The car on the left in the photo and in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.

$v = 3t^2 + 2t \;\to\; s,\; a$

$s \xrightarrow{\frac{d}{dt}} v \xrightarrow{\frac{d}{dt}} a$, $\quad a \xrightarrow{\int} v \xrightarrow{\int} s$

① s:

$$\boxed{s - s_0 = \int_{t_0}^{t} v\,dt}$$

$$s - 0 = \int_0^t 3t^2 + 2t\,dt$$

$$s = \frac{3t^3}{3} + \frac{2t^2}{2}\Big|_0^t$$

$$\boxed{s = t^3 + t^2}$$

$$s(3) = 3^3 + 3^2 = 36 \text{ ft}$$

② a:

$$\boxed{a = \frac{dv}{dt}}$$

$$a = \frac{d}{dt}\left[3t^2 + 2t\right]$$

$$\boxed{a = 6t + 2}$$

$$a(3) = 6*3 + 2 = 20 \text{ ft/s}^2$$

ملاحظة

$\underbrace{s \quad v \quad a}_{\text{دوال في } t}$

معطى ① s (دالة في t) $\longrightarrow v = \frac{ds}{dt} \longrightarrow a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

معطى ② v (دالة في t) $\to s$: $\quad s - s_0 = \int_{t_0}^{t} v\,dt$

$\to a$: $\quad a = \frac{dv}{dt}$

معطى ③ $a \longrightarrow v - v_0 = \int_{t_0}^{t} a\,dt \qquad v = \boxed{}$ دالة

$$s - s_0 = \int_{t_0}^{t} v\,dt$$

---

a دالة في s: حالة خاصة

$a = s^2 - 2s \longrightarrow v$??

$$a\,ds = v\,dv$$

$$\int s^2 - 2s\,ds = \int v\,dv \leftarrow$$

12–18. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)$ m/s^2, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.

$\int x\,dx = \frac{x^2}{2}$

$a\,ds = v\,dv$

$$\int_0^s (6 + 0.02s)\,ds = \int_0^v v\,dv$$

$$6s + 0.02\frac{s^2}{2}\Big|_0^s = \frac{v^2}{2}\Big|_0^v$$

$$6s + 0.01 s^2 = \frac{v^2}{2}$$

$$v^2 = 12s + 0.02s^2$$

$$\cancel{v} = \sqrt{12s + 0.02s^2}$$

$$\frac{ds}{dt} = \sqrt{12s + 0.02s^2}$$

$$\frac{1}{\sqrt{12s + 0.02s^2}}\,ds = dt$$

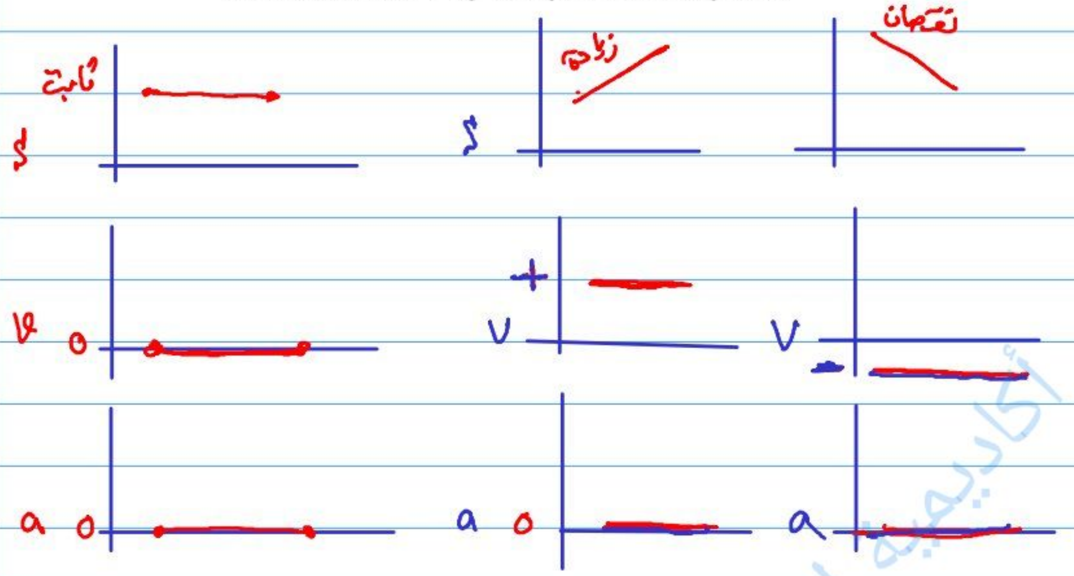
$$\int_0^s \frac{1}{\sqrt{12s + 0.02s^2}}\,ds = \int_0^t dt$$

$$\int_0^{100} \frac{1}{\sqrt{12s + 0.02s^2}}\,ds = t$$

$$\boxed{t = 5.6}$$

1. How to construct s-t, v-t and a-t graphs ?

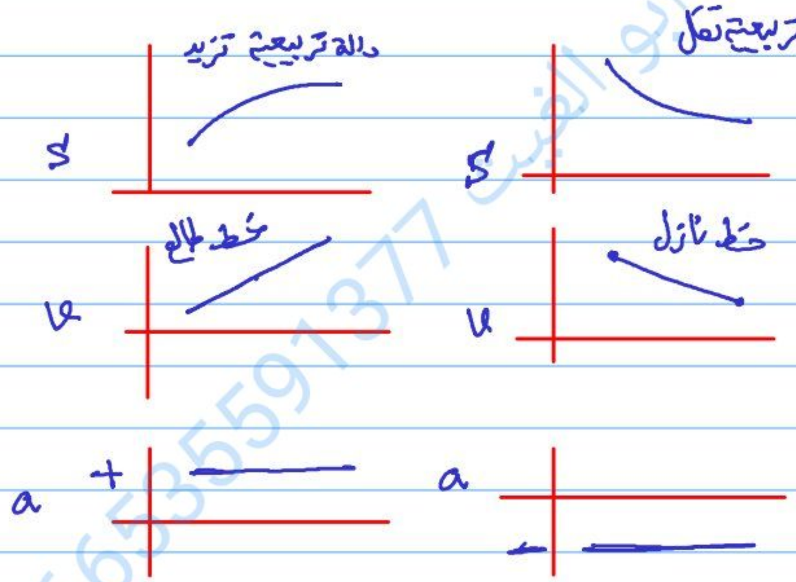
- To construct the v-t graph given the s-t graph.



$s = t^2 + 1$

$v = 2t$

$a = 2$



$s = -t^2 + 1$

$v = -2t$

$$\boxed{s=t^2}$$

$$v=\frac{ds}{dt}$$

$$\boxed{v=2t}$$

$$v=2*10=20$$

$$a=\frac{dv}{dt}$$

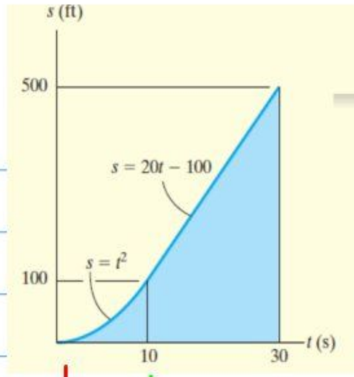
$$\boxed{a=2}$$

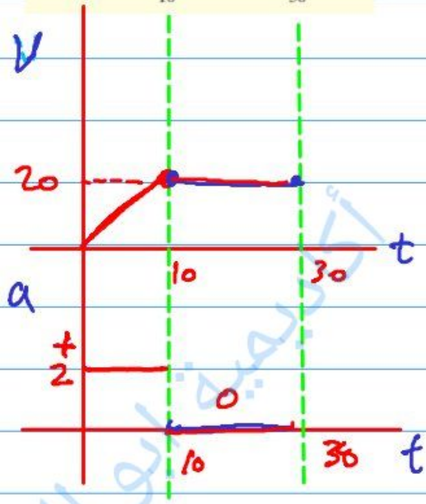
$$s=20t-100$$

$$v=20$$

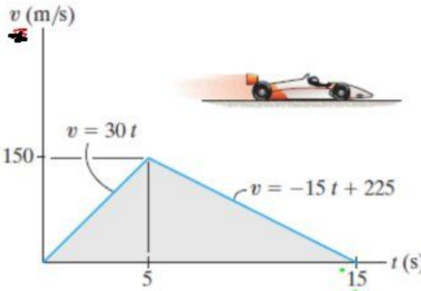
$$a=\frac{dv}{dt}$$

$$a=0$$





F12–14. The dragster starts from rest and has a velocity described by the graph. Construct the s–t graph during the time interval $0 \le t \le 15$ s. Also, determine the total distance traveled during this time interval.

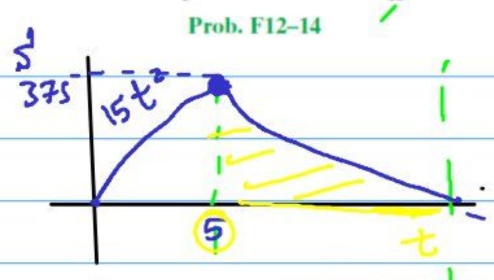


Prob. F12–14

$$s-\cancel{s_0}^{\,0}=\int v\,dt$$

$$s=\int 30t\,dt=15t^2$$

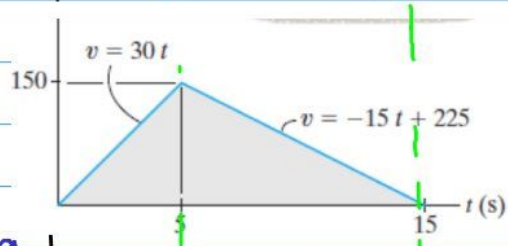
$$s=15(5)^2=375$$

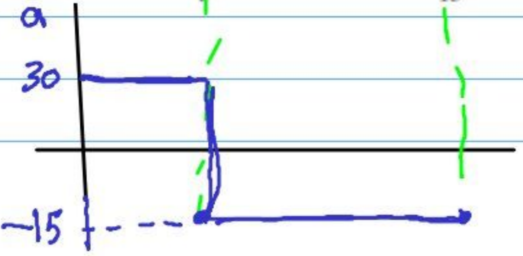


$$s-\cancel{s_0}^{\,375}=\int_5^t -15t+225\,dt$$

$$s-375=\frac{-15t^2}{2}+225t\Big|_5^t$$

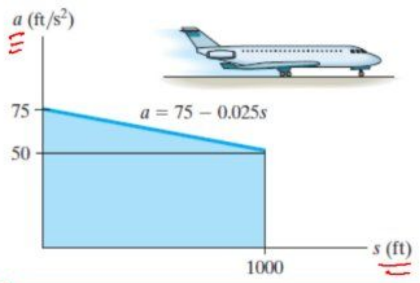
$$s=375-\frac{15}{2}t^2+225t-\left(-\frac{15}{2}(5)^2+225(5)\right)$$





$v = 0 \quad s = 0$

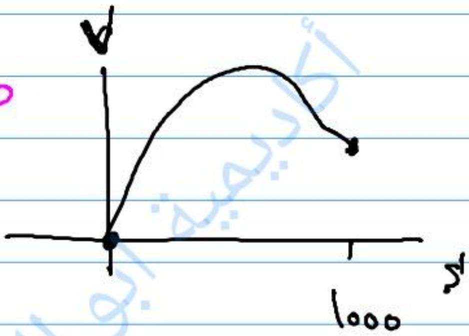
12–65. The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled 1000 ft. Also, how much time is required for it to travel 1000 ft?



$$a\,ds = v\,dv$$

$$\int_{s_0}^{s} 75 - 0.025\,ds = \int_{v_0}^{v} v\,dv$$

$$75s - 0.02s^2 \Big|_{\cancel{s_0}\,0}^{s} = \frac{v^2}{2}\Big|_{\cancel{v_0}\,0}^{v}$$



$$75s - 0.02s^2 = \frac{v^2}{2}$$

$$v^2 = 150s - 0.04s^2$$

$$v = \sqrt{150\underset{1000}{s} - 0.04\underset{1000}{s^2}}$$

$$= \sqrt{150 \times 1000 - 0.04 \times 1000^2} = \checkmark$$

Motion of a Projectile مقذوف

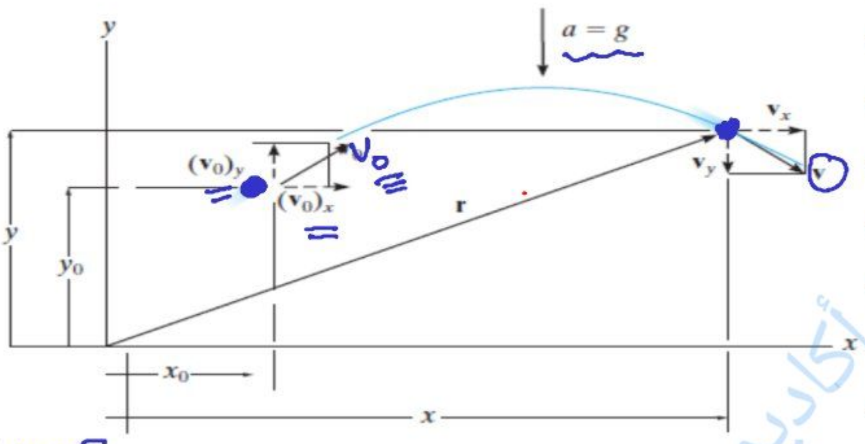


Fig. 12–20

الحركة الأفقية

Horizontal Motion. Since $a_x = 0$, application of the constant acceleration equations, 12–4 to 12–6, yields

$$(\xrightarrow{+}) \quad v = v_0 + \cancel{a_c} t \qquad\qquad v_x = (v_0)_x$$

$$(\xrightarrow{+}) \quad x = x_0 + v_0 t + \tfrac{1}{2}\cancel{a_c} t^2; \qquad\qquad x = x_0 + (v_0)_x t$$

$$(\xrightarrow{+}) \quad v^2 = v_0^2 + \cancel{2a_c}(x - x_0); \qquad\qquad v_x = (v_0)_x$$

The first and last equations indicate that *the horizontal component of velocity always remains constant during the motion.*

الحركة الرأسية

Vertical Motion. Since the positive y axis is directed upward, then $a_y = -g$. Applying Eqs. 12–4 to 12–6, we get

$$(+\uparrow) \quad v = v_0 + \overset{-g}{\cancel{a_c}} t; \qquad\qquad v_y = (v_0)_y - gt$$

$$(+\uparrow) \quad y = y_0 + v_0 t + \tfrac{1}{2}a_c t^2; \qquad\qquad y = y_0 + (v_0)_y t - \tfrac{1}{2}gt^2$$

$$(+\uparrow) \quad v^2 = v_0^2 + 2a_c(y - y_0); \qquad\qquad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$