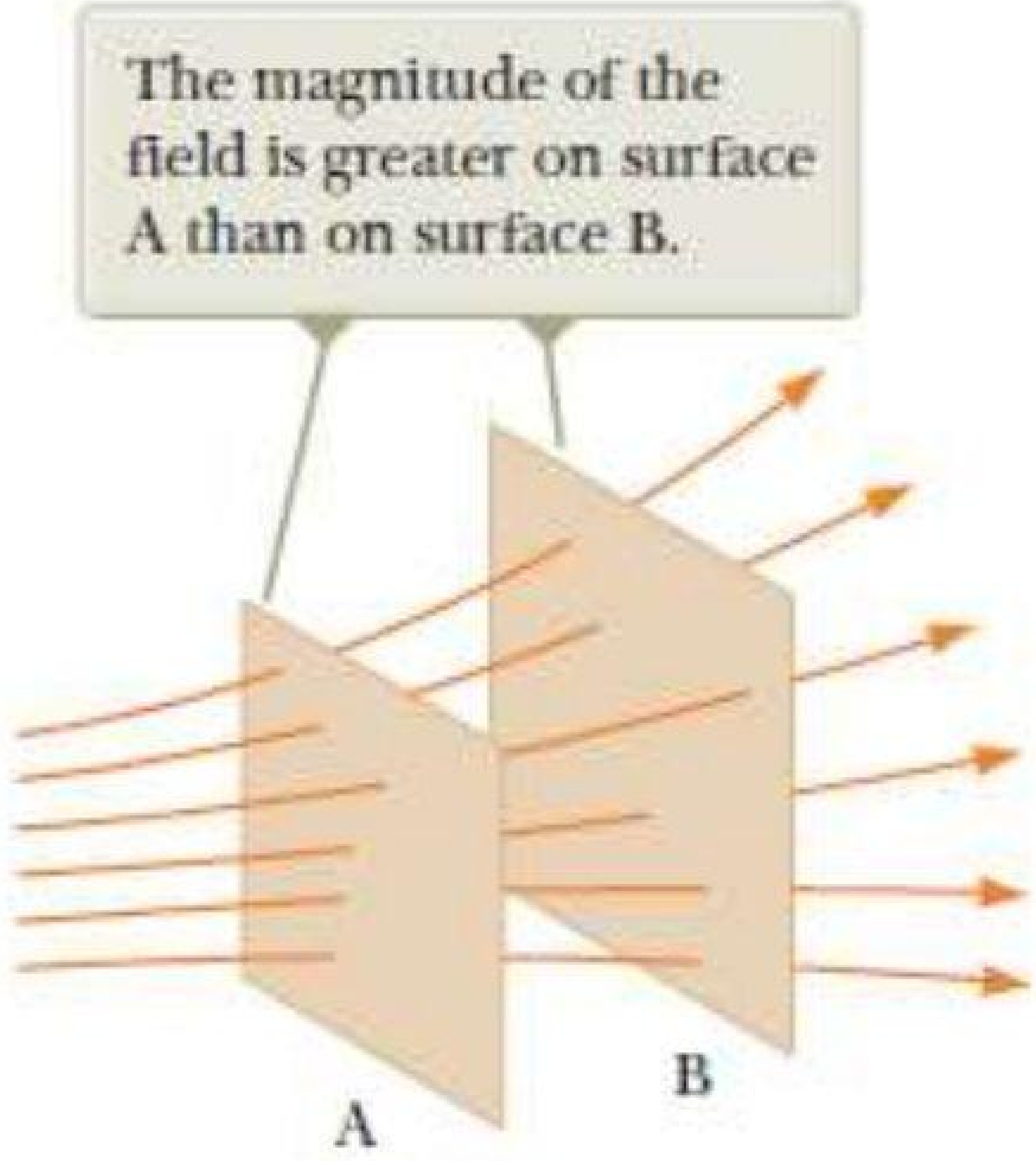
$$\boxed{E_t = -600\,i - 2700\,j}$$

(b) F on 5nC /

$$\boxed{F = qE = 5\times10^{-9}\left[-600\,i - 2700\,j\right] = -3\times10^{-6}\,i - 1.35\times10^{-6}\,j}$$

23.4 Electric Field Lines

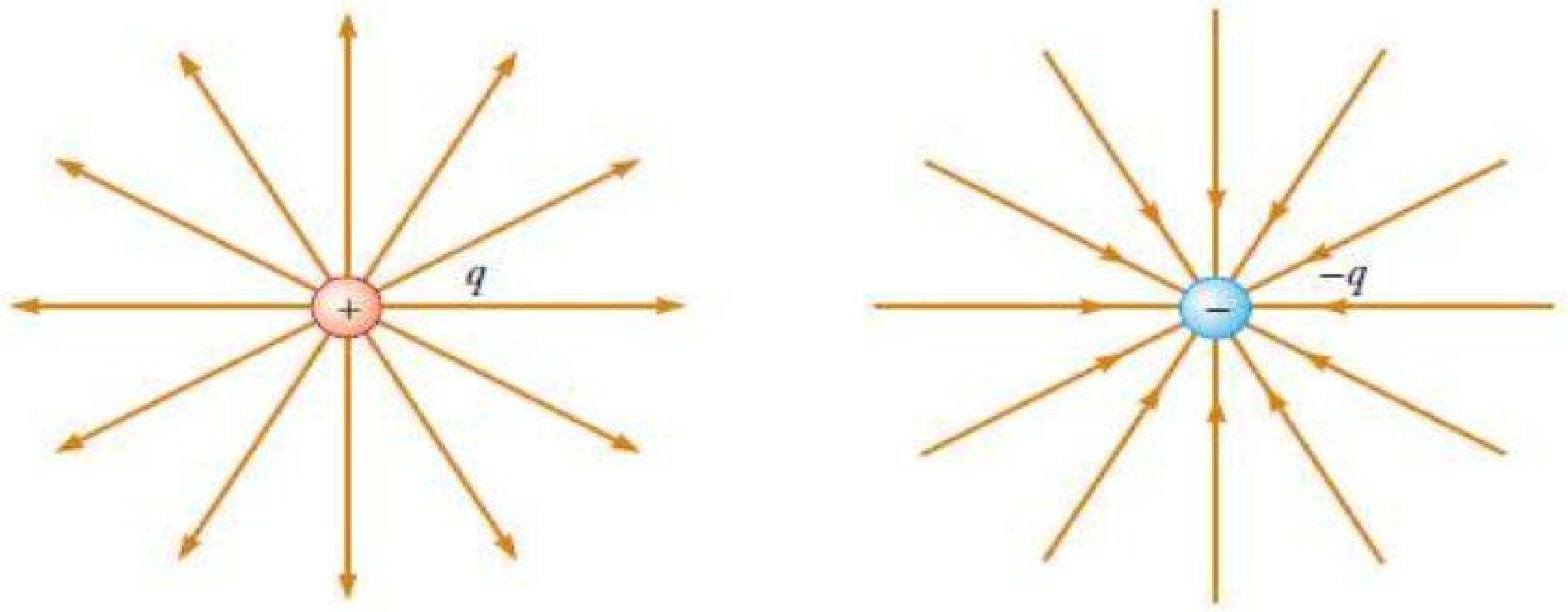
- Electric field lines describe an electric field in any region of space.
- The electric field vector $\vec{\boldsymbol{E}}$ is tangent to the electric field line at each point.
- The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, the field lines are close together where the electric field is strong, and far apart where the field is weak.



23.4 Electric Field Lines

- **The rules for drawing electric field lines are as follows:**
  - The lines must begin on a positive charge and terminate on a negative charge.
  - The number of lines is proportional to the magnitude of the charge.
  - No two field lines can cross.

- Point charge:

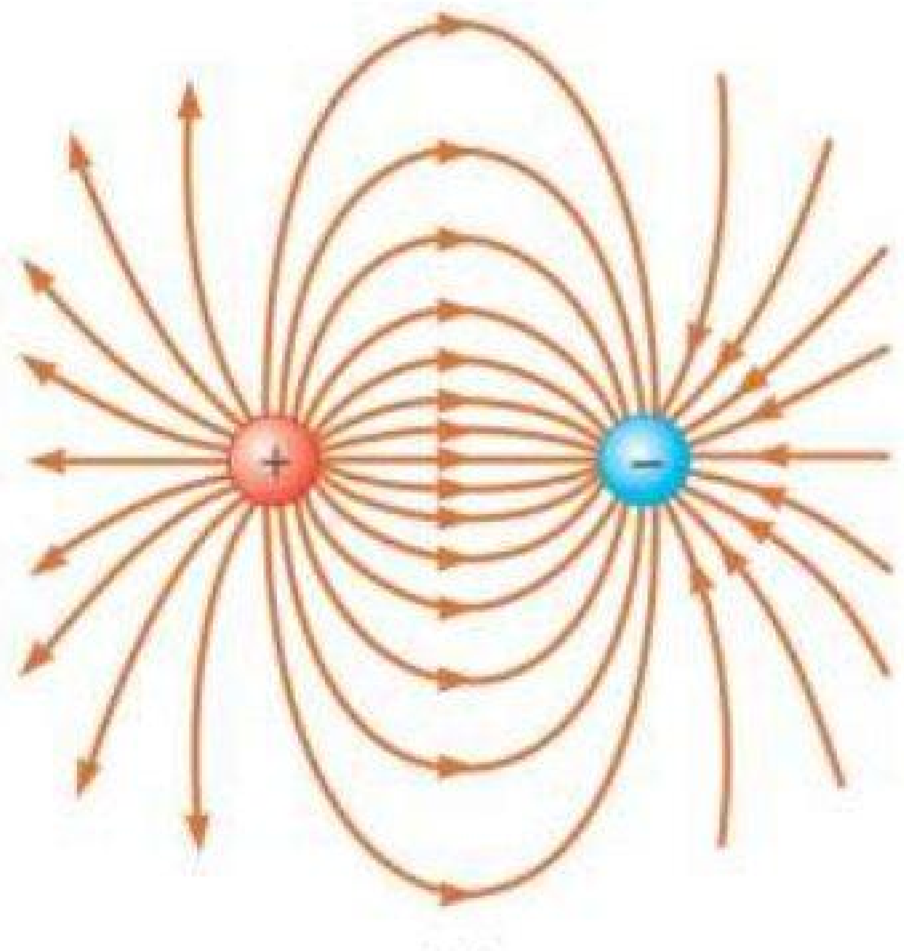


(a) the field lines are directed radially outward.

(b) the field lines are directed radially inward.

23.4 Electric Field Lines

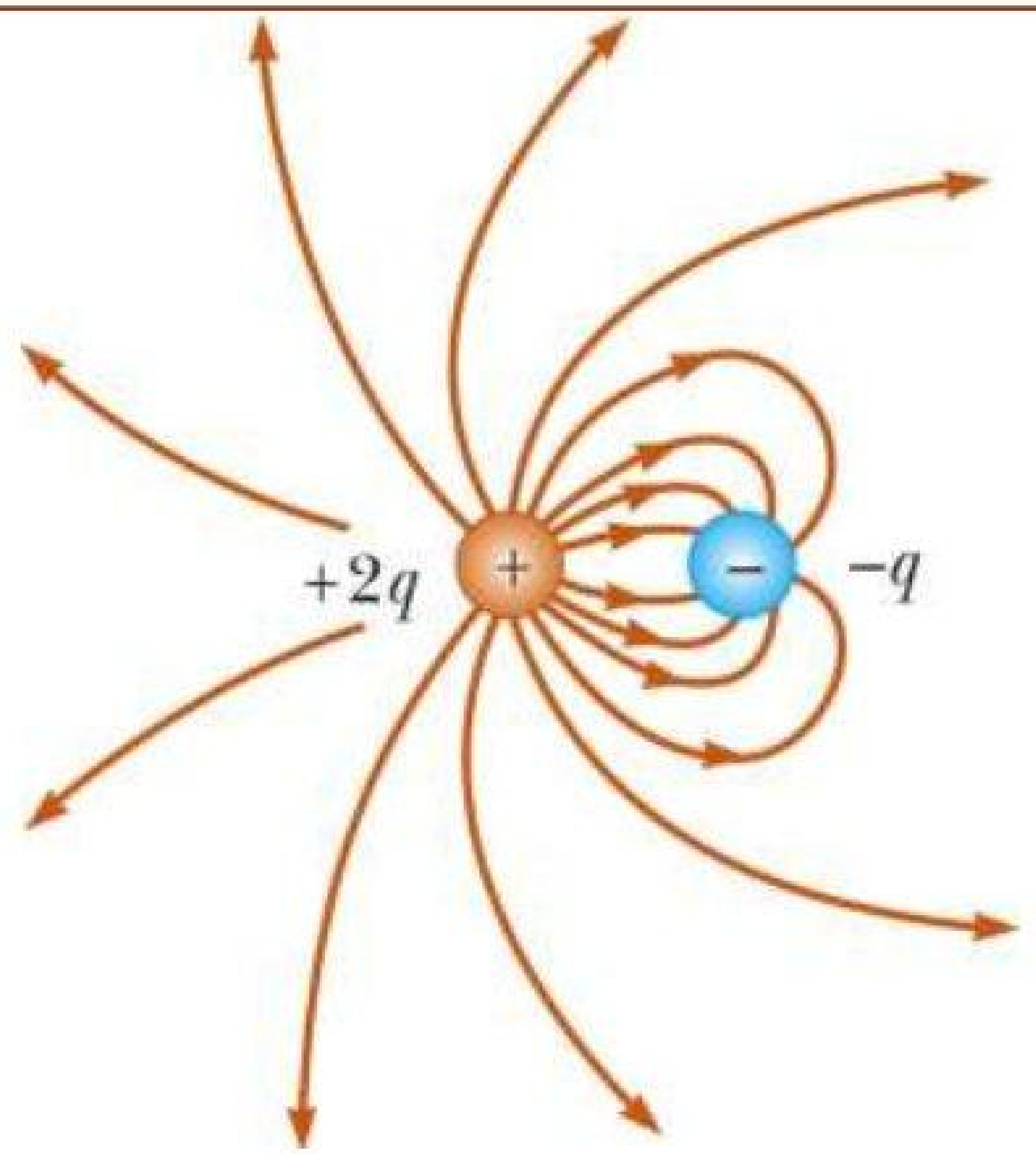
- **Electric dipole**



(a)

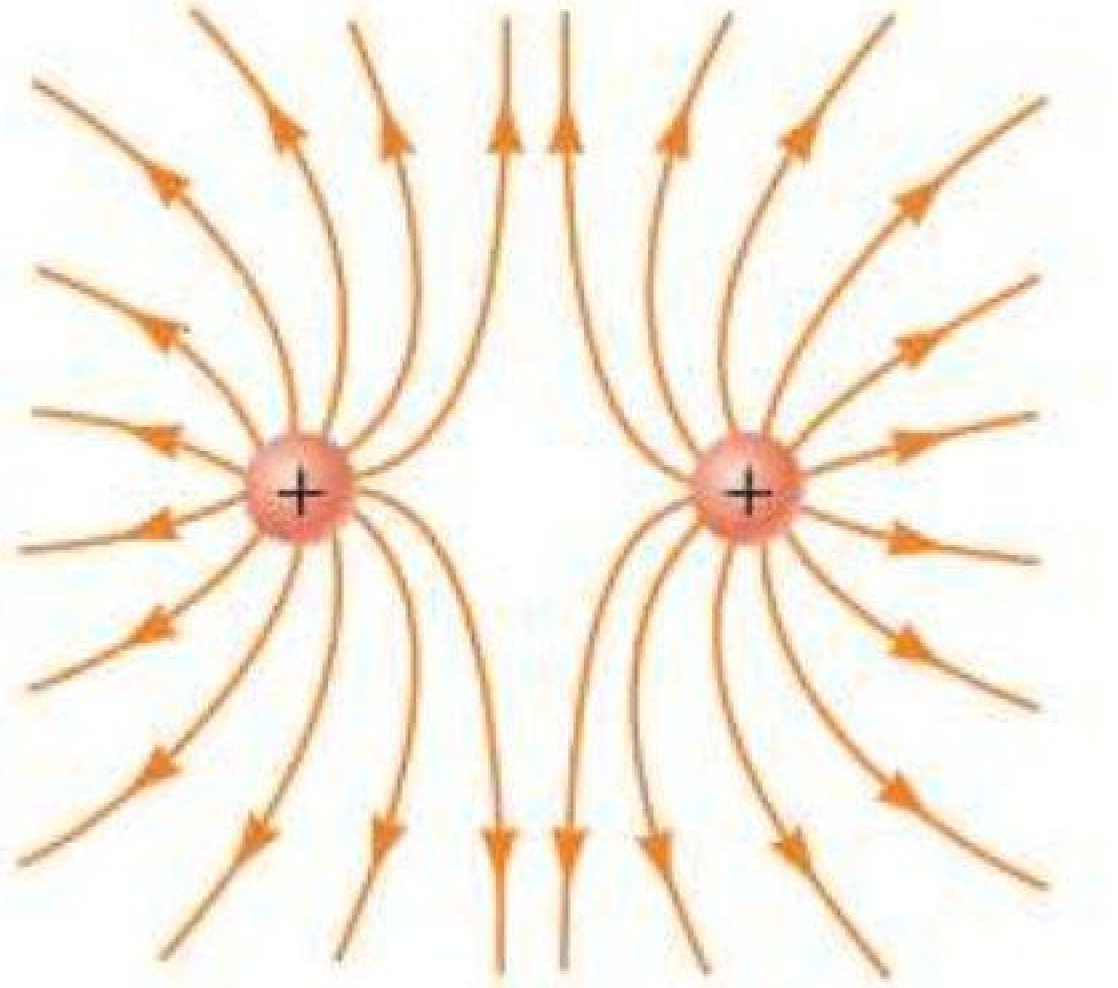
- The charges are equal and opposite
- The number of field lines leaving the positive charge equals the number terminating at the negative charge.

- Unequal and unlike point charges



- Two field lines leave $2q$ for every one that terminates on q.
- The positive charge is twice the magnitude of the negative charge
- At a great distance, the field would be approximately the same as that due to a single charge of +q

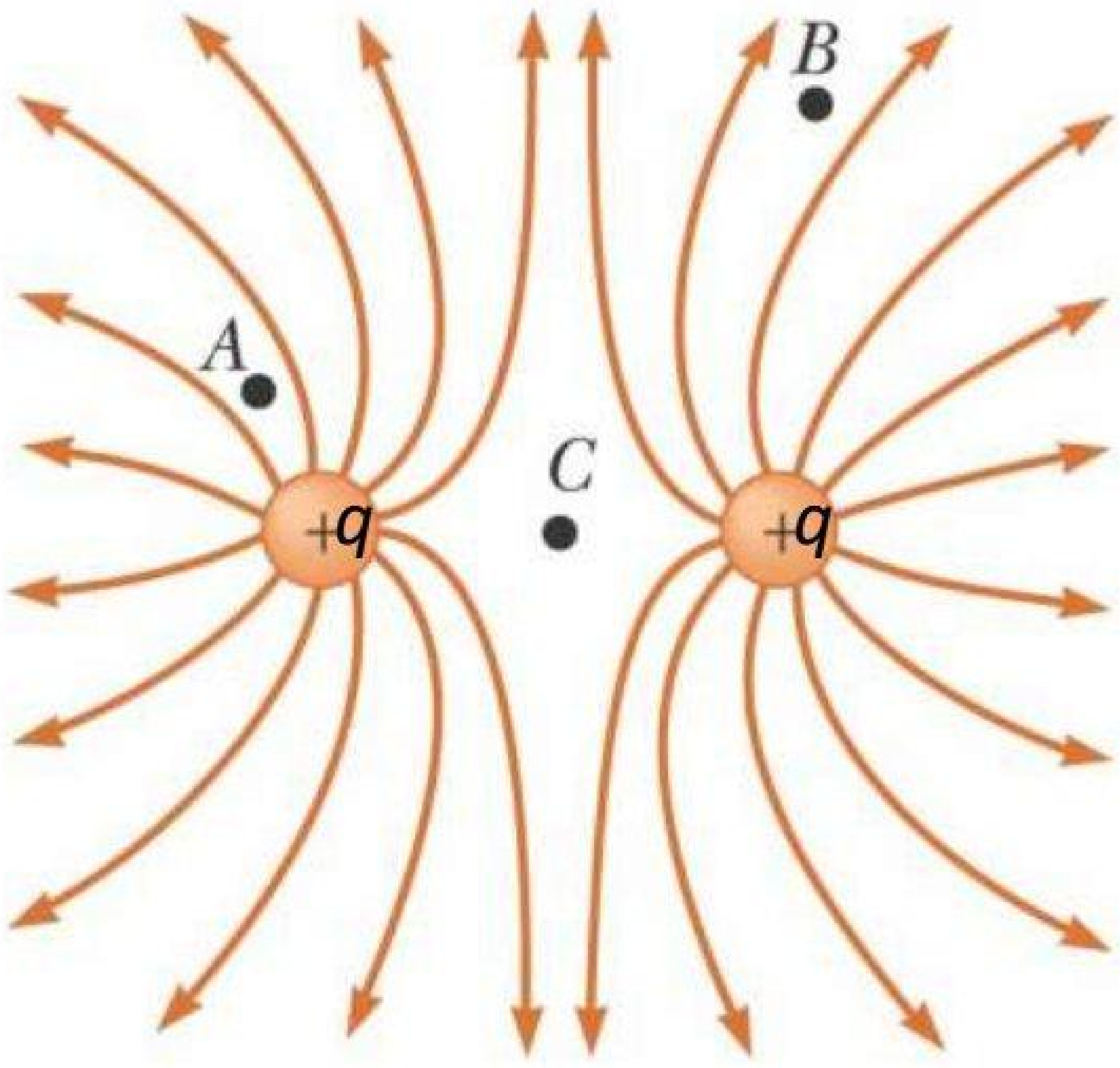
- Equal and like point charges



Electric Field Lines:

Like Charges

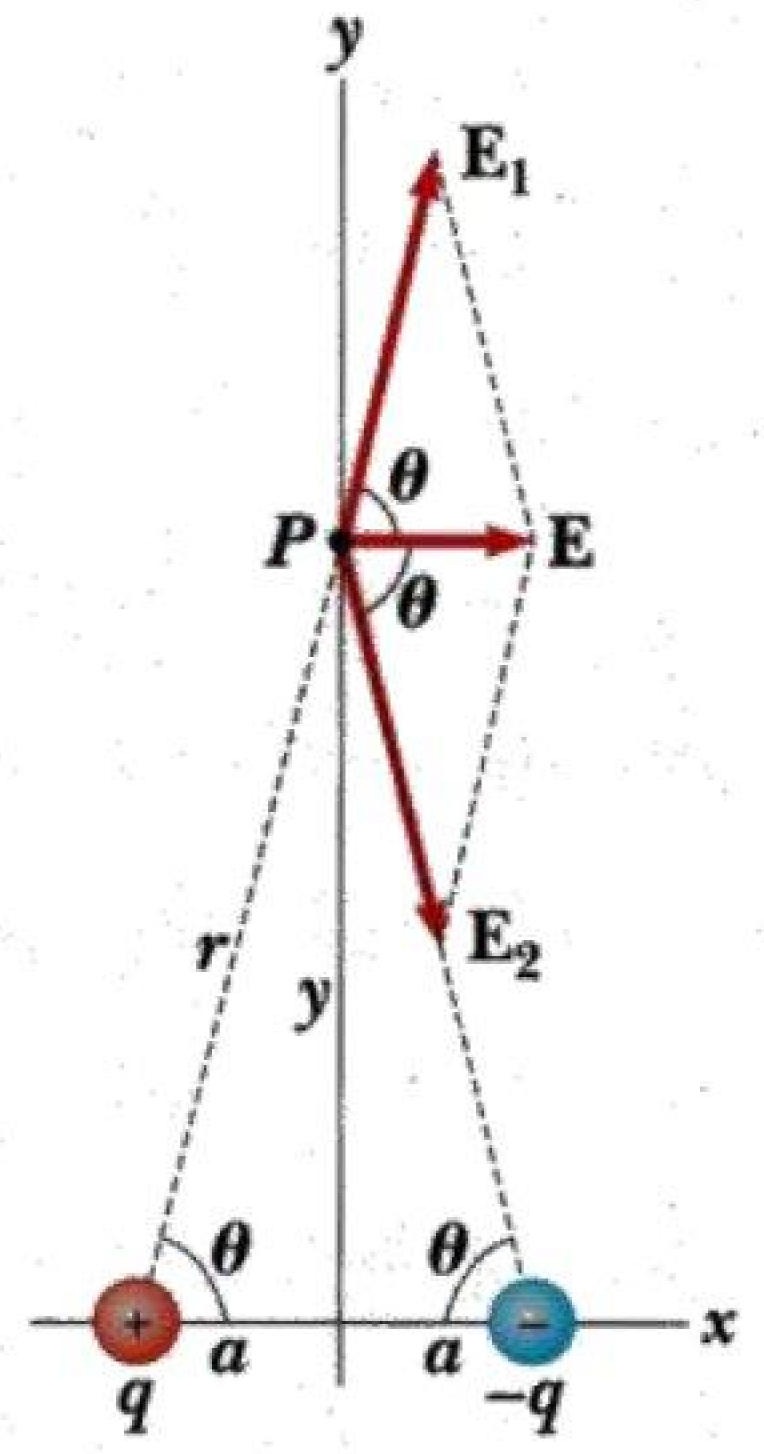
- The charges are equal and positive
- The same number of lines leave each charge since they are equal in magnitude
- At a great distance, the field is approximately equal to that of a single charge of $2q$



(a)

Example 5 : ***(a New method to solve the question)***

Electric Field of a dipole



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = k_e \frac{|q|}{r^2}\hat{r}_1 + k_e \frac{|q|}{r^2}\hat{r}_2$$

$$|E_1| = |E_2|$$

$$E_y = E_1 \sin\theta + E_2 \sin(-\theta) = E_1 \sin\theta - E_1 \sin\theta = 0$$

$$E_x = E = E_1 \cos\theta + E_2 \cos\theta = 2E_1 \cos\theta$$

$$E = 2k_e \frac{q}{r^2}\cos\theta$$

$$= 2k_e \frac{q}{(y^2+a^2)} \cdot \frac{a}{(y^2+a^2)^{1/2}}$$

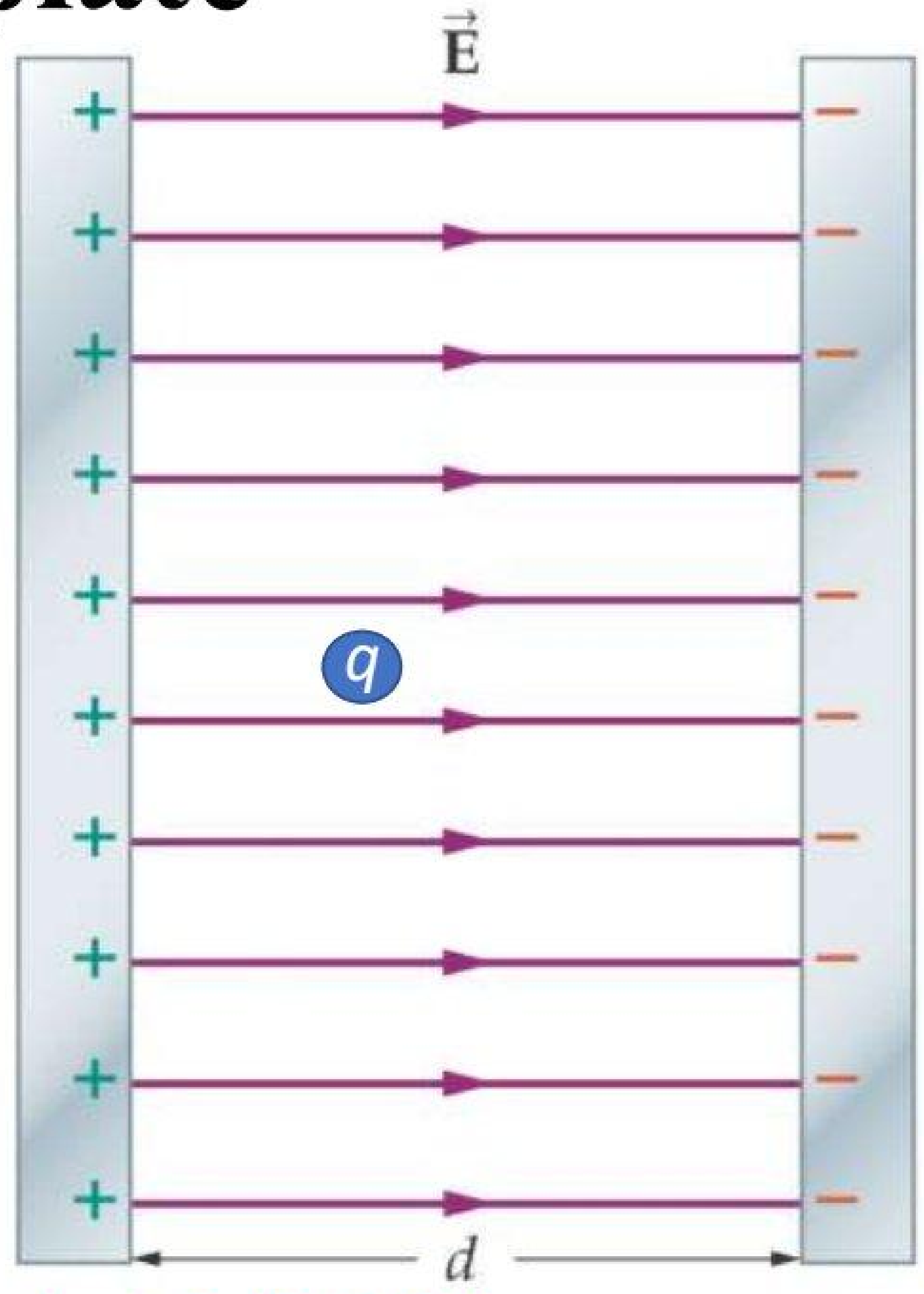
$$= k_e \frac{2qa}{(y^2+a^2)^{3/2}}$$

For r >> a:

$$\boxed{E = k_e \frac{2qa}{y^3}} \quad \propto \frac{1}{r^3}$$

Electric Field Lines: **A parallel-plate**

- A **parallel-plate capacitor** consists of two conducting plates with equal and opposite charges. Here is the electric field:



$$E = \frac{F_e}{q}$$

According to Newton's second law:

$\sum F = ma$, then

$$F_e = ma$$

$$qE = ma$$

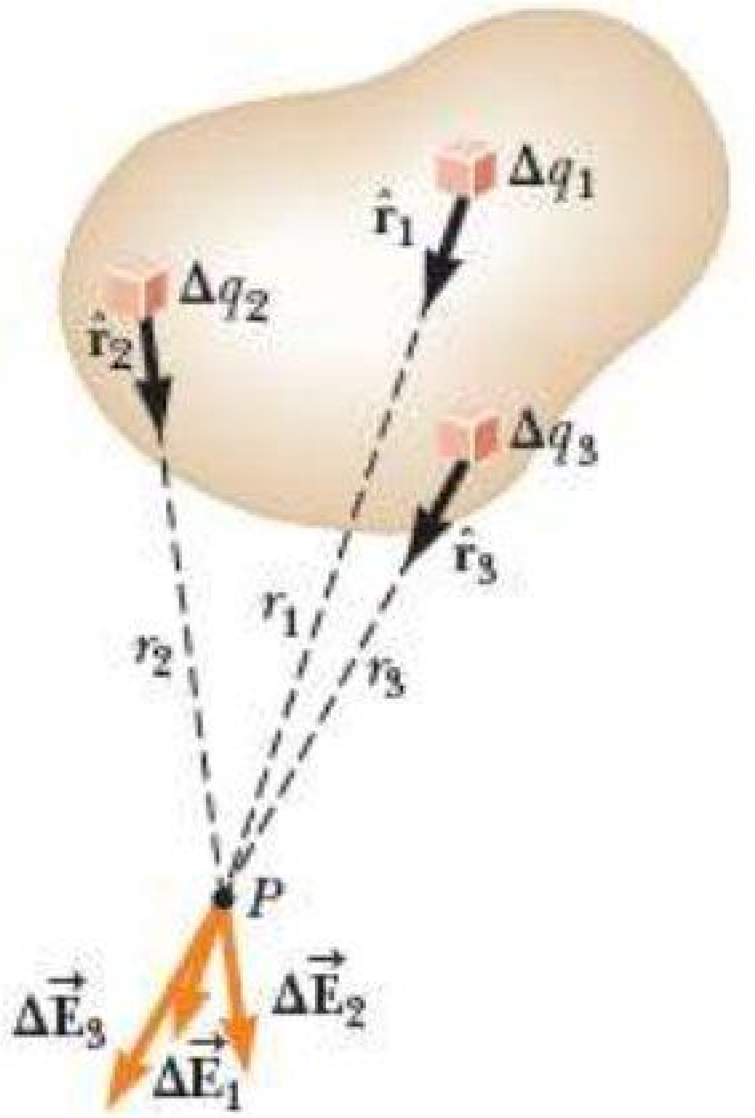
$$\rightarrow \boxed{a = \frac{qE}{m}}$$

*This is valid for a **uniform electric field (E).***

In Vector form: $\boxed{\vec{a} = \frac{q\vec{E}}{m}}$

- If q is positive, the acceleration of q ($\vec{a}$) and the electric field ($\vec{E}$) are in the same direction
- If q is negative, the acceleration of q and the electric field are in opposite directions.

Electric Field of a Continuous Charge Distribution



➢ The electric field at *P* due to one charge element carrying charge Δq is

$$\Delta \vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

where *r* is the distance from the charge element to point *P* and $\hat{\boldsymbol{r}}$ is a unit vector directed from the element toward *P*. The total electric field at *P* due to all elements in the charge distribution is approximately:

$$\vec{\mathbf{E}} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i$$

where the index *i* refers to the *i*th element in the distribution. Because the number of elements is very large and the charge distribution is modeled as continuous, the total field at *P* in the limit $\Delta q \to 0$ is

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i = \boxed{k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}}$$

Charge Density (Three Types):

- **Volume charge density**: when a charge Q is distributed evenly throughout a volume
  - $\boxed{\rho \equiv Q / V}$; units C/m^3

- **Surface charge density**: when a charge Q is distributed evenly over a surface area
  - $\boxed{\sigma \equiv Q / A}$; units C/m^2

- **Linear charge density**: when a charge Q is distributed along a line
  - $\boxed{\lambda \equiv Q / \ell}$; units C/m

Example 23.11 An Accelerated Electron **AM**

An electron enters the region of a uniform electric field as shown in Figure 23.24, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

(A) Find the acceleration of the electron while it is in the electric field.

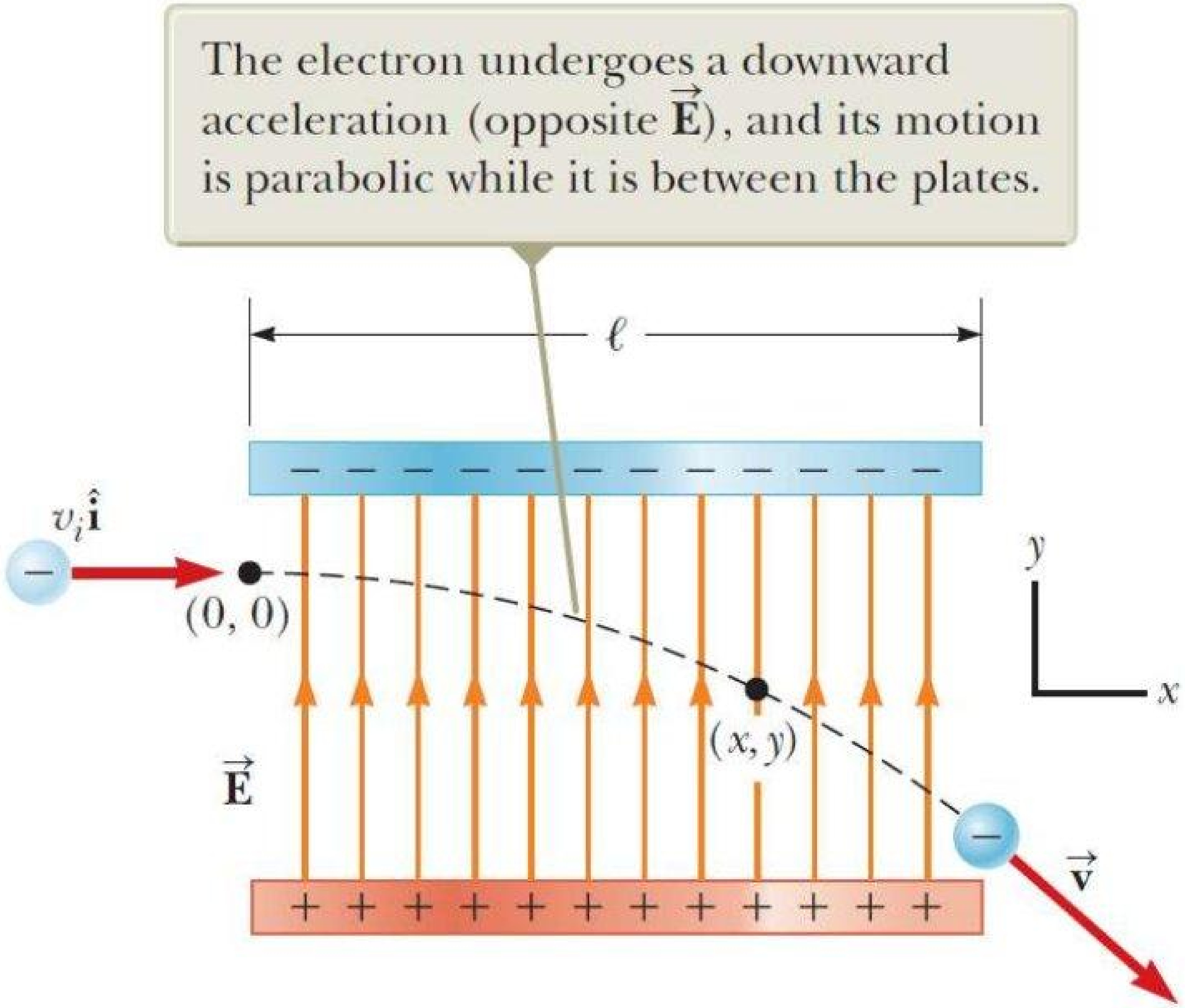


Figure 23.24 (Example 23.11) An electron is projected horizontally into a uniform electric field produced by two charged plates.

SOLUTION

Conceptualize This example differs from the preceding one because the velocity of the charged particle is initially perpendicular to the electric field lines. (In Example 23.10, the velocity of the charged particle is always parallel to the electric field lines.) As a result, the electron in this example follows a curved path as shown in Figure 23.24. The motion of the electron is the same as that of a massive particle projected horizontally in a gravitational field near the surface of the Earth.

Categorize The electron is a *particle in a field (electric)*. Because the electric field is uniform, a constant electric force is exerted on the electron. To find the acceleration of the electron, we can model it as a *particle under a net force*.

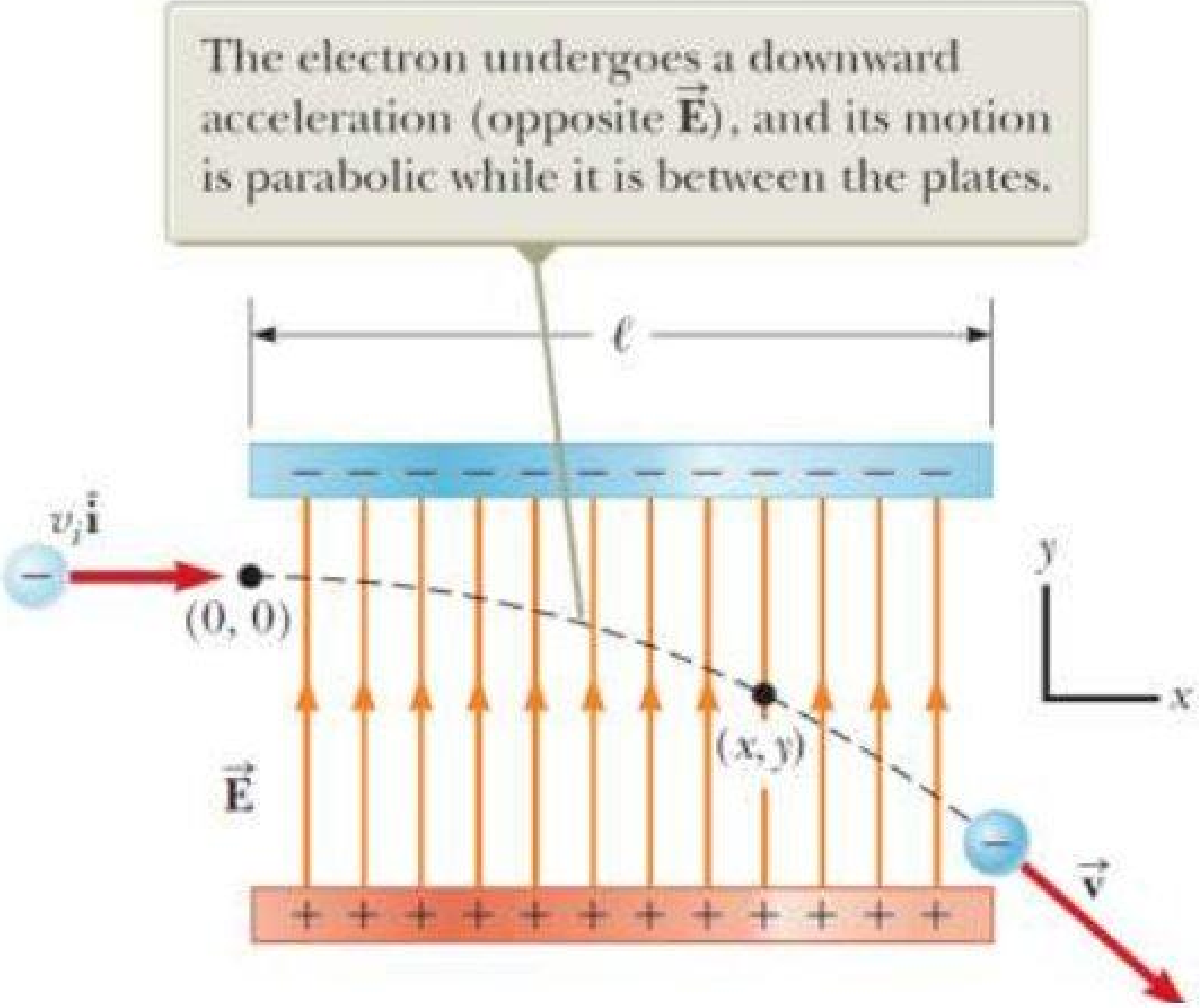
Analyze From the particle in a field model, we know that the direction of the electric force on the electron is downward in Figure 23.24, opposite the direction of the electric field lines. From the particle under a net force model, therefore, the acceleration of the electron is downward.

- We know that the **electron charge** (e = $1.6 \times 10^{-19} C$)
- And the **electron mass** ($m_e = 9.11 \times 10^{-31} kg$)

$$a = \frac{qE}{m} \rightarrow \boxed{a = \frac{e\,E}{m_e}}$$

$$a_y = -\frac{eE}{m_e}$$

$$a_y = -\frac{(1.60 \times 10^{-19}\ \mathrm{C})(200\ \mathrm{N/C})}{9.11 \times 10^{-31}\ \mathrm{kg}} = -3.51 \times 10^{13}\ \mathrm{m/s^2}$$



(B) Assuming the electron enters the field at time $t = 0$, find the time at which it leaves the field.

Analyze Solve Equation 2.7 for the time at which the electron arrives at the right edges of the plates:

$$\boxed{x_f = x_i + v_x t} \rightarrow \quad t = \frac{x_f - x_i}{v_x}$$

Substitute numerical values:

$$t = \frac{\ell - 0}{v_x} = \frac{0.100 \text{ m}}{3.00 \times 10^6 \text{ m/s}} = \boxed{3.33 \times 10^{-8} \text{ s}}$$

(C) Assuming the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

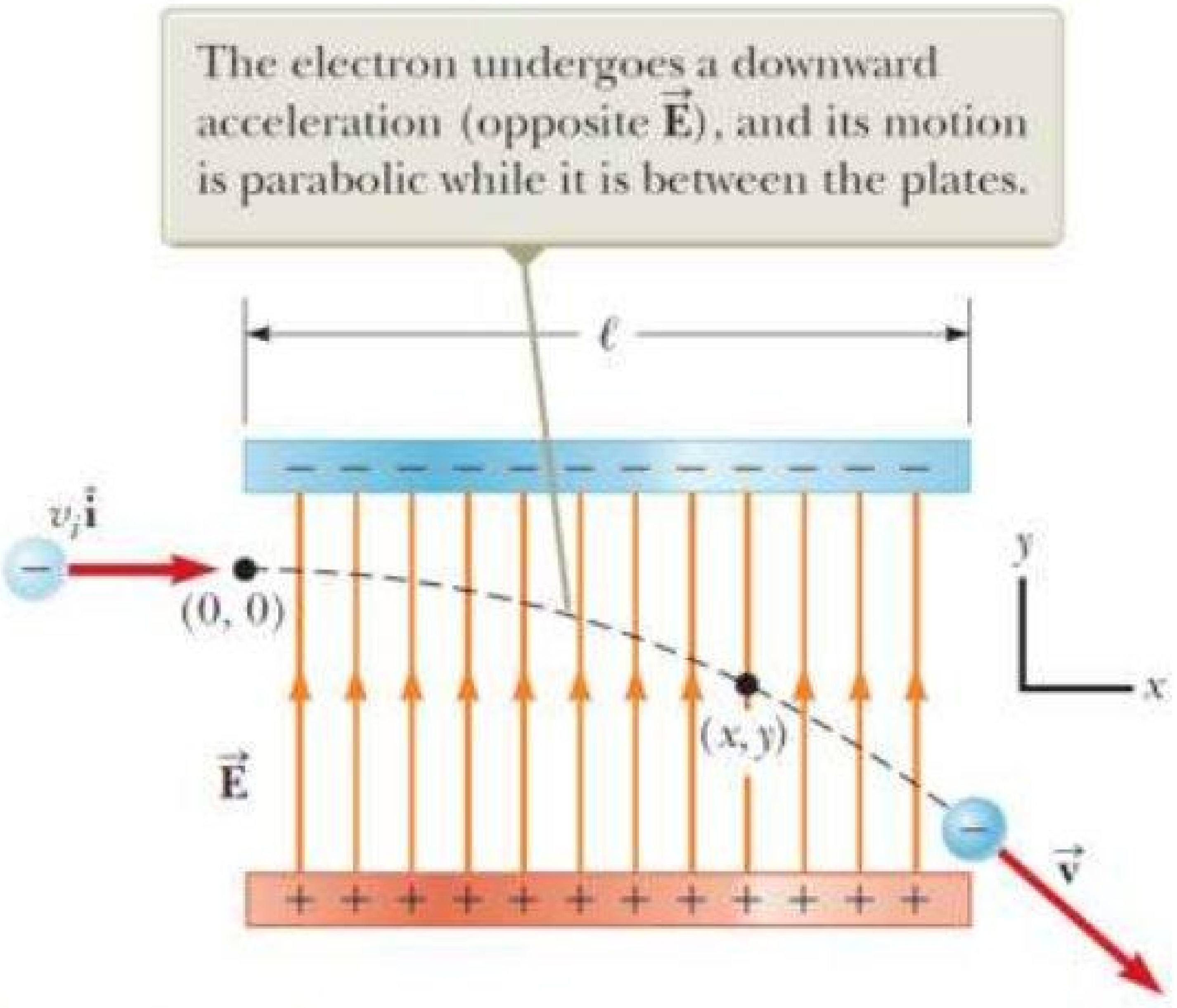


Figure 23.24 (Example 23.11) An electron is projected horizontally into a uniform electric field produced by two charged plates.

SOLUTION

Categorize Because the electric force is constant in Figure 23.24, the motion of can be analyzed by modeling it as a *particle under constant acceleration.*

Analyze Use Equation 2.16 to describe the position of the particle at any time t:

$$\boxed{y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2}$$

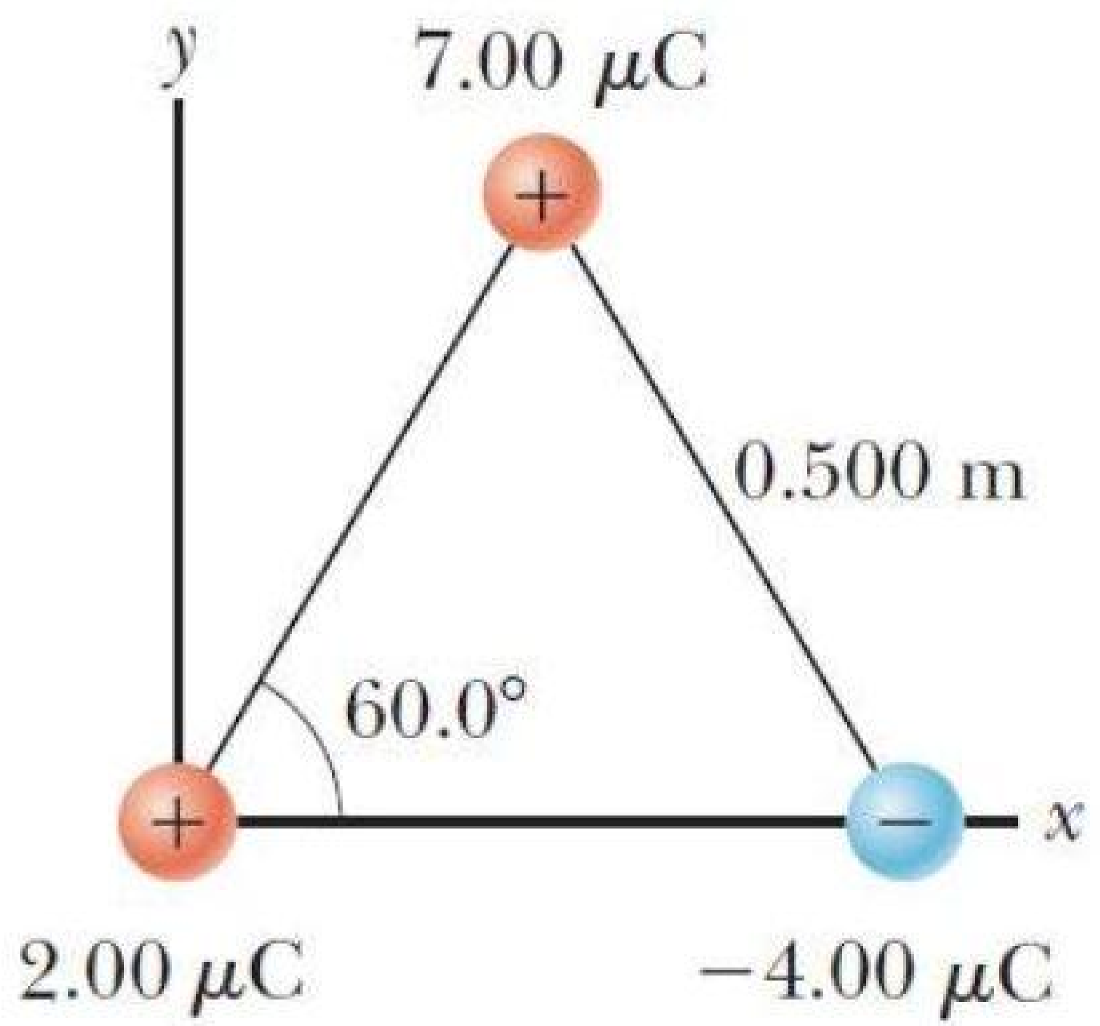
Substitute numerical values:

$$y_f = 0 + 0 + \tfrac{1}{2}(-3.51 \times 10^{13}\ \mathrm{m/s^2})(3.33 \times 10^{-8}\ \mathrm{s})^2$$

$$= -0.019\,5\ \mathrm{m} = -1.95\ \mathrm{cm}$$

*Problem1:

15. Three charged particles are located at the corners of an equilateral triangle as shown in Figure P23.15. Calculate the total electric force on the $7.00\text{-}\mu C$ charge.



(d) Find the speed of the electron as it emerges from the field. ($v_f =?$)

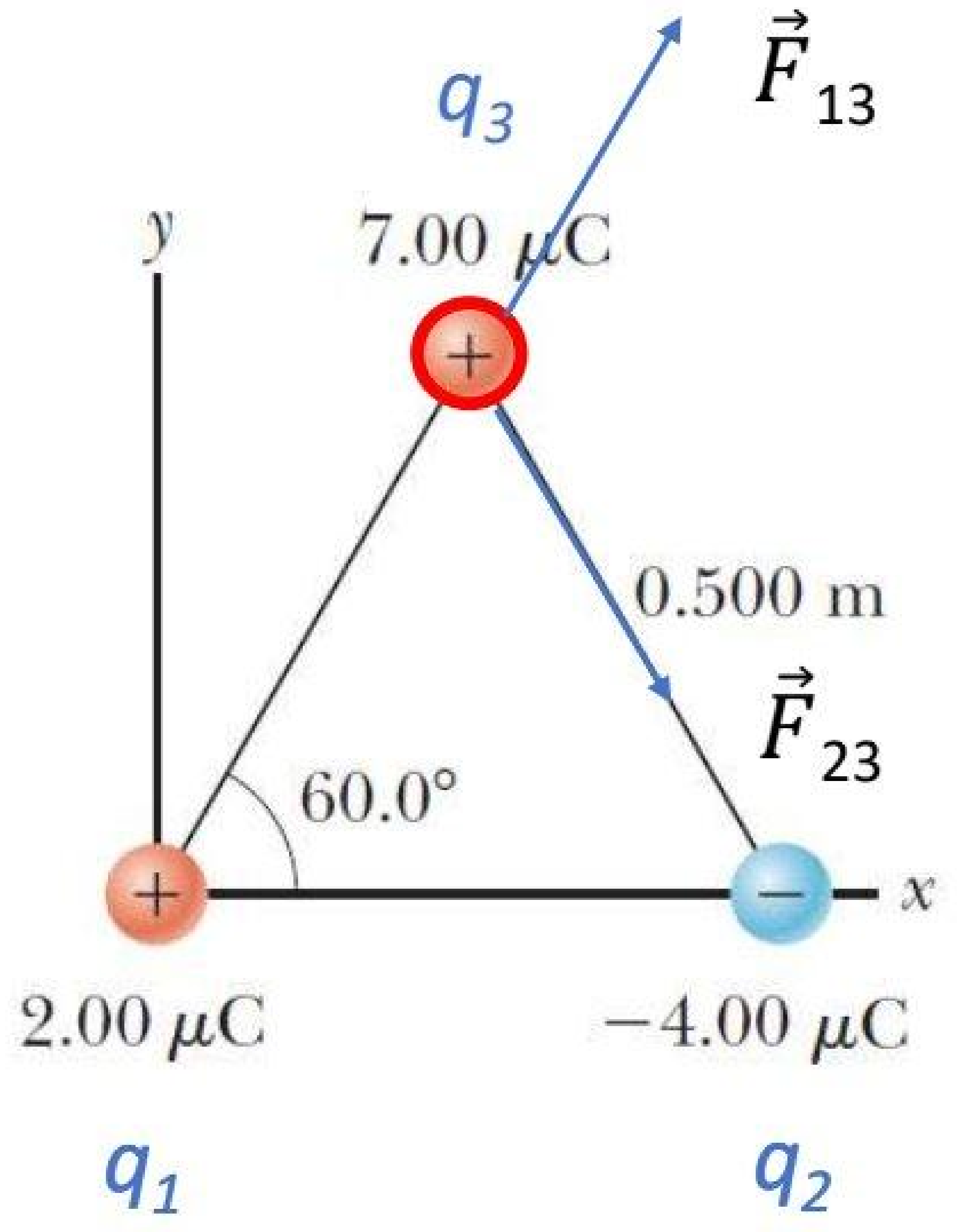
$$\boxed{v_f = v_i + a\,t}$$

$$v_f = \ldots$$

For you to practice:

- *Solve Example **22.7** in the textbook!*

Solution:



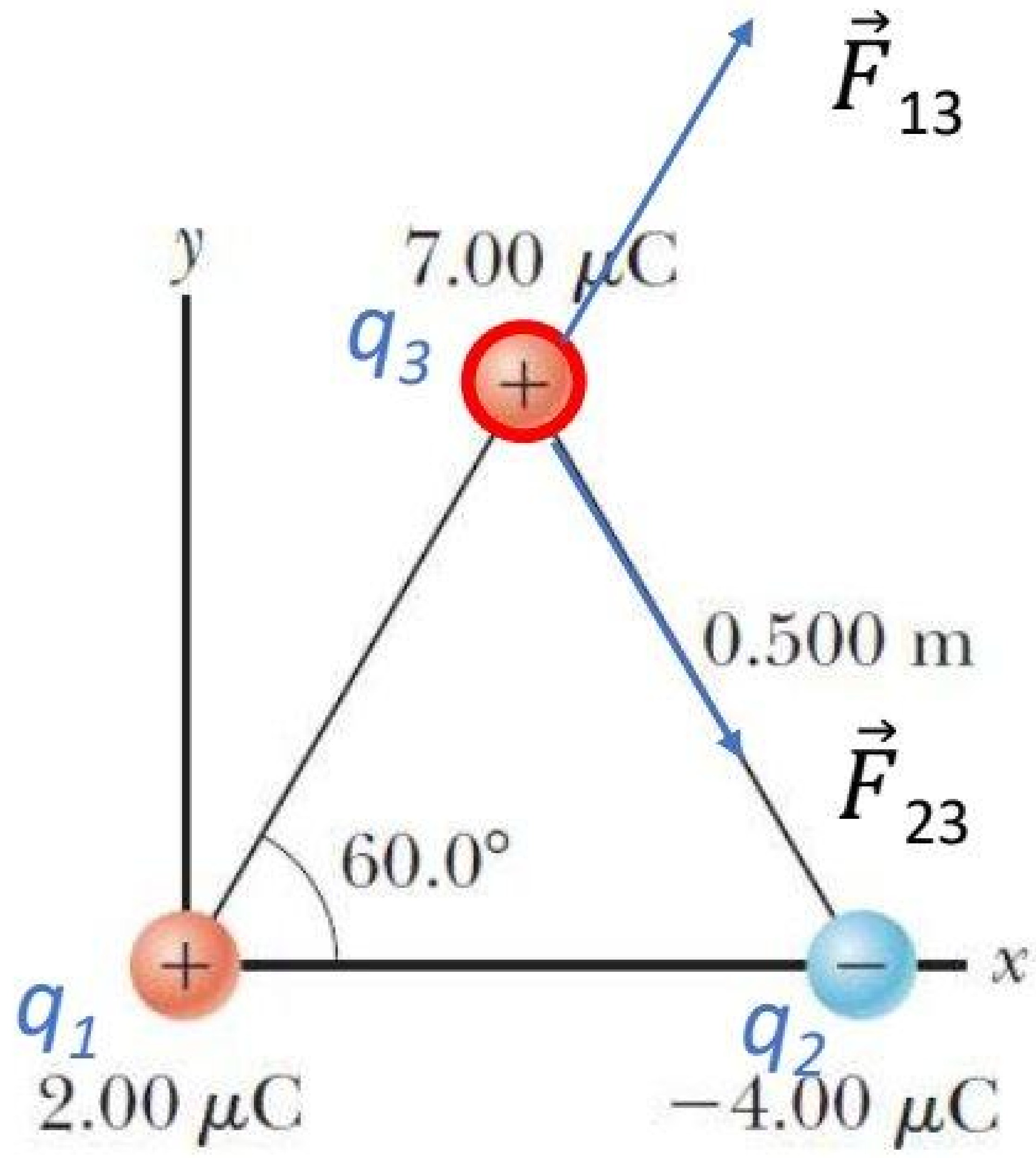
- $\vec{F}_{13}$: repulsive force
- $\vec{F}_{23}$: attractive force

$$F_{13} = k_e \frac{|q_1||q_3|}{r^2} \quad \text{(Coulomb's law)}$$

We have an equilateral triangle (each side = 0.5 m and each angle is 60^0).

$$F_{13} = \frac{(9\times10^9)(2\times10^{-6})(7\times10^{-6})}{(0.5)^2} = 0.503 \text{ N}$$

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2} = \frac{(9\times10^9)(4\times10^{-6})(7\times10^{-6})}{(0.5)^2} = 1.01 \text{ N}$$



- The resultant force $\overrightarrow{F_3}$:

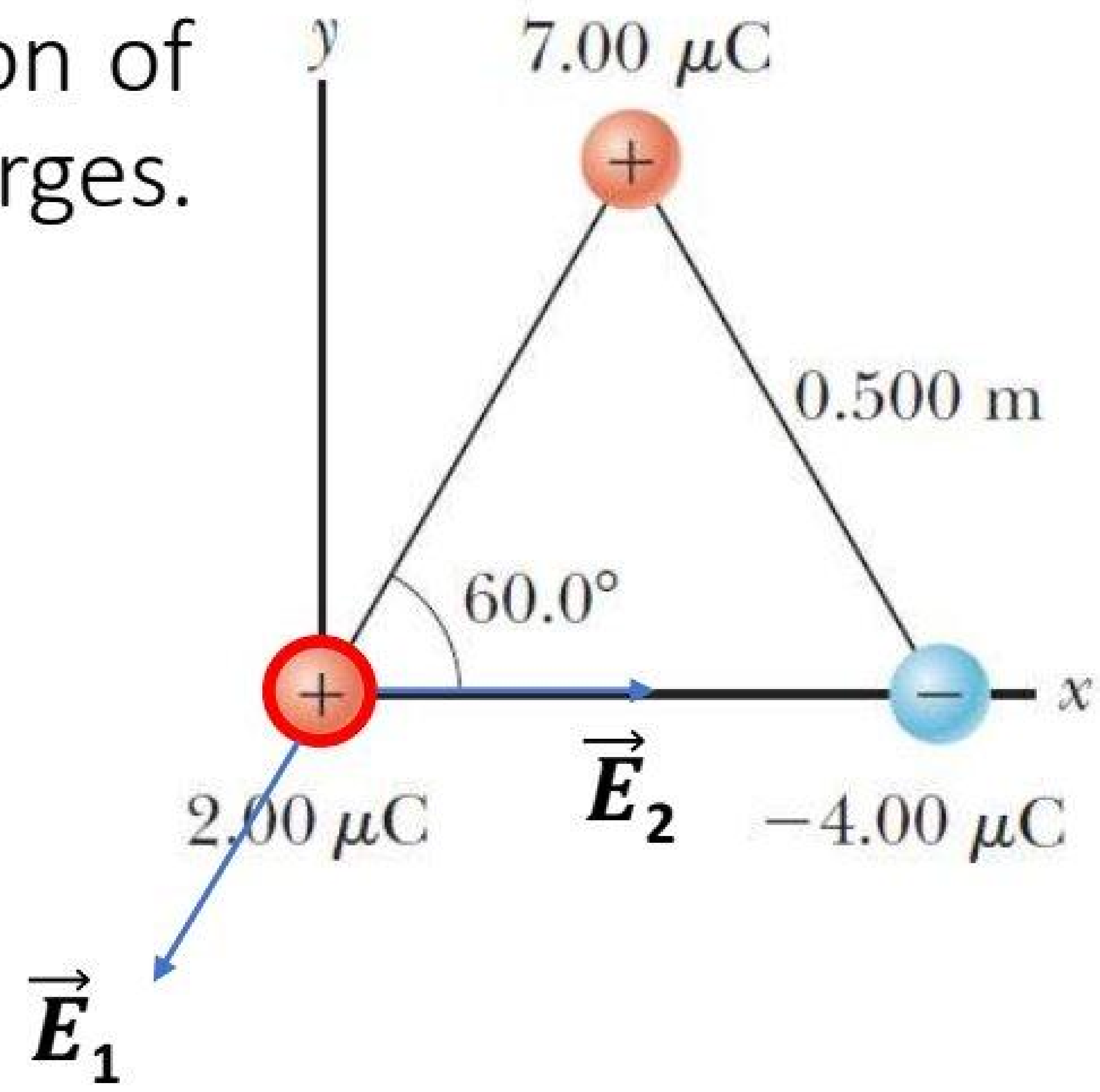
$$F_x = F_{13}\cos 60^{\circ} + F_{23}\cos 60^{\circ} = 0.503\left(\frac{1}{2}\right) + 1.01\left(\frac{1}{2}\right) = 0.76\ N$$

$$F_y = F_{13}\sin 60^{\circ} - F_{23}\sin 60^{\circ} = 0.503\left(\frac{\sqrt{3}}{2}\right) - 1.01\left(\frac{\sqrt{3}}{2}\right) = -0.44\ N$$

$\overrightarrow{F_3} = (0.76\ \hat{\imath} - 0.44\ \hat{\jmath})$ N

Then, $\boxed{|\vec{F}_3| = \sqrt{F_x^{\,2} + F_y^{\,2}} = 0.878 \text{ N and } \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 330°}$

***Problem2:** Three charges are at the corners of an equilateral triangle. **A)** Calculate the electric field at the position of the $2\mu C$ charge due to the $7\ \mu C$ and -4 μC charges. **B)** Determine the electric force on the $2\mu C$ charge.



$$E = k_e \frac{|q|}{r^2}$$ (Electric field)

.

We have an equilateral triangle (each side = 0.5 m and each angle is 60 degrees).

$$E_1 = k_e \frac{|q1|}{r^2} = \frac{(9\times10^9)\,(7\times10^{-6})}{(0.5)^2} = 2.52\times10^5 \text{ N/C}$$

$$E_2 = k_e \frac{|q2|}{r^2} = \frac{(9\times10^9)(4\times10^{-6})}{(0.5)^2} = 1.44\times10^5 \text{ N/C}$$

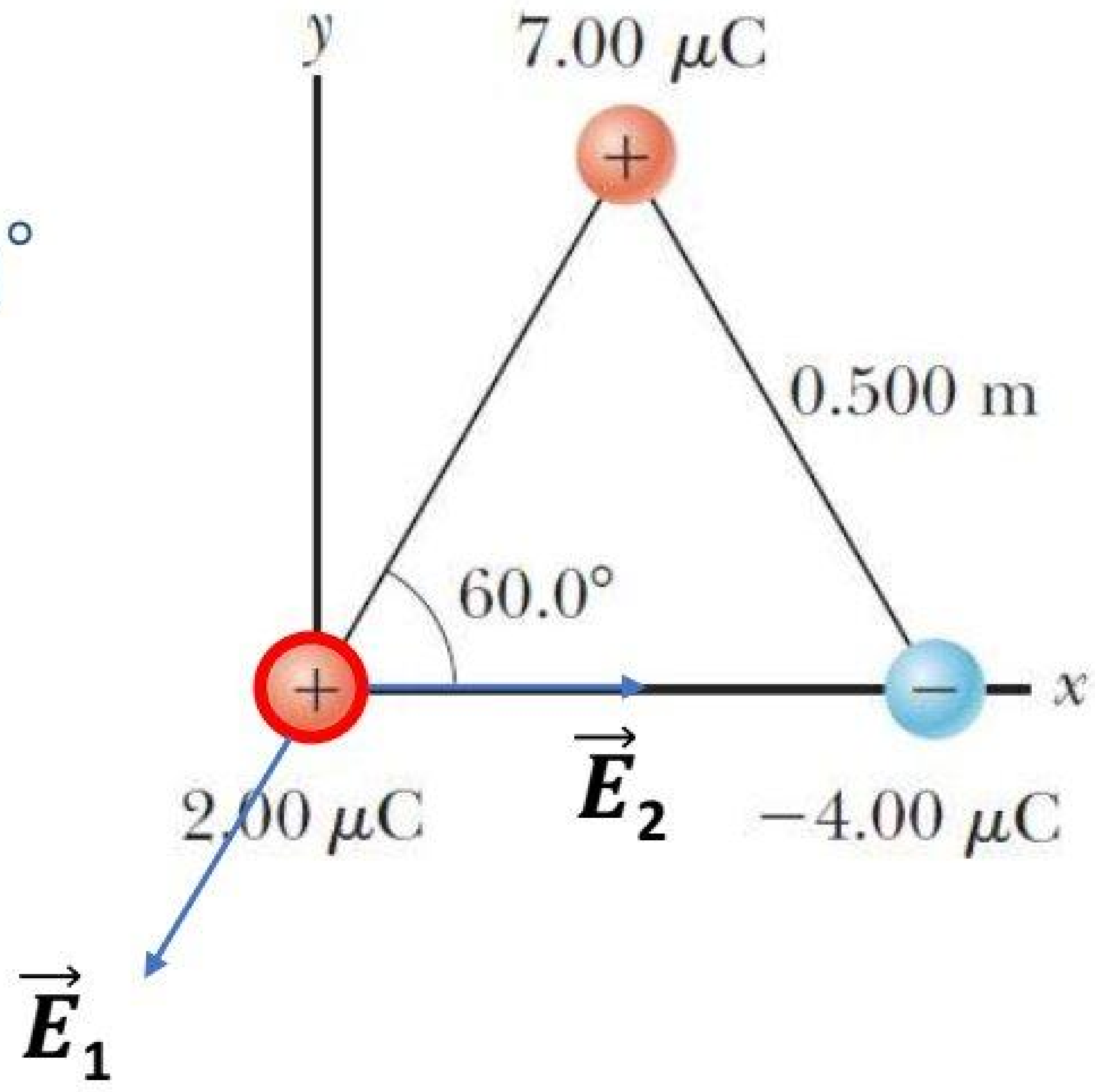
- **The electric field $\vec{E}$:**

$E_x = E_2 \cos 0^{\circ} - E_1 \cos 60^{\circ}$ = (1.44×10^5) − (2.52×105) cos 60°

= 18×10³ N/C = 18 kN/C

$E_y = -E_1 \sin 60^{\circ} = -218{\times}10^3\ N/C = -218\ kN/C$

- Then, $\vec{E} = [E_x\,\hat{\imath} + E_y\,\hat{\jmath}] = [18\hat{\imath} - 218\hat{\jmath}]\ kN/C$



- **The electric force $\vec{F}_e$:**
- $\vec{F}_e = q\vec{E}$ = (2×10⁻⁶C) $[18\hat{\imath} - 218\hat{\jmath}] \times 10^3 N/C$

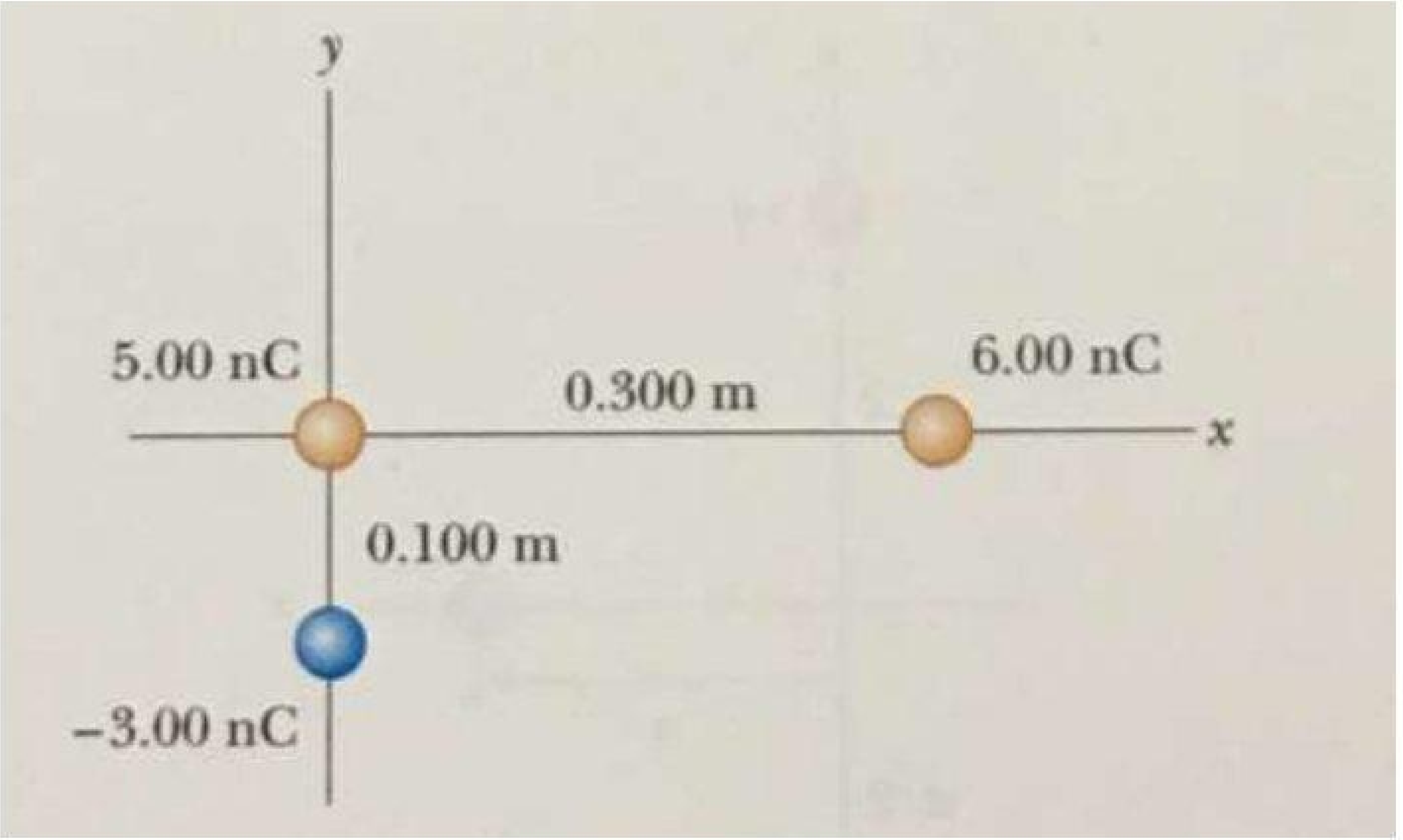
= $[36\ \hat{\imath}$ - $436\ \hat{\jmath}]\times 10^{-3} N$

*Problem3:

- Three point charges are arranged as shown in Figure.

a) Find **the electric field vector** that the *6 nC* and *-3 nC* charges together create at the origin.

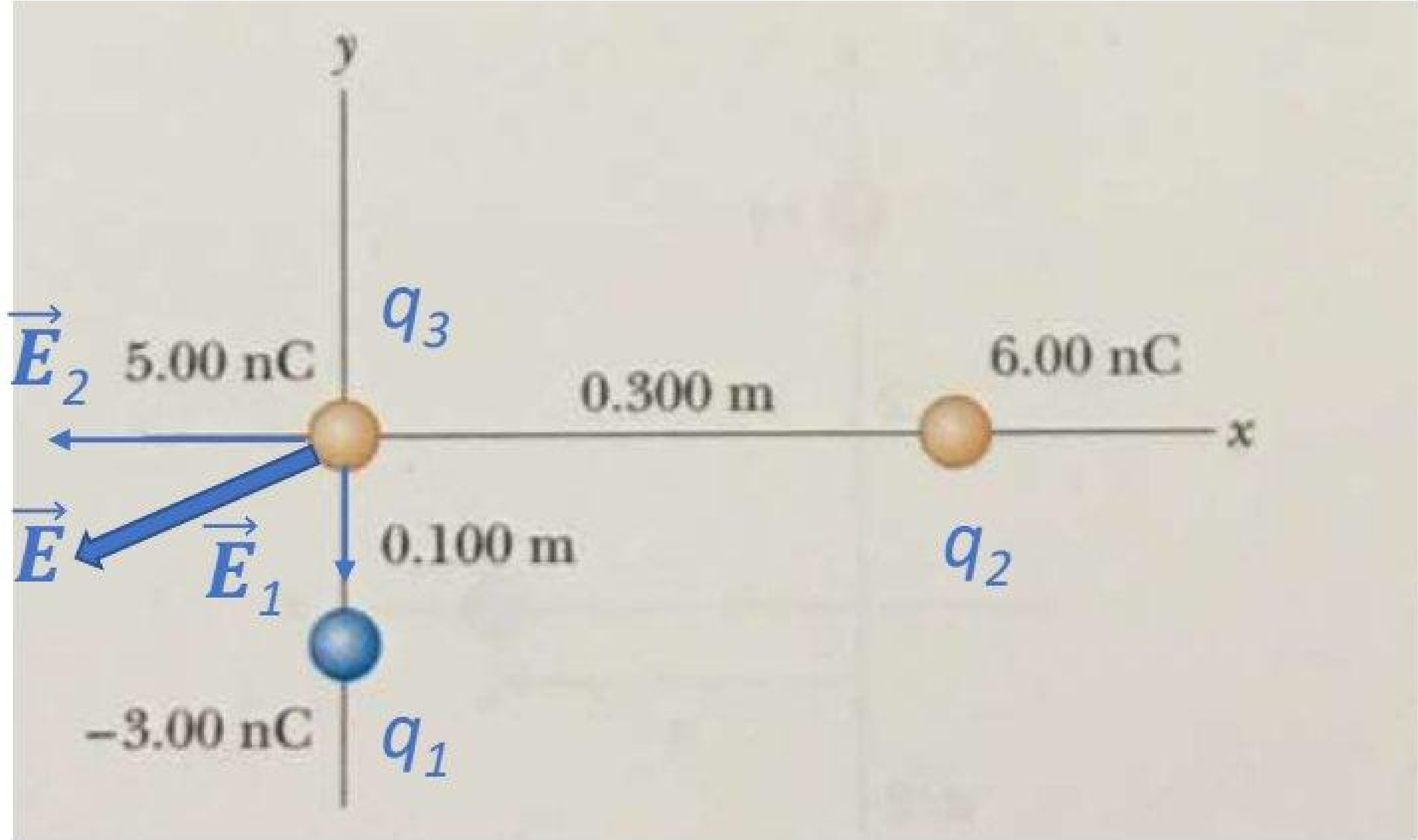
b) Find **the electric force vector** on the *5 nC* charge.



a) **Vector Electric field**: $\vec{E} = k_e \frac{|q|}{r^2}\hat{r}$

$\vec{E}_1 = k_e \frac{|q_1|}{r^2}(-\hat{j}) = \frac{(9\times10^9)\,(3\times10^{-9})}{(0.1)^2}(-\hat{j})$

$= -(2.7\times10^3)\,\hat{j}$ N/C



$\vec{E}_2 = k_e \frac{|q_2|}{r^2}(-\hat{\imath}) = \frac{(9\times10^9)(6\times10^{-9})}{(0.3)^2}(-\hat{\imath})$

$= -(6\times10^2)\,\hat{\imath}$ N/C

$\vec{E} = \vec{E}_1 + \vec{E}_2 = [-(6\times10^2)\,\hat{\imath} - (2.7\times10^3)\,\hat{j}\,]$N/C

b) **Vector Electric Force**:

$\vec{F}_e = q\vec{E} = (5\times10^{-9})\,[-(6\times10^2)\,\hat{\imath} - (2.7\times10^3)\,\hat{j}\,]$

$= [(-3\times10^{-6})\,\hat{\imath} - (13.5\times10^{-6})\,\hat{j}\,]\ N$

Electric Field of a Continuous Charge Distribution:

Example 23.7 The Electric Field Due to a Charged Rod

A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end (Fig. 23.15).

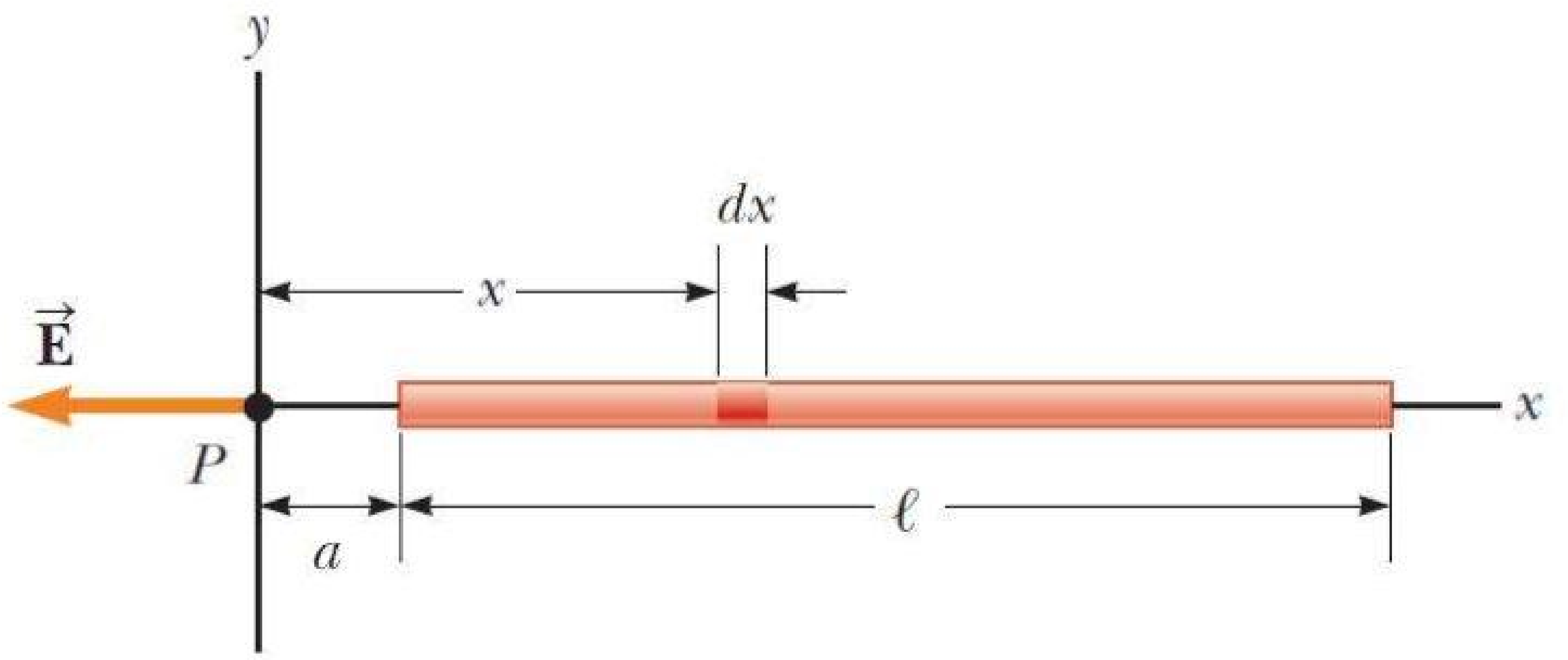


Figure 23.15 (Example 23.7) The electric field at P due to a uniformly charged rod lying along the x axis.

SOLUTION

Conceptualize The field $d\vec{\mathbf{E}}$ at P due to each segment of charge on the rod is in the negative x direction because every segment carries a positive charge. Figure 23.15 shows the appropriate geometry. In our result, we expect the electric field to become smaller as the distance a becomes larger because point P is farther from the charge distribution.

continued

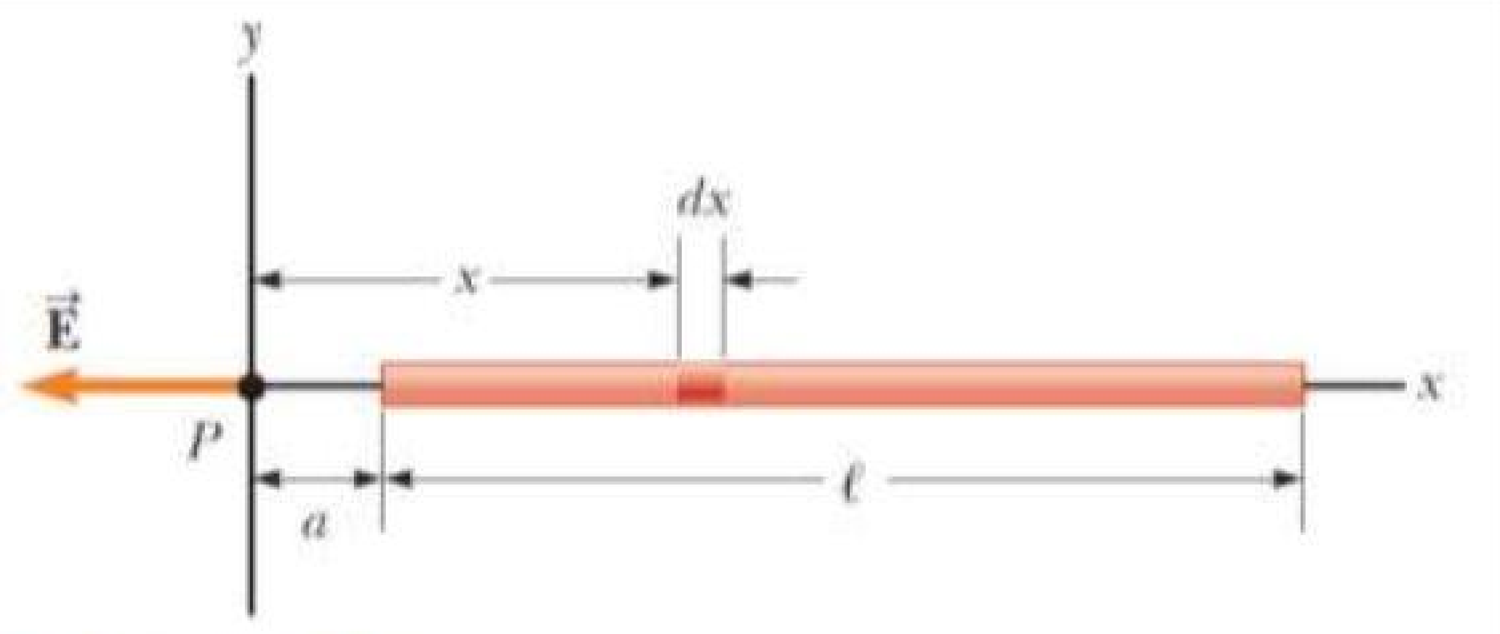


Figure 23.15 (Example 23.7) The electric field at P due to a uniformly charged rod lying along the x axis.

Categorize Because the rod is continuous, we are evaluating the field due to a continuous charge distribution rather than a group of individual charges. Because every segment of the rod produces an electric field in the negative x direction, the sum of their contributions can be handled without the need to add vectors.

Analyze Let's assume the rod is lying along the x axis, dx is the length of one small segment, and dq is the charge on that segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is $dq = \lambda\, dx$.

Find the magnitude of the electric field at P due to one segment of the rod having a charge dq:

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda\, dx}{x^2}$$

Find the total field at P using[4] Equation 23.11:

$$E = \int_a^{\ell + a} k_e \lambda \frac{dx}{x^2}$$

Noting that k_e and $\lambda = Q/\ell$ are constants and can be removed from the integral, evaluate the integral:

$$E = k_e \lambda \int_a^{\ell + a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell + a}$$

$$(1) \quad E = k_e \frac{Q}{\ell}\left(\frac{1}{a} - \frac{1}{\ell + a}\right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$

Example 23.8 The Electric Field of a Uniform Ring of Charge

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring (Fig. 23.16a).

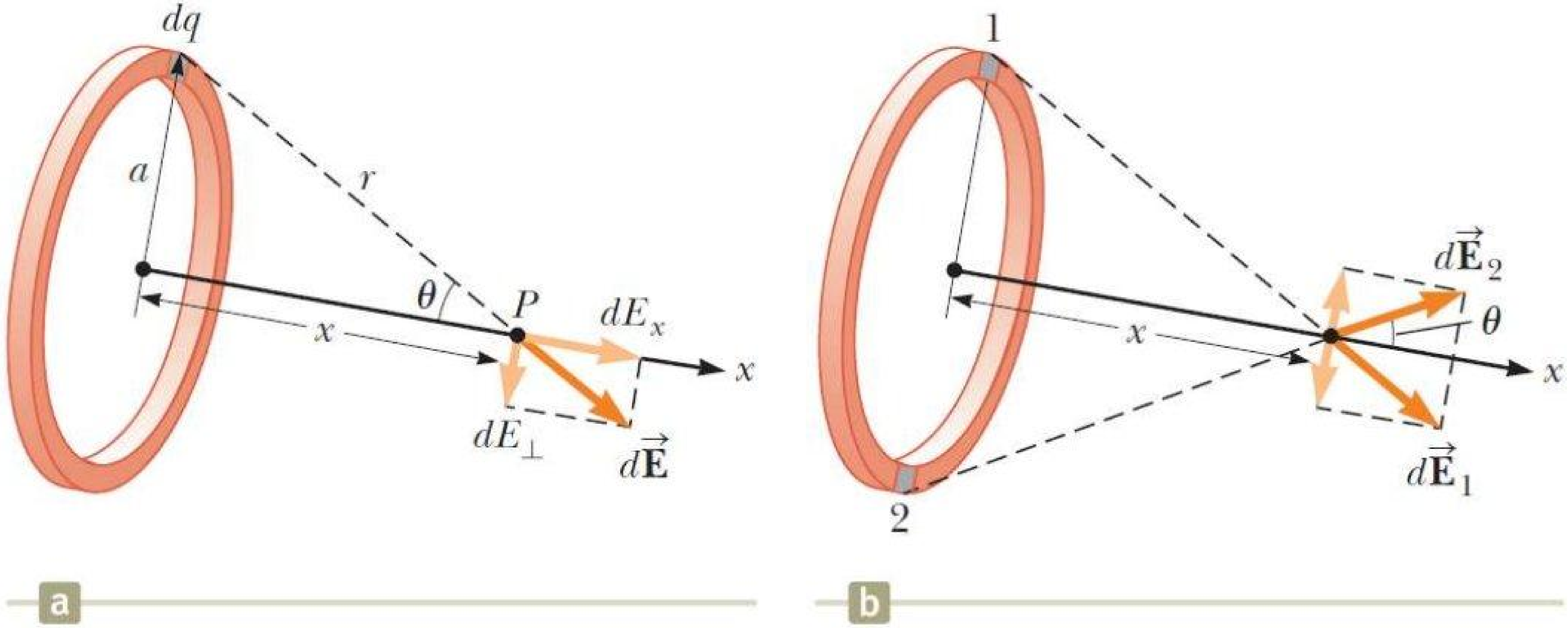


Figure 23.16 (Example 23.8) A uniformly charged ring of radius a. (a) The field at P on the x axis due to an element of charge dq. (b) The total electric field at P is along the x axis. The perpendicular component of the field at P due to segment 1 is canceled by the perpendicular component due to segment 2.

SOLUTION

Conceptualize Figure 23.16a shows the electric field contribution $d\vec{\mathbf{E}}$ at P due to a single segment of charge at the top of the ring. This field vector can be resolved into components dE_x parallel to the axis of the ring and $dE_\perp$ perpendicular to the axis. Figure 23.16b shows the electric field contributions from two segments on opposite sides of the ring. Because of the symmetry of the situation, the perpendicular components of the field cancel. That is true for all pairs of segments around the ring, so we can ignore the perpendicular component of the field and focus solely on the parallel components, which simply add.

Categorize Because the ring is continuous, we are evaluating the field due to a continuous charge distribution rather than a group of individual charges.

Analyze Evaluate the parallel component of an electric field contribution from a segment of charge dq on the ring:

$$(1)\quad dE_x = k_e \frac{dq}{r^2}\cos\theta = k_e \frac{dq}{a^2 + x^2}\cos\theta$$

From the geometry in Figure 23.16a, evaluate $\cos\theta$:

$$(2)\quad \cos\theta = \frac{x}{r} = \frac{x}{(a^2 + x^2)^{1/2}}$$

Substitute Equation (2) into Equation (1):

$$dE_x = k_e \frac{dq}{a^2 + x^2}\left[\frac{x}{(a^2 + x^2)^{1/2}}\right] = \frac{k_e x}{(a^2 + x^2)^{3/2}}\,dq$$

All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Integrate over the circumference of the ring to obtain the total field at P:

$$E_x = \int \frac{k_e x}{(a^2 + x^2)^{3/2}}\,dq = \frac{k_e x}{(a^2 + x^2)^{3/2}}\int dq$$

$$(3)\quad E = \boxed{\frac{k_e x}{(a^2 + x^2)^{3/2}}\,Q}$$

For you to practice:

- *Solve Example* ***23.3 (The Electric Field of a Uniformly Charged Disk)*** *in TEXTBOOK!*

Example 23.3 The Electric Field of a Uniformly Charged Disk

A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk (Fig. 23.4).

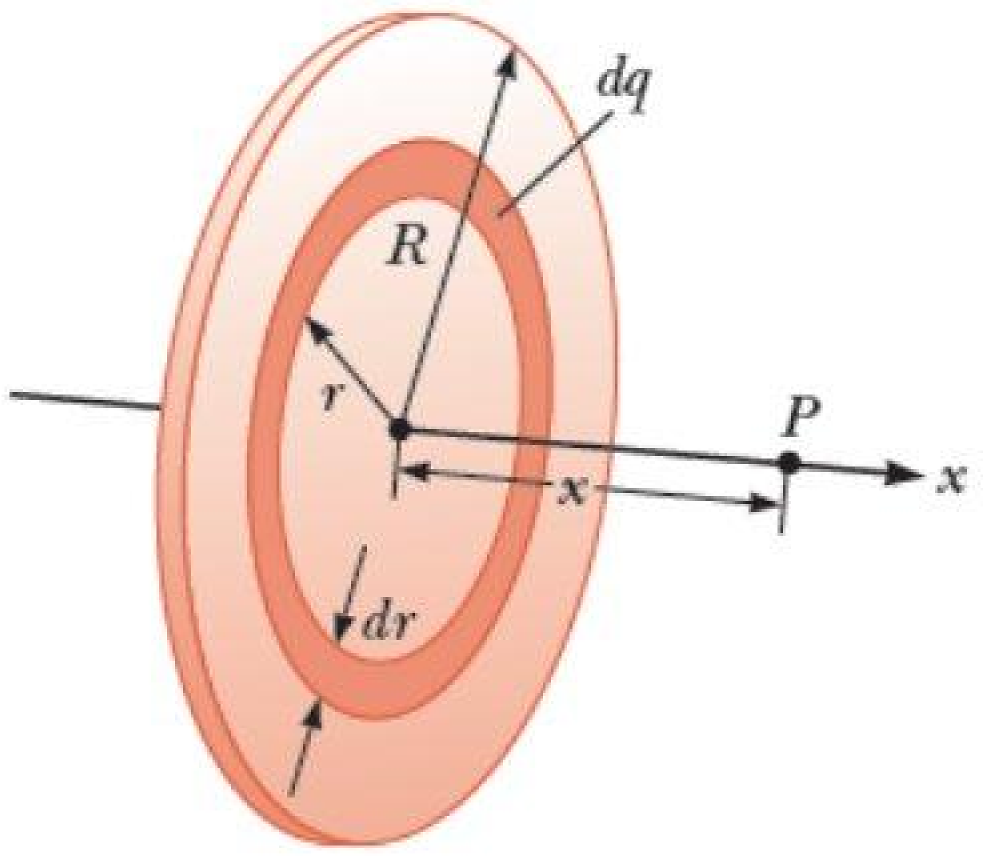


Figure 23.4 (Example 23.3) A uniformly charged disk of radius R. The electric field at an axial point P is directed along the central axis, perpendicular to the plane of the disk.

SOLUTION

Conceptualize If the disk is considered to be a set of concentric rings, we can use our result from Example 23.2—which gives the field created by a single ring of radius a—and sum the contributions of all rings making up the disk. By symmetry, the field at an axial point must be along the central axis.

Categorize Because the disk is continuous, we are evaluating the field due to a continuous charge distribution rather than a group of individual charges.