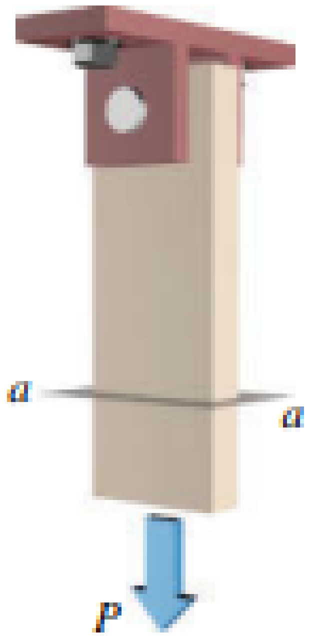
Chapter 1
Stress

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$\sigma = \frac{F}{A}$$



شد أو [illegible]

$$\frac{N}{m^2} \longrightarrow \text{Pascal}$$

$$\frac{N}{mm^2} \longrightarrow \text{MPa}$$

$$\text{KPa} \rightarrow 10^3 \text{ Pa}$$

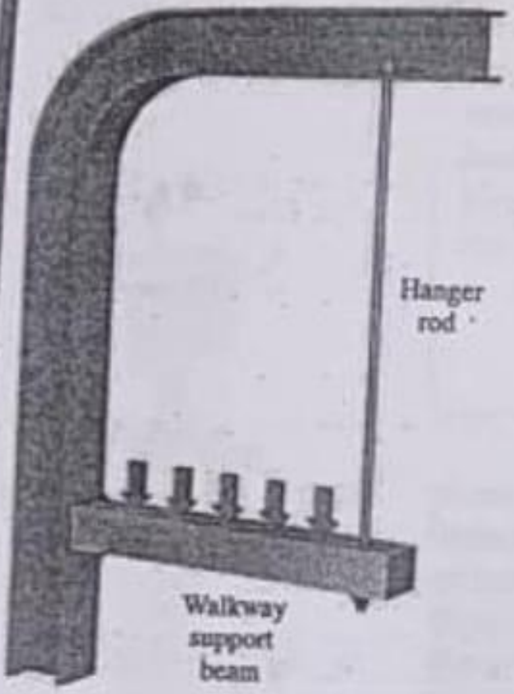
$$\text{MPa} \rightarrow 10^6 \text{ Pa}$$

$$\text{GPa} \rightarrow 10^9 \text{ Pa}$$

$$\text{Area of circle} = \pi r^2 = \frac{\pi d^2}{4}$$

EXAMPLE 1.1

A solid 14 mm diameter steel hanger rod is used to hold up one end of a walkway support beam. The force carried by the rod is 21 kN. Determine the normal stress in the rod. (Disregard the weight of the rod.)



Free-body diagram of hanger rod.

SOLUTION

A free-body diagram of the rod is shown. The solid rod has a circular cross section, and its area is computed as

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(14\text{ mm})^2 = 153.938\,04\text{ mm}^2$$

where d = rod diameter.

Since the force in the rod is 21 kN, the normal stress in the rod can be computed as

$$\sigma = \frac{F}{A} = \frac{(21\text{ kN})(1000\text{ N/kN})}{153.938\,04\text{ mm}^2} = 136.418\,52\text{ MPa}$$

Although this answer is numerically correct, it would not be proper to report a stress of 136.418 52 MPa as the final answer. A number with this many digits implies an accuracy that we have no right to claim. In this instance, both the rod diameter and the force are given with only two significant digits of accuracy; however, the stress value we have computed here has 8 significant digits.

In engineering, it is customary to round the final answers to three significant digits (if the first digit is not 1) or four significant digits (if the first digit is 1). Using this guideline, the normal stress in the rod would be reported as:

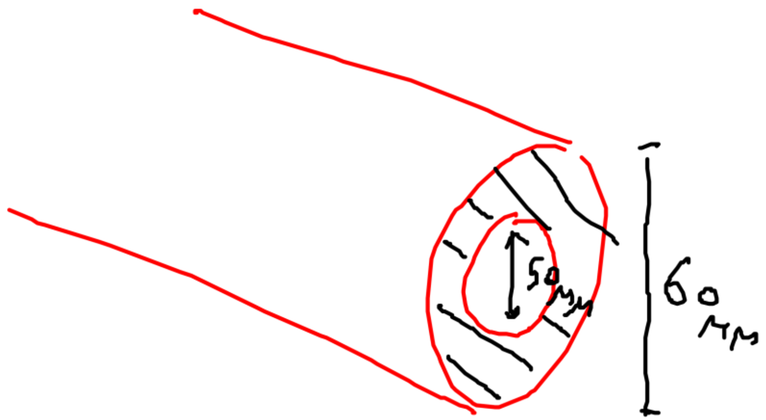
$$\sigma = 136.4\text{ MPa} \qquad \textbf{Ans.}$$

$F = 21\text{ kN}$

$d = 14\text{ mm}$

$\sigma = \dfrac{21 * 10^3}{\frac{\pi}{4}(14)^2} = 136{,}4\text{ MPa}$

P1.1 A stainless steel tube with an outside diameter of 60 mm and a wall thickness of 5 mm is used as a compression member. If the axial normal stress in the member must be limited to 200 MPa, determine the maximum load P that the member can support.

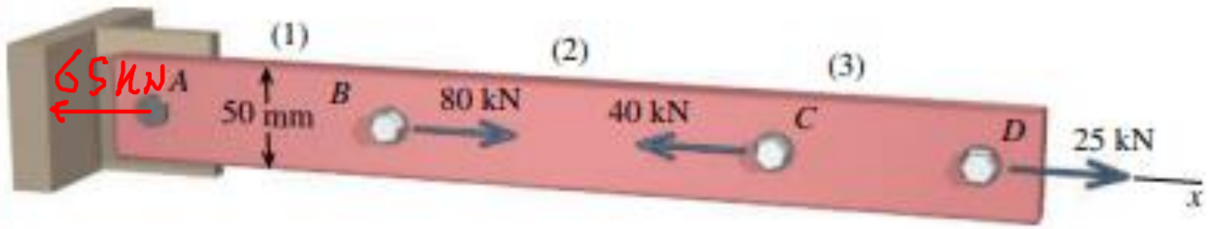


$$\sigma = P/A$$

$$200 = \frac{P}{\frac{\pi}{4}\left[60^2 - 50^2\right]}$$

$$P = 172788\,N$$

EXAMPLE 1.3



A 50 mm wide steel bar has axial loads applied at points *B*, *C*, and *D*. If the normal stress magnitude in the bar must not exceed 60 MPa, determine the minimum thickness that can be used for the bar.

Area = 50t

(1) 65KN شد

(2) 15 KN ضغط

(3) 25KN شد

$$\sigma = \frac{F}{A}$$

$$60 = \frac{65 * 10^3}{50\,t}$$

$$t = 21{,}7 \text{ mm}$$

1.4 ✓ Two solid cylindrical rods (1) and (2) are joined together at flange B and subjected to loads of $P = 44$ kN and $Q = 27$ kN, as shown in Figure P1.3/4. The diameter of rod (1) is 15 mm and the diameter of rod (2) is 30 mm. Determine the normal stresses in rods (1) and (2).

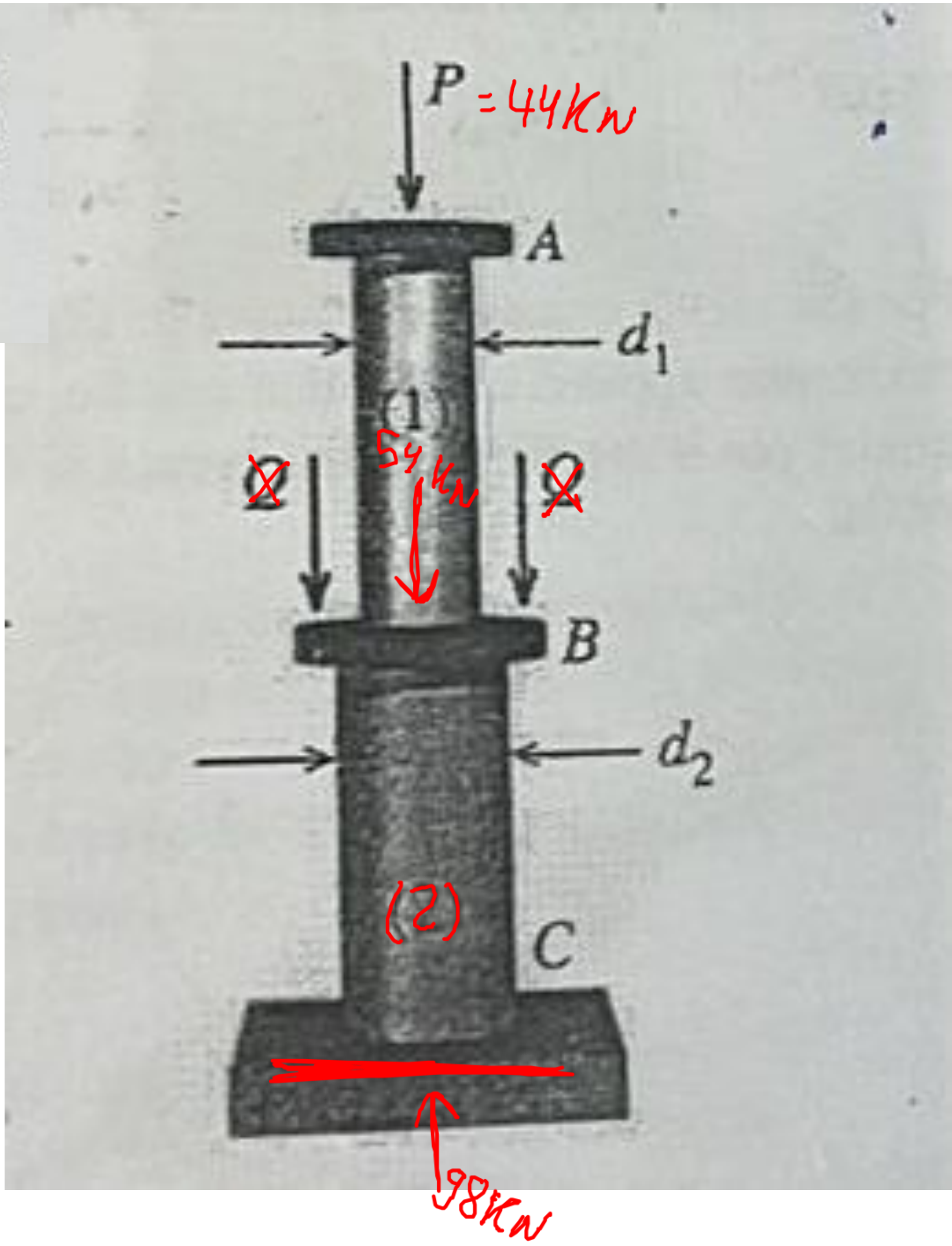
$d_1 = 15$ MM, $d_2 = 30$ MM

(1) 44 KN ضغط

(2) 98 KN ضغط

$$\sigma_1 = \frac{44 * 10^3}{\frac{\pi}{4}(15)^2} = 248,98 \text{ MPa}$$

$$\sigma_2 = \frac{98 * 10^3}{\frac{\pi}{4}(30)^2} = 138,64 \text{ MPa}$$



P1.3 Two solid cylindrical rods (1) and (2) are joined together at flange B and loaded with loads of $P = 70$ kN and $Q = 50$ kN as shown in Figure P1.3/4. If the normal stress in each rod must be limited to 210 MPa, determine the minimum diameter required for each rod.

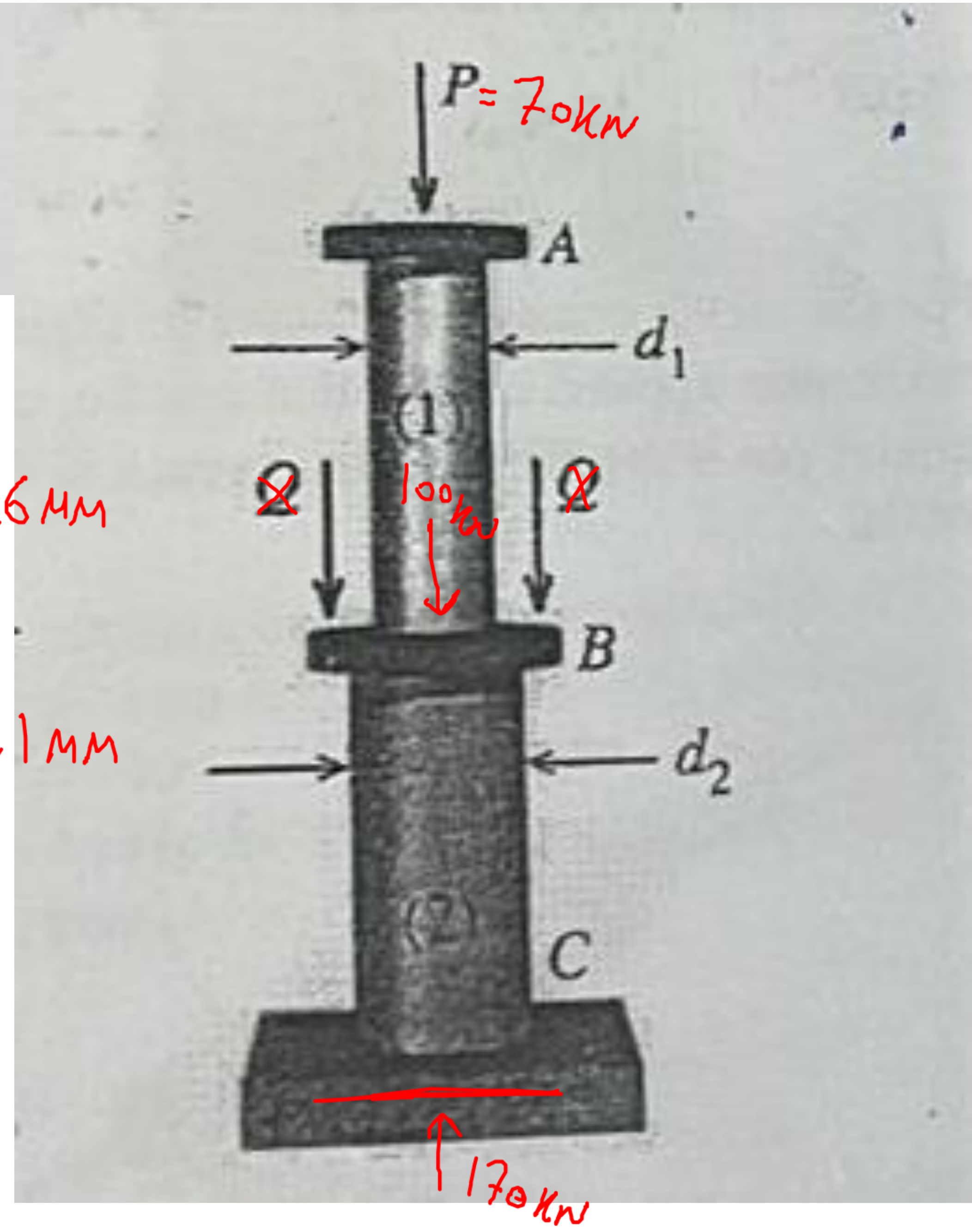
$\sigma = 210 MPa$

(1) 70kN ضغط

(2) 170kN ضغط

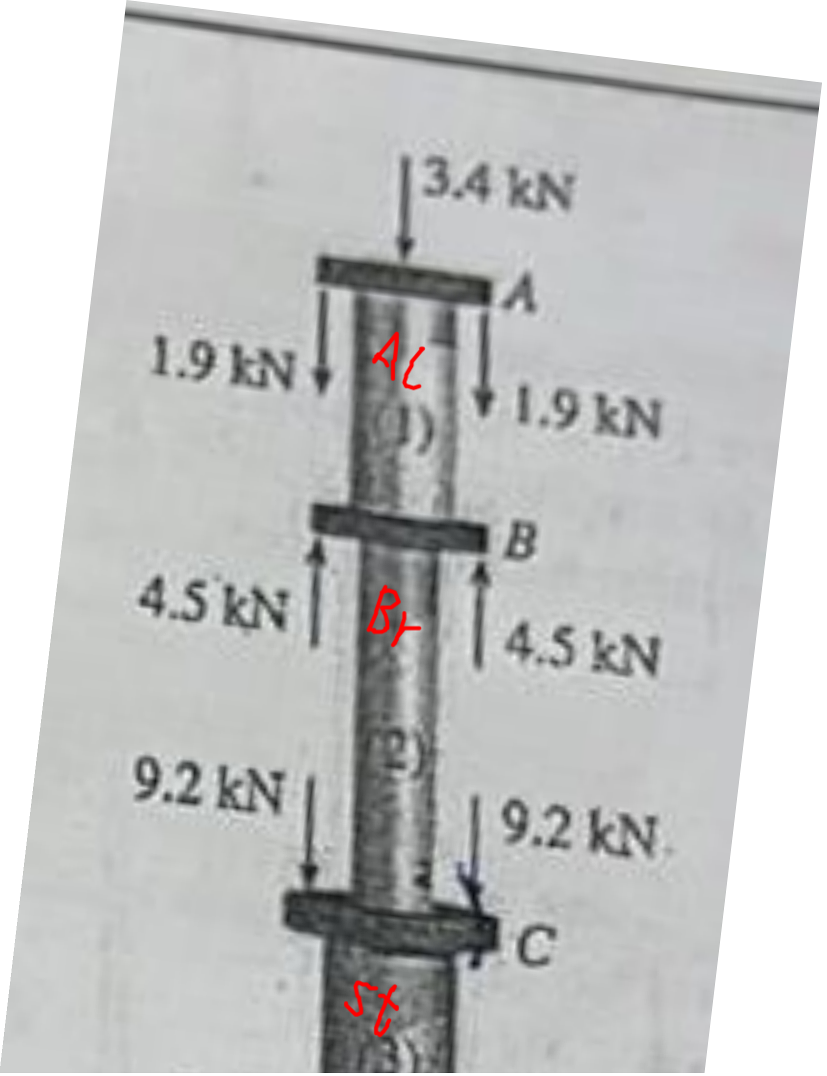
$$210 = \frac{70*10^3}{\frac{\pi}{4} d_1^2} \qquad d_1 = \sqrt{\frac{70*10^3}{210*\frac{\pi}{4}}} = 20{,}6 \text{ mm}$$

$$210 = \frac{170*10^3}{\frac{\pi}{4} d_2^2} \qquad d_2 = \sqrt{\frac{170*10^3}{210*\frac{\pi}{4}}} = 32{,}1 \text{ mm}$$



P1.5 Axial loads are applied with rigid bearing plates to the solid cylindrical rods shown in Figure P1.5/6. The diameter of aluminum rod (1) is 9 mm, the diameter of brass rod (2) is 6 mm, and the diameter of steel rod (3) is 11 mm. Determine the normal stress in each of the three rods.

$d_1 = 9\,mm$, $d_2 = 6\,mm$, $d_3 = 11\,mm$



(1) 7,2 kN ضغط

(2) 1,8 kN شد

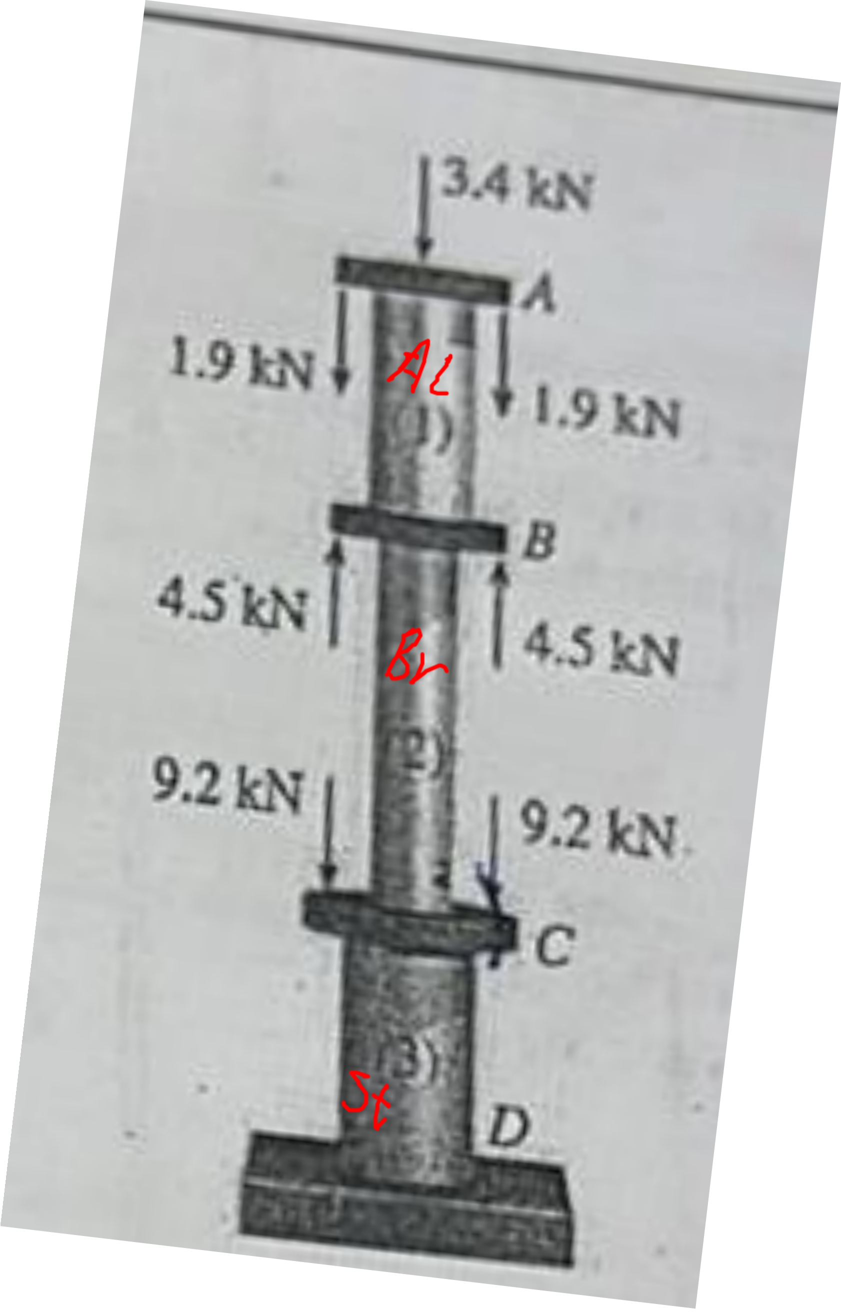
(3) 16,6 kN ضغط

$$\sigma_1 = \frac{7{,}2 * 10^3}{\frac{\pi}{4}(9)^2} = 113{,}17\,MPa$$

$$\sigma_2 = \frac{1{,}8 * 10^3}{\frac{\pi}{4}(6)^2} = 63{,}66\,MPa$$

$$\sigma_3 = \frac{16{,}6 * 10^3}{\frac{\pi}{4}(11)^2} = 174{,}67\,MPa$$

P1.6 Axial loads are applied with rigid bearing plates to the solid cylindrical rods shown in Figure P1.5/6. The normal stress in aluminum rod (1) must be limited to 125 MPa, the normal stress in brass rod (2) must be limited to 140 MPa, and the normal stress in steel rod (3) must be limited to 175 MPa. Determine the minimum diameter required for each of the three rods.



$\sigma_1 = 125\,MPa$; $\sigma_2 = 140\,MPa$, $\sigma_3 = 175\,MPa$

(1) 7,2 kN ضغط

(2) 1,8 kN شد

(3) 16,6 kN ضغط

$$125 = \frac{7{,}2 * 10^3}{\frac{\pi}{4} d_1^2}$$

$d_1 = 8{,}56\,mm$

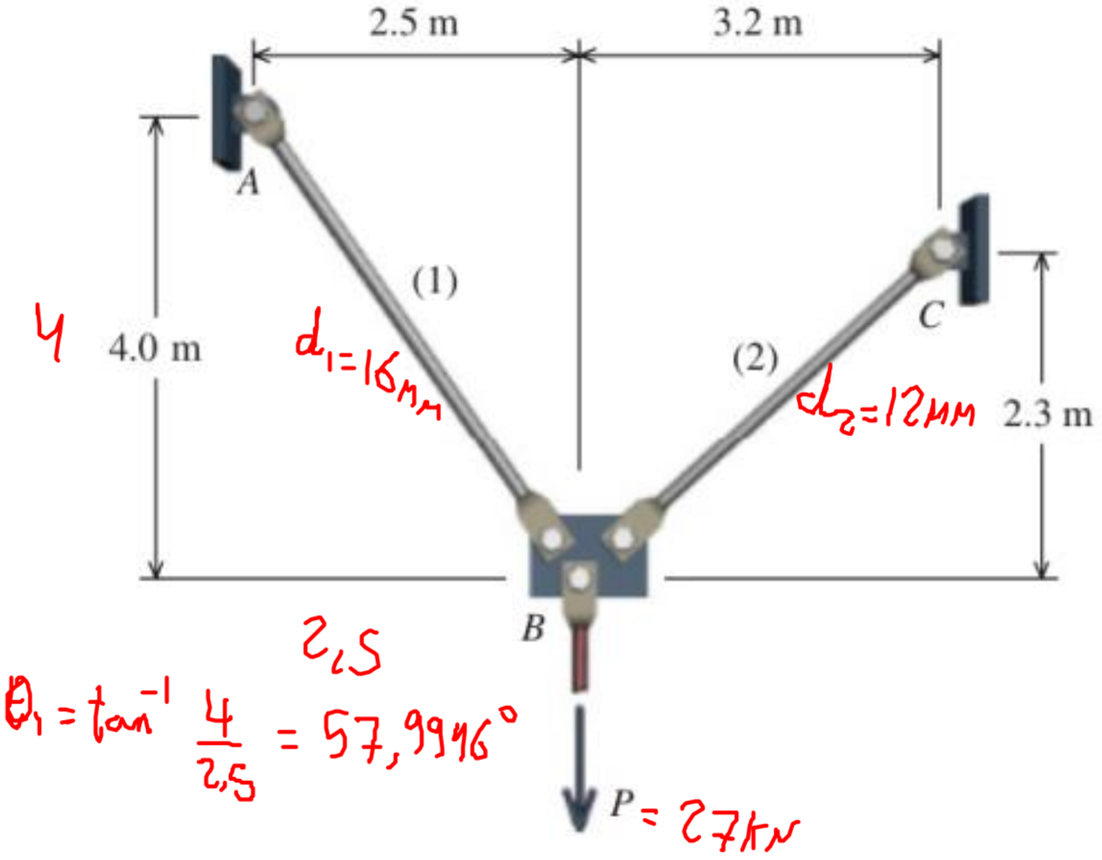
$$140 = \frac{1{,}8 * 10^3}{\frac{\pi}{4} d_2^2}$$

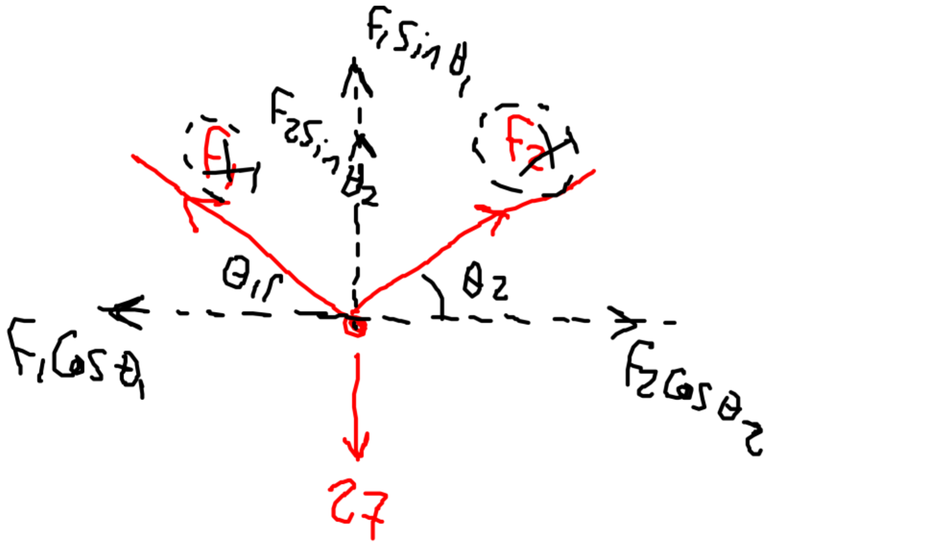
$d_2 = 4\,mm$

$$175 = \frac{16{,}6 * 10^3}{\frac{\pi}{4} d_3^2}$$

$d_3 = 10{,}98\,mm$

P1.8 Two solid cylindrical rods support a load of $P = 27$ kN, as shown in Figure P1.7/8. Rod (1) has a diameter of 16 mm, and the diameter of rod (2) is 12 mm. Determine the normal stress in each rod.





$$\theta_1 = \tan^{-1}\frac{4}{2{,}5} = 57{,}9946^\circ$$

$$\theta_2 = \tan^{-1}\frac{2{,}3}{3{,}2} = 35{,}7067^\circ$$

$\Sigma f_x = \text{zero}$

$$F_2 \cos(35{,}7067) - F_1 \cos(57{,}9946) = \text{zero}$$

$$F_2 = \frac{\cos(57{,}9946)}{\cos(35{,}7067)} F_1$$

$$\boxed{F_2 = 0{,}6526 F_1}$$

$\Sigma f_y = \text{zero}$

$$F_1 \sin(57{,}9946) + 0{,}6526 F_1 \sin(35{,}7067) - 27 = \text{zero}$$

$$F_1 = \frac{27}{\sin(57{,}9946 + 0{,}6526 * \sin(35{,}7067)} = 21{,}97 \text{ kN}$$

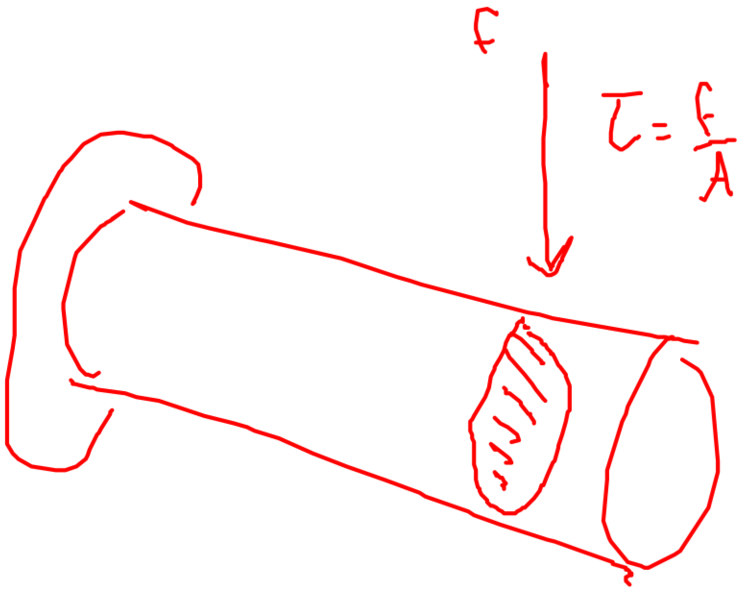
$$F_2 = 0{,}6526 * 21{,}97 = 14{,}34 \text{ kN}$$

$$\sigma_1 = \frac{21{,}97*10^3}{\frac{\pi}{4}(16)^2} = 109{,}26 \text{ MPa}$$

$$\sigma_2 = \frac{14{,}34*10^3}{\frac{\pi}{4}(12)^2} = 126{,}79 \text{ MPa}$$

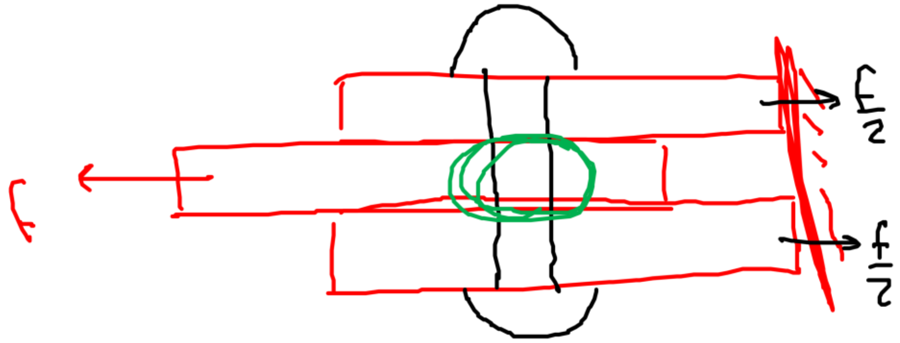
إجهاد القص

* Shear Stress (τ)



$$\tau = \frac{F}{A}$$

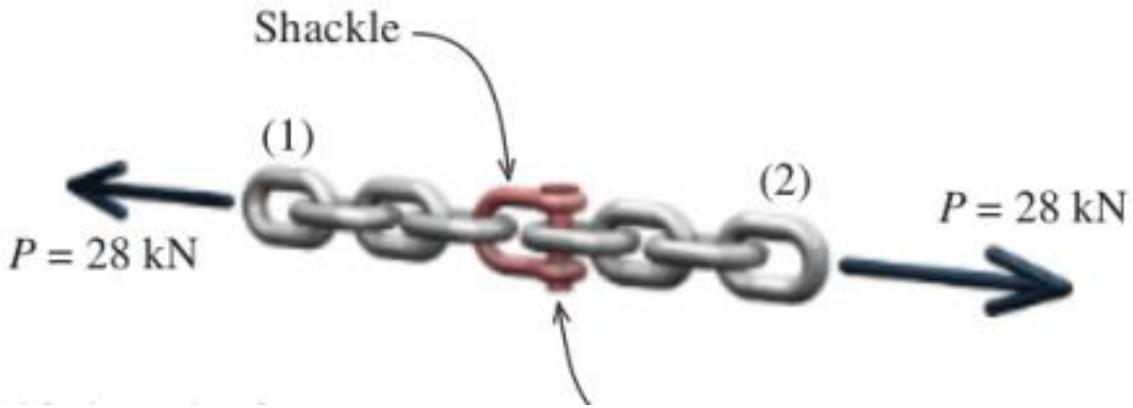
$$\tau = \frac{F}{A} \quad (\text{Single shear})$$

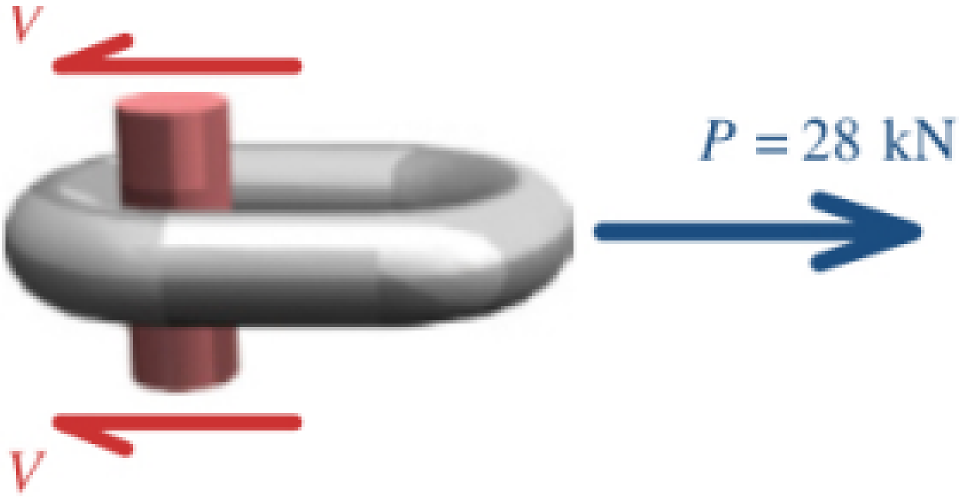


$$\tau = \frac{F}{2A} \quad (\text{double shear})$$

EXAMPLE 1.4

Chain members (1) and (2) are connected by a shackle and pin. If the axial force in the chains is $P = 28$ kN and the allowable shear stress in the pin is $\tau_{allow} = 90$ MPa, determine the minimum acceptable diameter d for the pin.





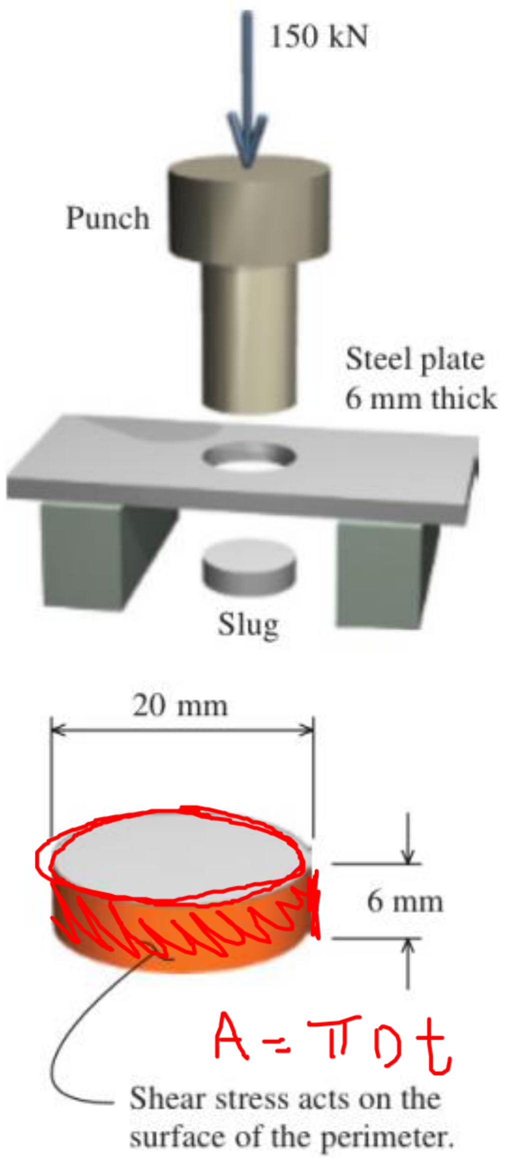
$$\tau = \frac{F}{2A}$$

$$90 = \frac{28 * 10^3}{2 * \frac{\pi}{4} (d)^2}$$

$$d = \sqrt{\frac{28 * 10^3}{2 * 90 * \frac{\pi}{4}}} = 14,07 \text{ mm}$$

$\hookrightarrow \simeq \underline{\underline{15 \text{ mm}}}$

EXAMPLE 1.6



A punch for making holes in steel plates is shown. A downward punching force of 150 kN is required to punch a 20 mm diameter hole in a steel plate that is 6 mm thick. Determine the average shear stress in the steel plate at the instant when the circular slug was torn away from the steel plate.

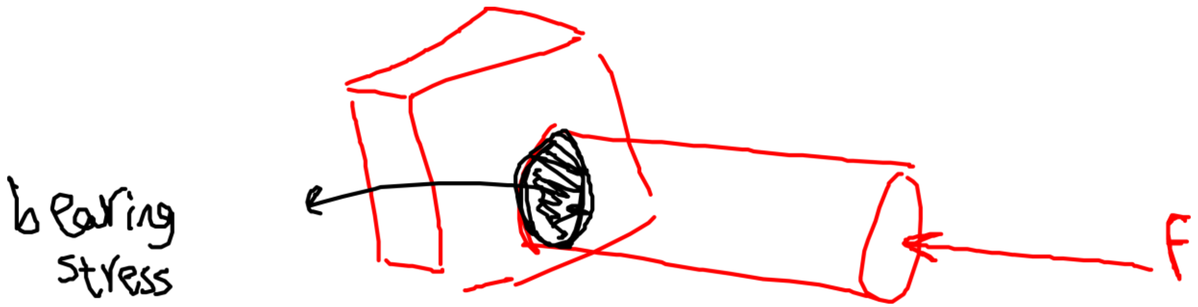
$$\tau = \frac{f}{A} = \frac{150 * 10^3}{\pi(20 * 6)} = 397{,}9\,\text{MPa}$$

1.4 Bearing Stress

A third type of stress, **bearing stress**, is actually a special category of normal stress. Bearing stresses are compressive normal stresses that occur on the surface of contact *between two separate interacting members*. This type of normal stress is defined in the same manner as normal and shear stresses (i.e., force per unit area); therefore, the average bearing stress σ_b is expressed as

$$\boxed{\sigma_b = \frac{F}{A_b}}$$

where A_b = area of contact between the two components.

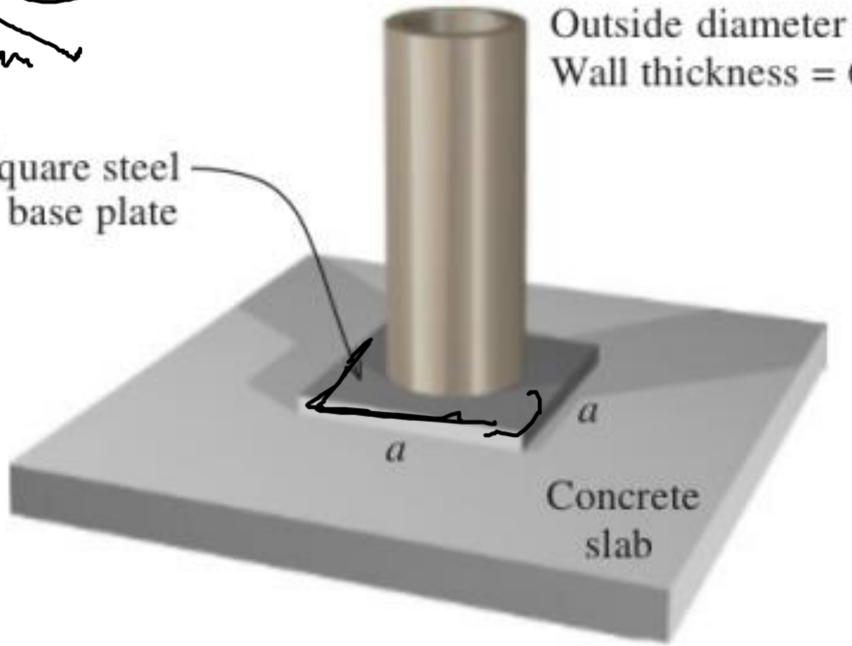


EXAMPLE 1.7

A steel pipe column (175 mm outside diameter; 6.5 mm wall thickness) supports a load of 54 kN. The steel pipe rests on a square steel base plate, which in turn rests on a concrete slab.

(a) Determine the bearing stress between the steel pipe and the steel plate.

(b) If the bearing stress of the steel base plate on the concrete slab must be limited to 0.65 MPa, what is the minimum allowable base plate dimension a?

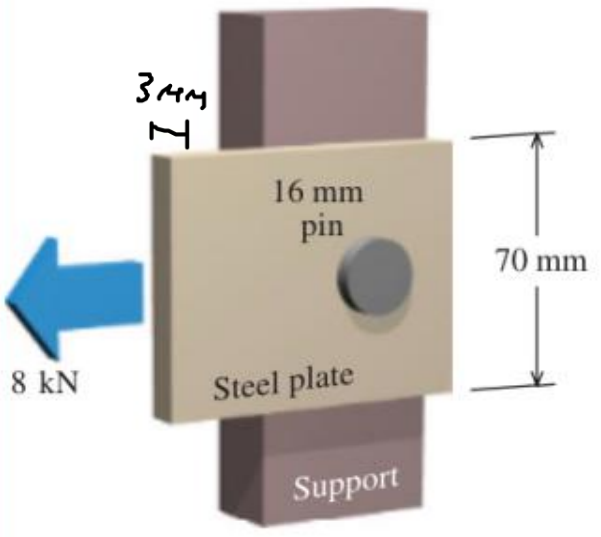


a) $$\sigma_b = \frac{54 * 10^3}{\frac{\pi}{4}\left[175^2 - 162^2\right]} = 15,69 \text{ MPa}$$

b) $$0,65 = \frac{54 * 10^3}{a^2}$$

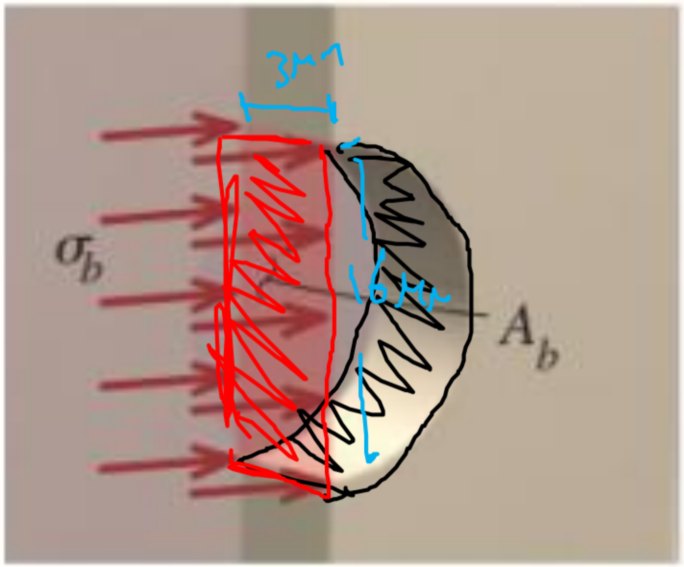
$$a = \sqrt{\frac{54 * 10^3}{0,65}} = 288 \text{ MM}$$

EXAMPLE 1.8



A 70 mm wide by 3 mm thick steel plate is connected to a support with a 16 mm diameter pin. The steel plate carries an axial load of 8 kN. Determine the bearing stress in the steel plate.

$$\sigma_b = \frac{8*10^3}{16*3} = 166.7\ \text{MPa}$$



* دائماً في ال bearing stress نتعامل في المساحة
مع ال flat surface

P1.20 An axial load P is supported by the short steel column shown in Figure P1.20. The column has a cross-sectional area of 14 500 mm^2. If the average normal stress in the steel column must not exceed 75 MPa, determine the minimum required dimension a so that the bearing stress between the base plate and the concrete slab does not exceed 8 MPa. Assume $b = 420$ mm.

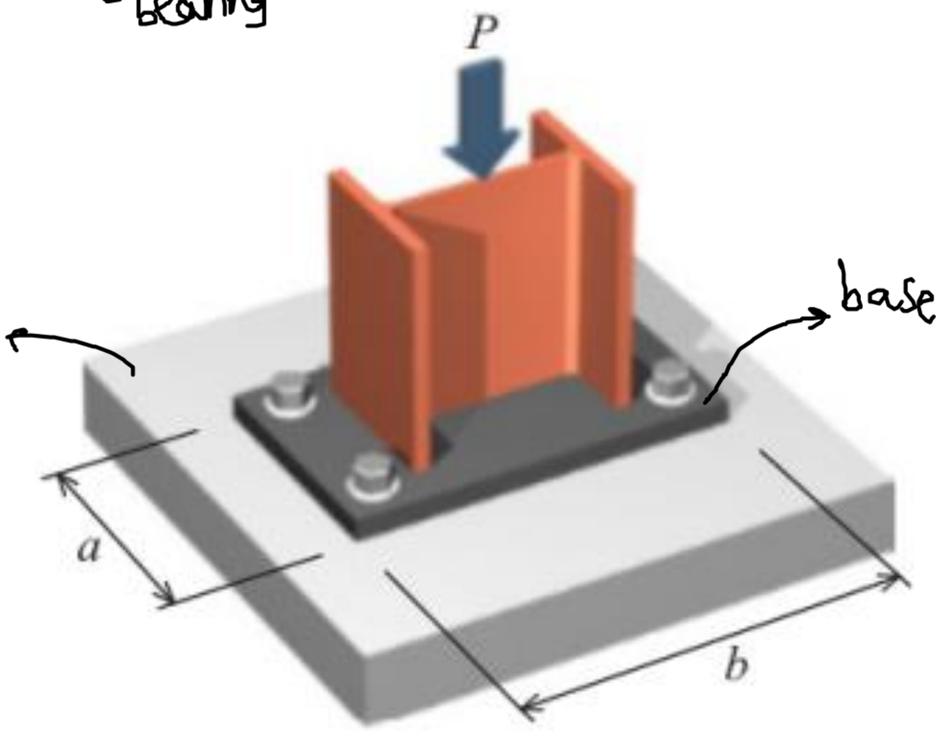
$\sigma_{bearing}$

$A = 14500\ MM^2$

$\sigma = 75\ MPa$

Concrete Slap

base Plate



For I Column

$$\sigma = \frac{P}{A} \qquad 75 = \frac{P}{14500}$$

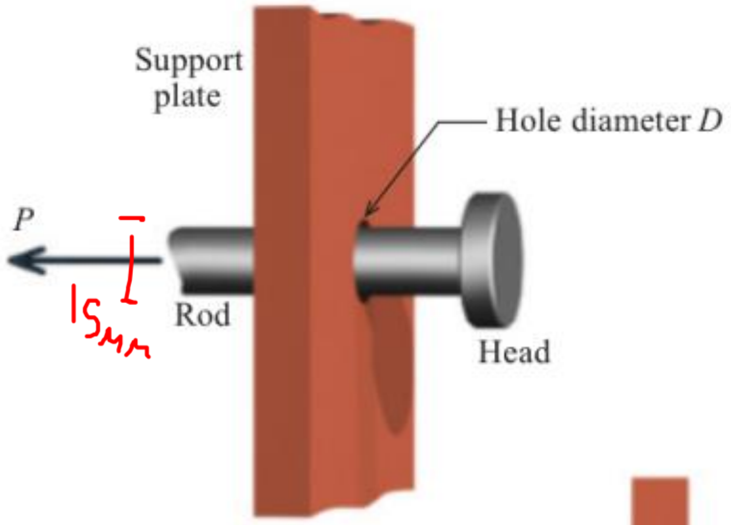
$$P = 1087500\ N$$

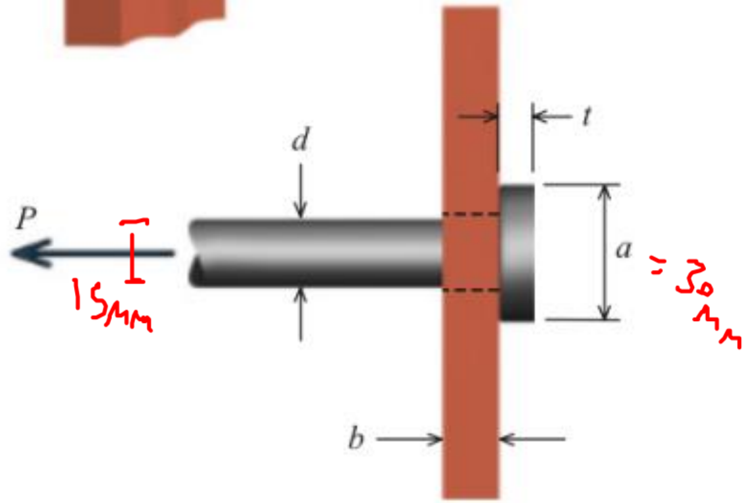
$$\sigma_b = \frac{P}{\underset{a*b}{A}} \qquad 8 = \frac{1087500}{a*420}$$

$$a = \frac{1087500}{8*120} = \boxed{324\ MM}$$

P1.23 The solid rod (d = 15 mm) shown in Figure P1.23 passes through a hole (D = 20 mm) in the support plate. When a load P is applied to the rod, the rod head rests on the support plate. The support plate has a thickness of b = 12 mm. The rod head has a diameter of a = 30 mm, and the head has a thickness of t = 10 mm. If the normal stress produced in the rod by load P is 225 MPa, determine

(a) the bearing stress acting between the support plate and the rod head.
(b) the average shear stress produced in the rod head.
(c) the punching shear stress produced in the support plate by the rod head.





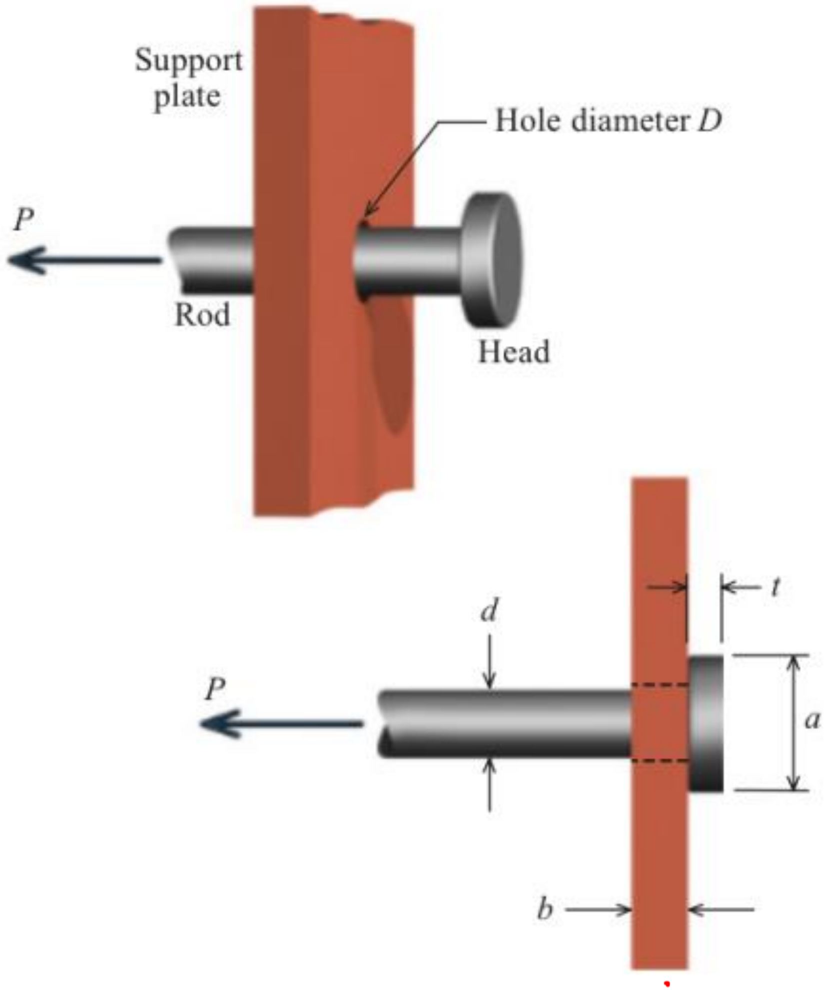
$$\sigma = \frac{P}{A} \qquad 225 = \frac{P}{\frac{\pi}{4}(15)^2} \qquad P = 39760,78\,N$$

a)

$$\sigma_b = \frac{39760,78}{\frac{\pi}{4}\left[30^2 - 20^2\right]} = 101,3\ MPa$$

P1.23 The solid rod (d = 15 mm) shown in Figure P1.23 passes through a hole (D = 20 mm) in the support plate. When a load P is applied to the rod, the rod head rests on the support plate. The support plate has a thickness of b = 12 mm. The rod head has a diameter of a = 30 mm, and the head has a thickness of t = 10 mm. If the normal stress produced in the rod by load P is 225 MPa, determine

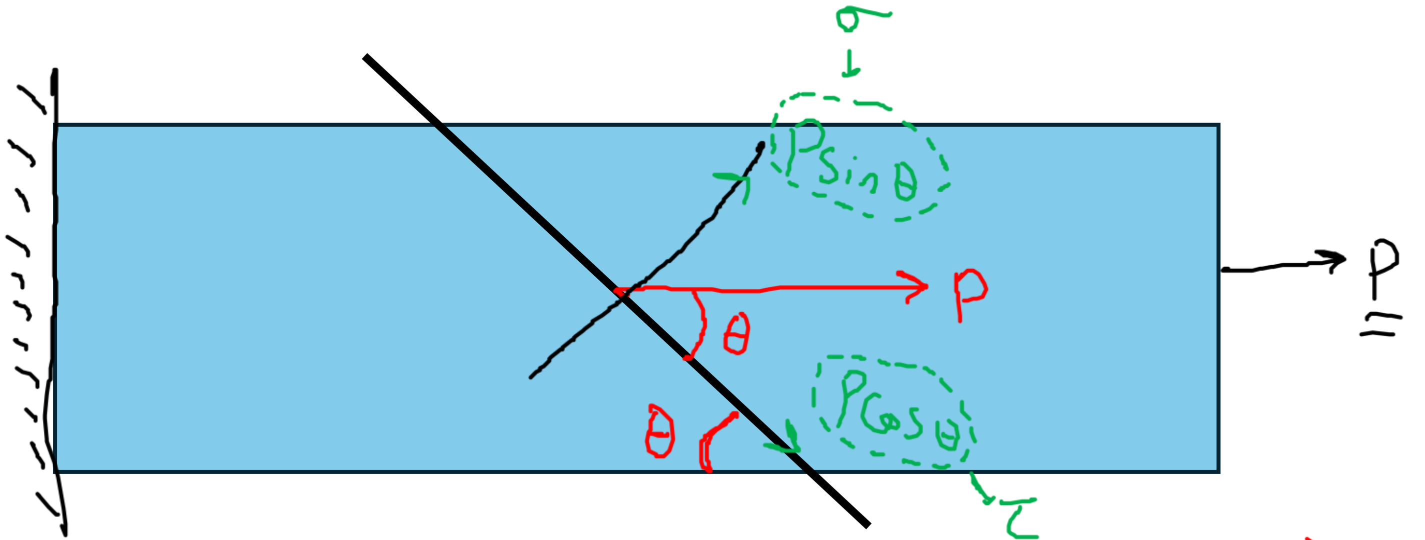
(a) the bearing stress acting between the support plate and the rod head.
(b) the average shear stress produced in the rod head.
(c) the punching shear stress produced in the support plate by the rod head.



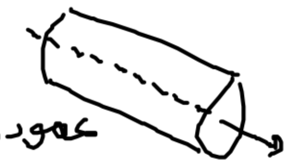
(b) $\tau = \dfrac{39760{,}78}{\pi * 15 * 10} = 84{,}4\ \text{MPa}$

(c) $\tau = \dfrac{39760{,}78}{\pi * 30 * 12} = 35{,}2\ \text{MPa}$

* Stress on inclined surface

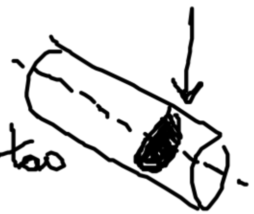


$$\sigma = \frac{F}{A}$$



عمودي على المساحه
موازيه للمحور

$$\tau = \frac{F}{A}$$



مماسيه للمساحه
عموديه على المحور

$$A' = \frac{A}{\sin\theta}$$

$$\sigma_\theta = \frac{P\sin\theta}{A'} = \frac{P\sin\theta}{A/\sin\theta} = \frac{P}{A}\sin^2\theta$$

$$\sigma_\theta = \frac{P}{2A}(1-\cos 2\theta)$$

$$\tau_\theta = \frac{P\cos\theta}{A'} = \frac{P\cos\theta}{A/\sin\theta} = \frac{P}{A}\sin\theta\cos\theta$$

$$\tau_\theta = \frac{P}{2A}\sin 2\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

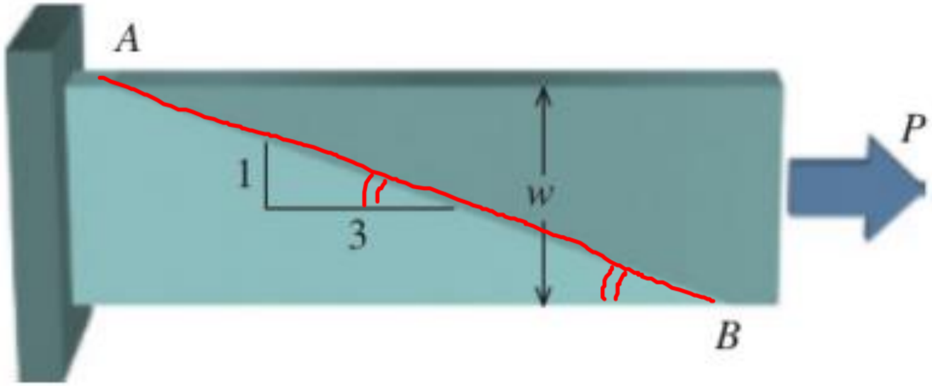
$$\sin\theta\cos\theta = \frac{\sin(2\theta)}{2}$$

P1.35 A rectangular bar having width w = 36 mm and thickness t = 8 mm is subjected to a tension load P as shown in Figure P1.35/36. The normal and shear stresses on plane AB must not exceed 140 MPa and 55 MPa, respectively. Determine the maximum load P that can be applied without exceeding either stress limit.

$\tan(\theta) = \frac{1}{3}$

$\boxed{\theta = 18{,}4349°}$

$\sigma = 140\,MPa$, $\tau = 55\,MPa$, $A = 36 * 8\ mm^2$



$$\sigma_\theta = \frac{P}{2A}\left[1 - \cos 2\theta\right]$$

$$140 = \frac{P}{2 * 36 * 8}\left[1 - \cos(2 * 18{,}4349)\right]$$

$P = 403\,202\ N$

$$\tau_\theta = \frac{P}{2A}\sin 2\theta$$

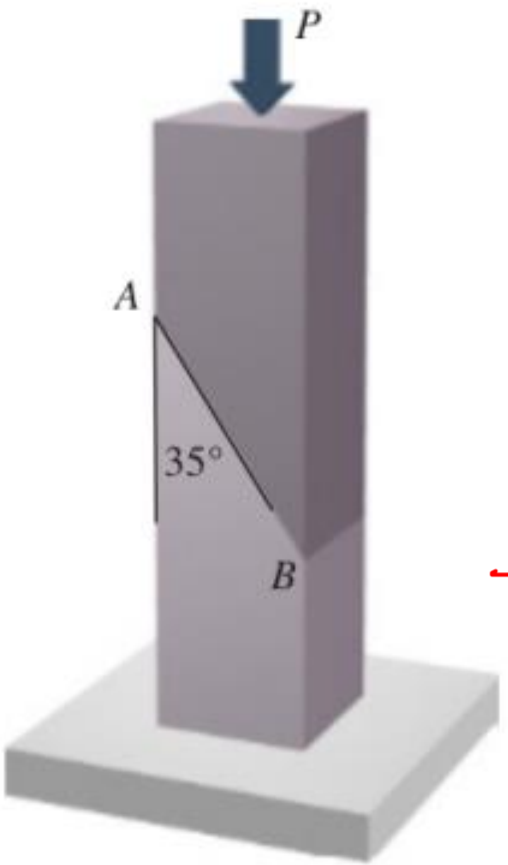
$$55 = \frac{P}{2 * 36 * 8}\sin(2 * 18{,}4349)$$

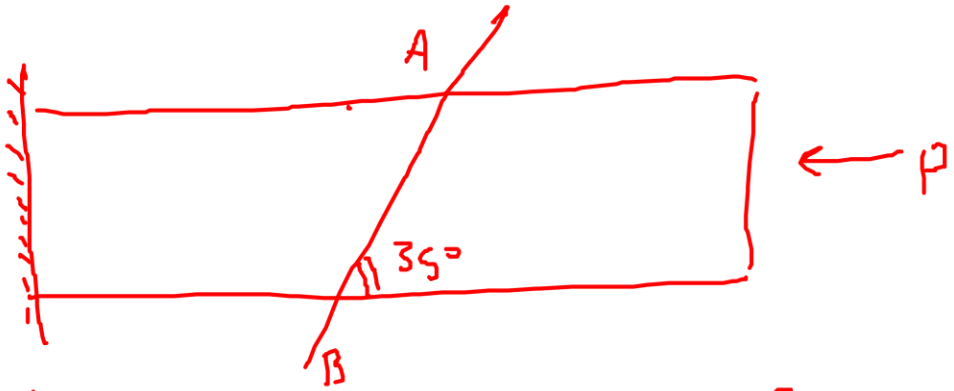
$\boxed{P = 52800\ N}$ ⇒ اختار القيمة الأصغر

P1.33 Specifications for the 50 mm by 50 mm square bar shown in Figure P1.33 require that the normal and shear stresses on plane *AB* not exceed 120 MPa and 90 MPa, respectively. Determine the maximum load *P* that can be applied without exceeding the specifications.

$\sigma = 120\,MPa$

$\tau = 90\,MPa$





$$\sigma = \frac{P}{2A}\left[1 - \cos 2\theta\right] \qquad \frac{P}{2*50*50}\left[1-\cos(2*35)\right] = 120$$

$$P = 91188\,2N$$

$$\tau = \frac{P}{2A}\sin 2\theta \qquad \frac{P}{2*50*50}\sin(2*35) = 90$$

$$P = 478880\,N$$

اختار القيمة → الأصغر

P1.37 The rectangular bar has a width of $w = 100$ mm and a thickness of $t = 75$ mm. The shear stress on plane AB of the rectangular block shown in Figure P1.37 is 12 MPa when the load P is applied. Determine

(a) the magnitude of load P.
(b) the normal stress on plane AB.
(c) the maximum normal and shear stresses in the block at any possible orientation.

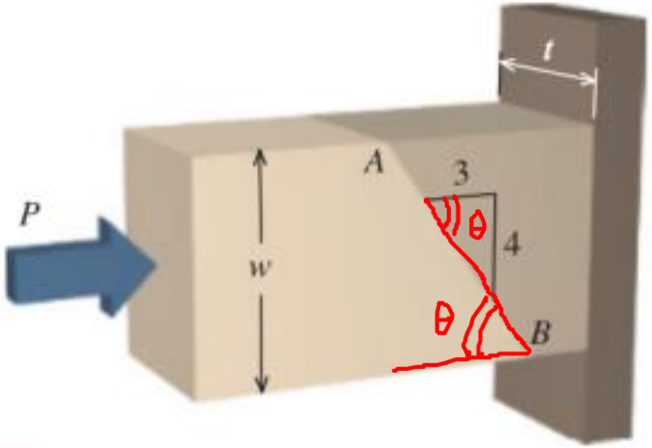
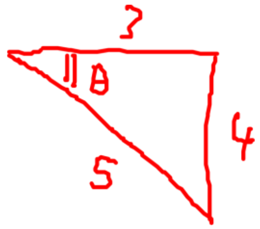


FIGURE P1.37



$\theta = \tan^{-1} \frac{4}{3}$

$\theta = 53{,}13°$

a) $Area = 100 * 75 = 7500\,mm^2$

$\tau_\theta = 12\,MPa$

$\tau_\theta = \frac{P}{2A} \sin 2\theta$

$12 = \frac{P}{2 * 7500} \sin(2 * 53{,}13)$

$P = \frac{12 * 2 * 7500}{\sin(2 * 53{,}13)} = \boxed{187499{,}8\,N}$

P1.37 The rectangular bar has a width of w = 100 mm and a thickness of t = 75 mm. The shear stress on plane AB of the rectangular block shown in Figure P1.37 is 12 MPa when the load P is applied. Determine

(a) the magnitude of load P.
(b) the normal stress on plane AB.
(c) the maximum normal and shear stresses in the block at any possible orientation.

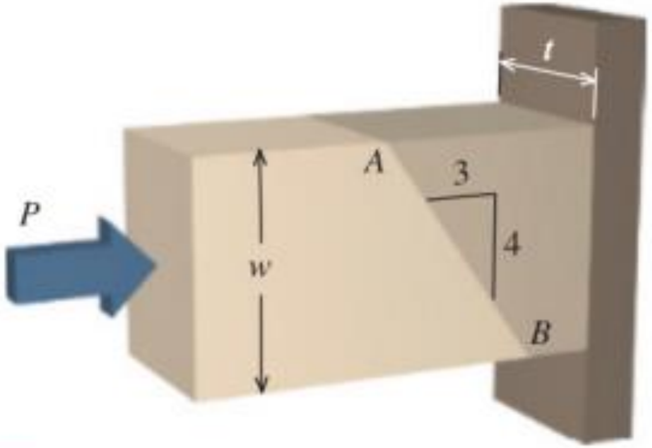
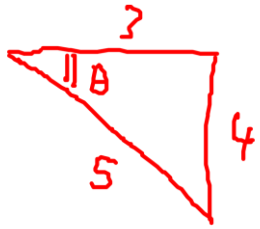


FIGURE P1.37



$\theta = 53{,}13$

b)

$$\sigma = \frac{P}{2A}(1 - \cos 2\theta) = \frac{187499{,}8}{2 * 7500}(1 - \cos 2 * 53{,}13) = 16\ MPa$$

c)

$$\sigma_{Max} = \frac{P}{A} \; ; \; \tau_{Max} = \frac{P}{2A}$$

$$\sigma_{Max} = \frac{187499{,}8}{7500} = 25\ MPa$$

$$\tau_{Max} = \frac{187499{,}8}{2 * 7500} = 12{,}5\ MPa$$

P1.31 A steel rod of circular cross section will be used to carry an axial load of 200 kN. The maximum stresses in the rod must be limited to 210 MPa in tension and 85 MPa in shear. Determine the minimum required diameter for the rod.

$P = 200\,kN$, $\sigma_{Max} = 210\,MPa$; $\tau_{Max} = 85\,MPa$

$$\sigma_{Max} = \frac{P}{A} \qquad 210 = \frac{200 * 10^3}{\frac{\pi}{4}(d)^2} \qquad d = \sqrt{\frac{200 * 10^3}{210 * \frac{\pi}{4}}} = 34{,}822\,mm$$

$$\tau_{Max} = \frac{P}{2A} \qquad 85 = \frac{200 * 10^3}{2 * \frac{\pi}{4}(d)^2} \qquad d = \sqrt{\frac{200 * 10^3}{2 * 85 * \frac{\pi}{4}}} = 38{,}7\,mm$$

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$\simeq 39\,mm$