

دوائر ثلاثه الوجه المتزنه

Balanced Three-phase Circuits

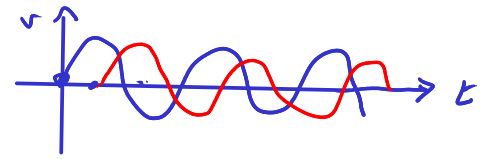
حليل الدوائر الكهربيه

EE 1222 Electrical Circuits Analysis

Dr. Hassan M. Hussein Farh

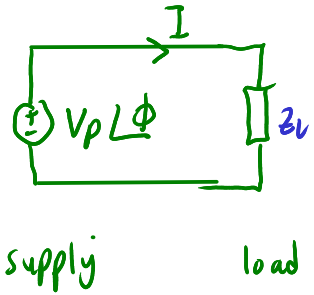
**Electrical Department
IMSIU**

Ac circuits

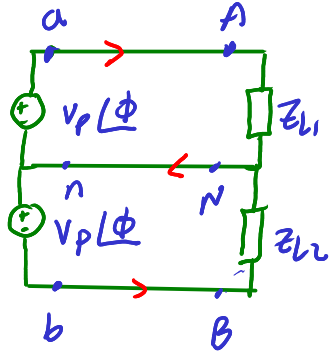


single phase

2 wire



3 wire

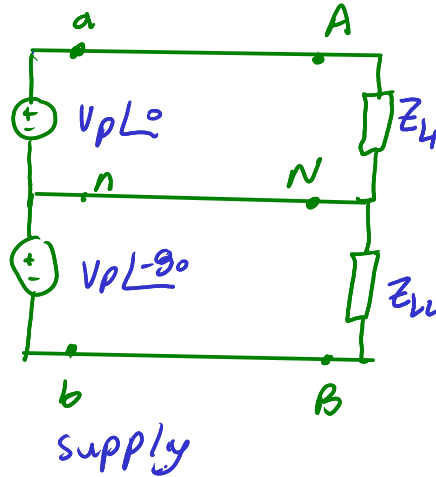


supply

فَسَ فَتَ الْجَمَدِ
وَفَتَ الزَّوْبِ

two phase

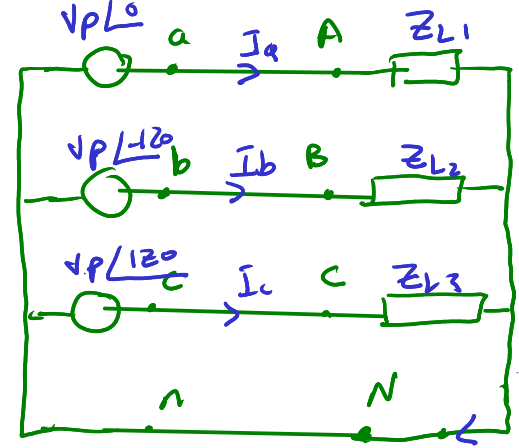
3 wire



فَسَ فَتَ الْجَمَدِ
الزَّوْبِ فَتَ فَتَ

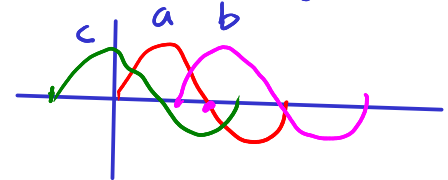
three phase

4 wire



supply

فَسَ فَتَ الْجَمَدِ
الزَّوْبِ فَتَ فَتَ

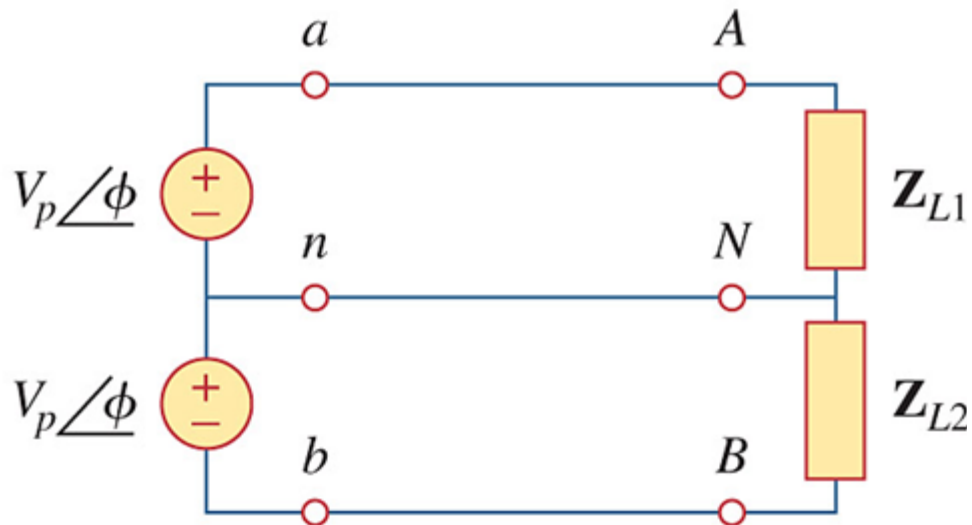


Balanced 3-phase systems



Single-phase two-wire system:

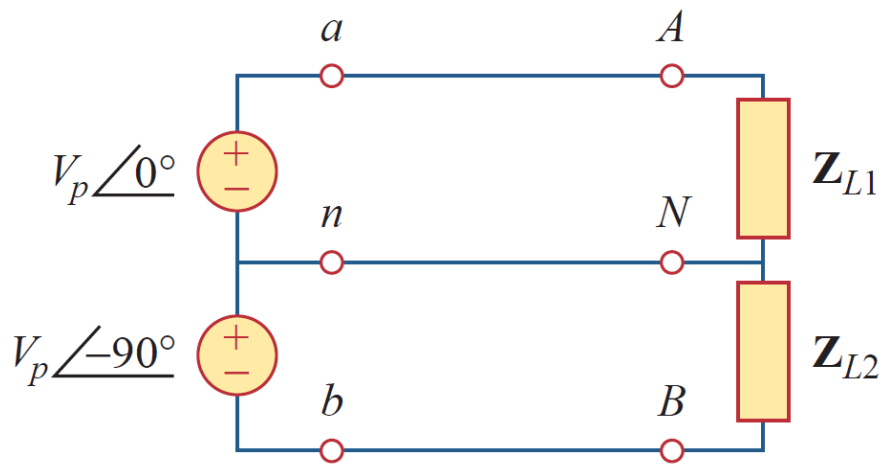
- Single source connected to a load using two-wire system



Single-phase three-wire system:

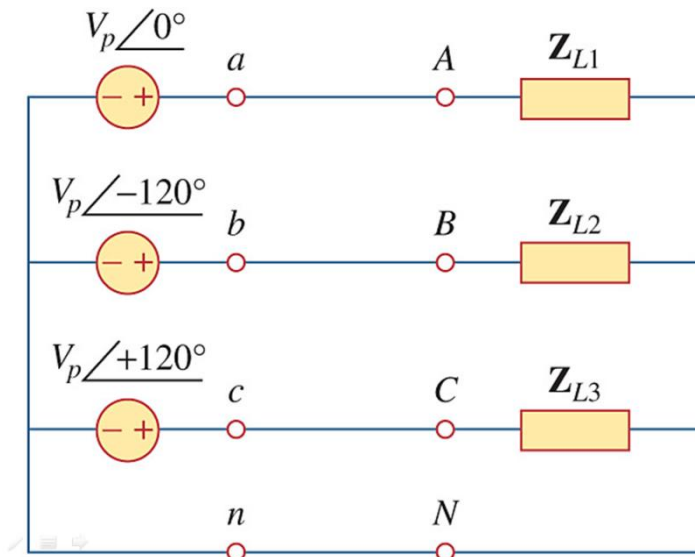
- Two sources connected to two loads using three-wire system
- Sources have EQUAL magnitude and are IN PHASE

Balanced 3-phase systems



Balanced Two-phase three-wire system:

- Two sources connected to two loads using three-wire system
- Sources have EQUAL frequency but DIFFERENT phases



Balanced Three-phase four-wire system:

- Three sources connected to 3 loads using four-wire system
- Sources have EQUAL frequency but DIFFERENT phases

Balanced 3-phase systems

WHY THREE PHASE SYSTEM ?

- ALL electric power system in the world used 3-phase system to GENERATE, TRANSMIT and DISTRIBUTE
- Instantaneous power is constant – thus smoother rotation of electrical machines
- More economical than single phase – less wire for the same power transfer

النظام الأكثر شهرة في الاستخدام هو النظام 3 phase

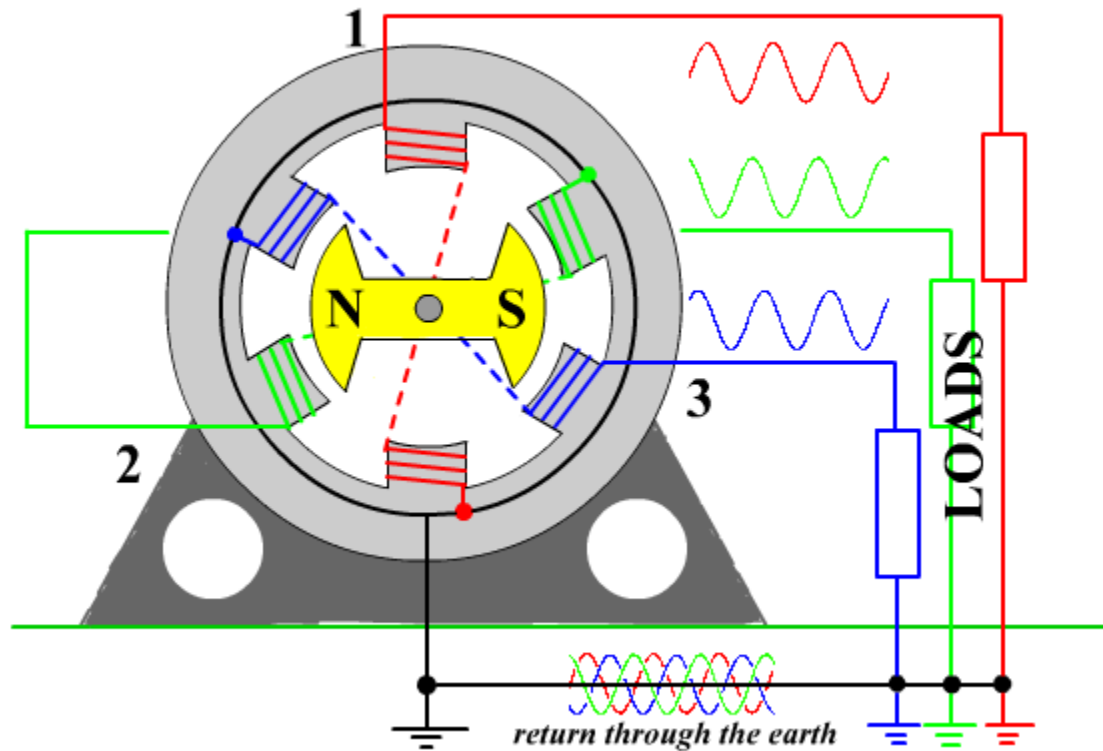
①. التكلفة أقل في حالة (3ph) من (1ph)

②. الدوائر حجمها أصغر في حالة (3ph)

③. القدرة للـ (3ph) متساوية وتأتي

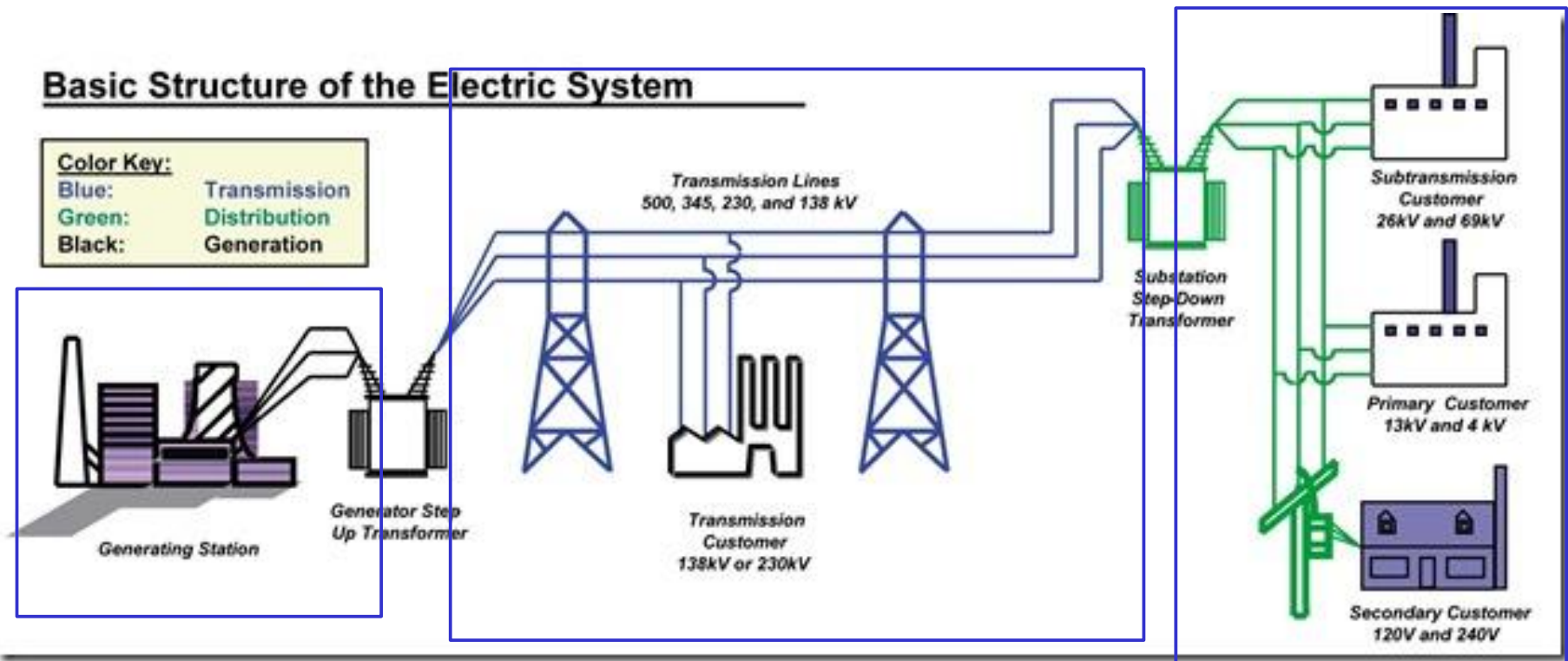
Balanced 3-phase systems

Generation of 3-phase voltage: Generator



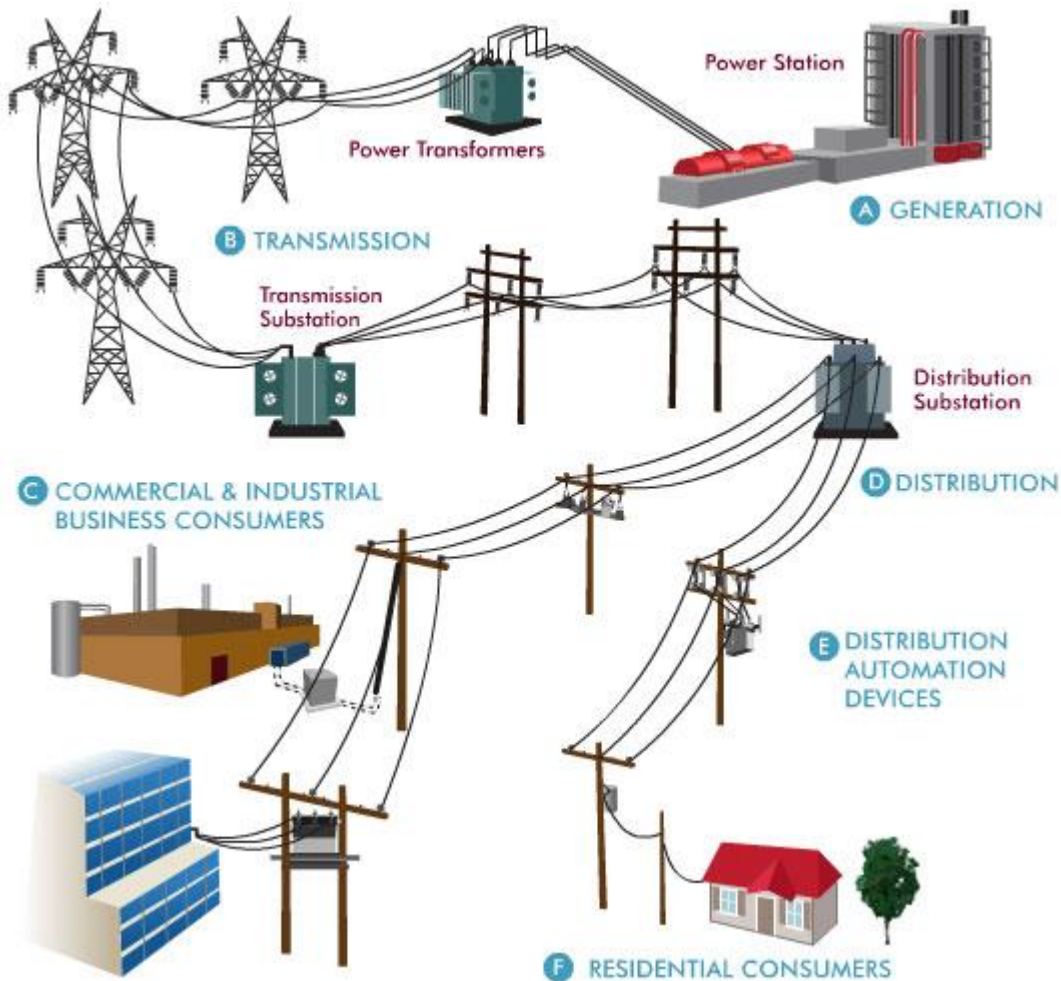
Balanced 3-phase systems

Generation, Transmission and Distribution



Balanced 3-phase systems

Generation, Transmission and Distribution



Balanced 3-phase systems

Y and Δ connections

Balanced 3-phase systems can be considered as 3 equal single phase voltage sources connected either as Y or Delta (Δ) to 3 single three loads connected as either Y or Δ

SOURCE CONNECTIONS

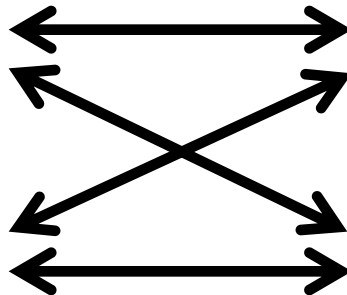
Y connected source

Δ connected source

LOAD CONNECTIONS

Y connected load

Δ connected load



Y-Y

Y- Δ

Δ -Y

Δ - Δ

التوصيل

star



delta



نظام (3Ph)

Source

load

①



②



③



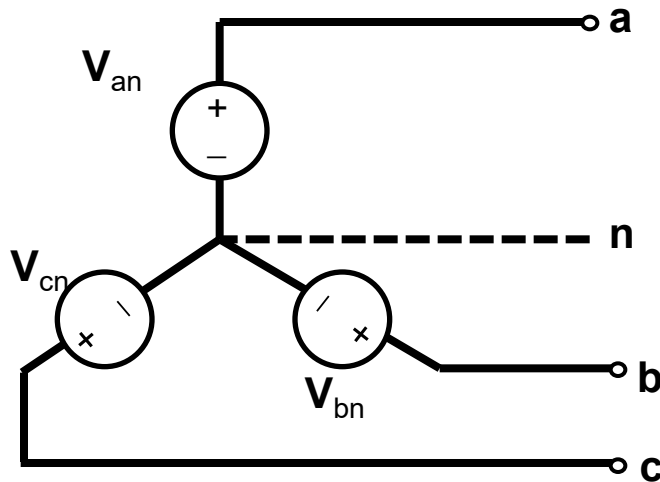
④



Balanced 3-phase systems

SOURCE CONNECTIONS

Source : Y connection

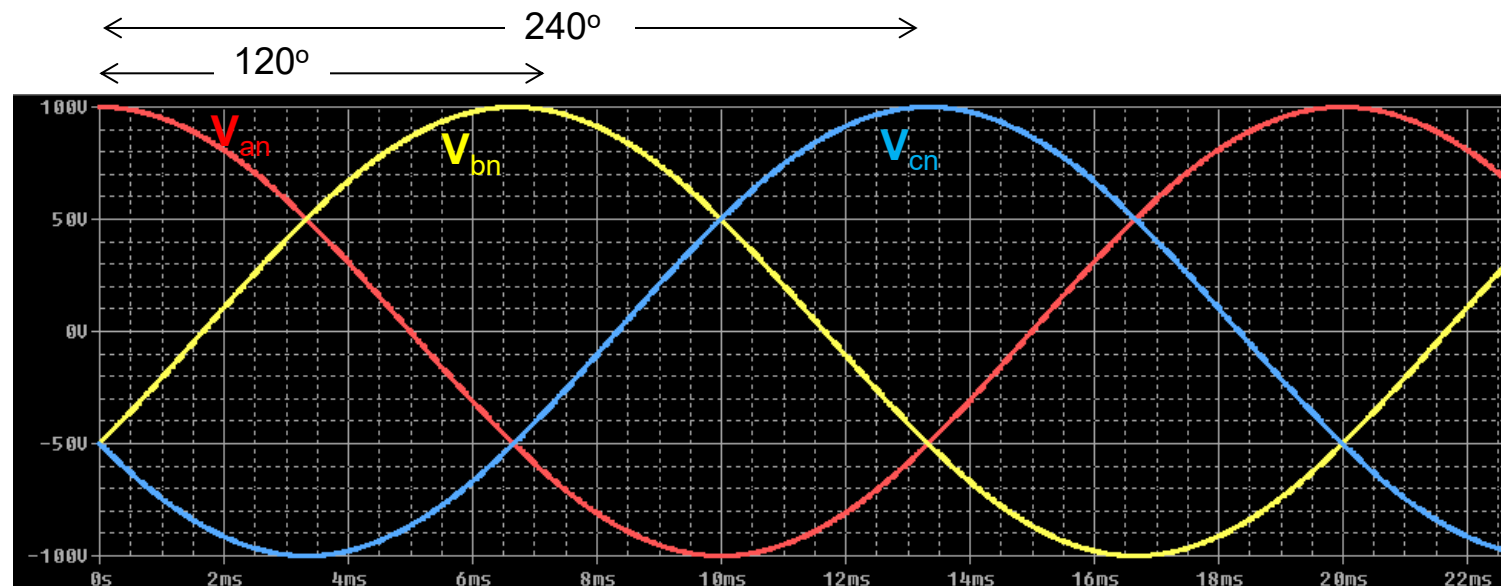


$$v_{an}(t) = \sqrt{2}V_p \cos(\omega t) \Rightarrow V_{an} = V_p \angle 0^\circ$$

$$v_{bn}(t) = \sqrt{2}V_p \cos(\omega t - 120^\circ) \Rightarrow V_{bn} = V_p \angle -120^\circ$$

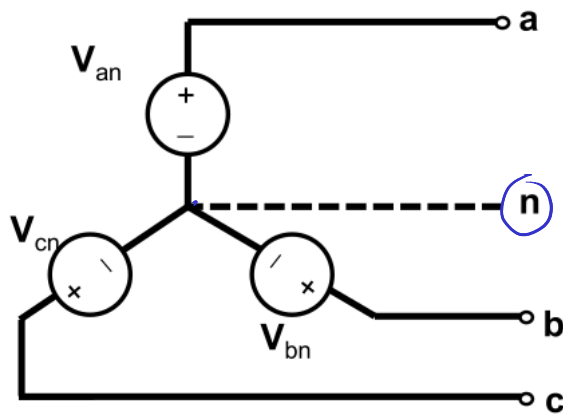
$$v_{cn}(t) = \sqrt{2}V_p \cos(\omega t + 120^\circ) \Rightarrow V_{cn} = V_p \angle 120^\circ$$

RMS phasors !



Source star

+ve sequence (abc) sequence

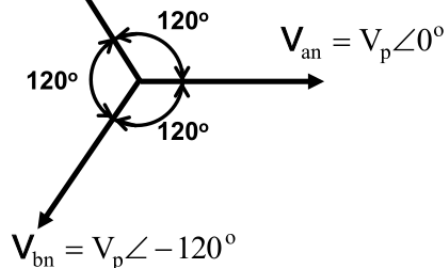


$$V_{an} = \sqrt{2} (V_p) \cos(\omega t) \rightarrow V_{an} = V_p \angle 0^\circ$$

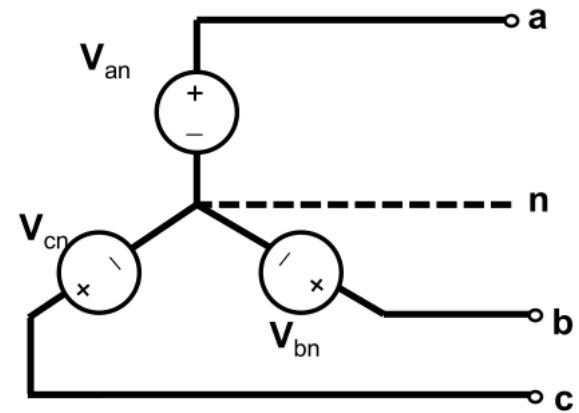
$$V_{bn} = \sqrt{2} (V_p) \cos(\omega t - 120^\circ) \rightarrow V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = \sqrt{2} (V_p) \cos(\omega t + 120^\circ) \rightarrow V_{cn} = V_p \angle 120^\circ$$

$$V_{cn} = V_p \angle 120^\circ = V_p \angle -240^\circ$$



-ve sequence (acb) sequence

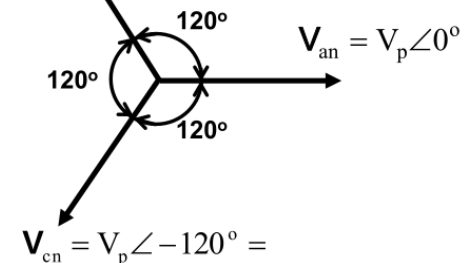


$$V_{an} = \sqrt{2} V_p \cos(\omega t) \rightarrow V_{an} = V_p \angle 0^\circ$$

$$V_{cn} = \sqrt{2} V_p \cos(\omega t - 120^\circ) \rightarrow V_{cn} = V_p \angle -120^\circ$$

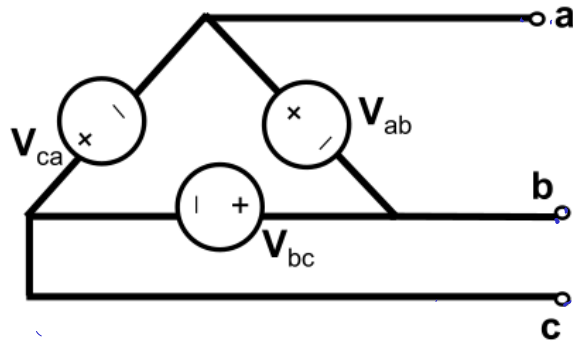
$$V_{bn} = \sqrt{2} V_p \cos(\omega t + 120^\circ) \rightarrow V_{bn} = V_p \angle +120^\circ$$

$$V_{bn} = V_p \angle 120^\circ$$



Source delta

+ve sequence (abc)

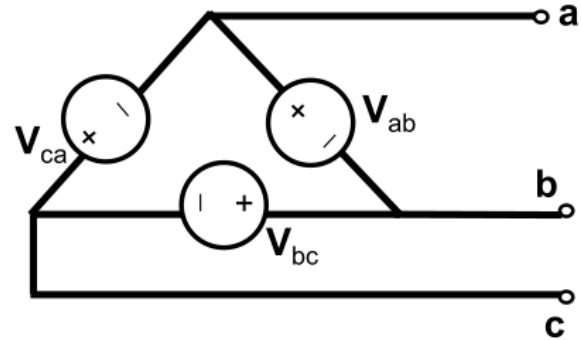


$$V_{ab} = \sqrt{2} V_p \cos(\omega t) \Rightarrow V_{ab} = V_p \angle 0^\circ$$

$$V_{bc} = \sqrt{2} V_p \cos(\omega t - 120^\circ) \Rightarrow V_{bc} = V_p \angle -120^\circ$$

$$V_{ca} = \sqrt{2} V_p \cos(\omega t + 120^\circ) \Rightarrow V_{ca} = V_p \angle 120^\circ$$

-ve sequence (acb)



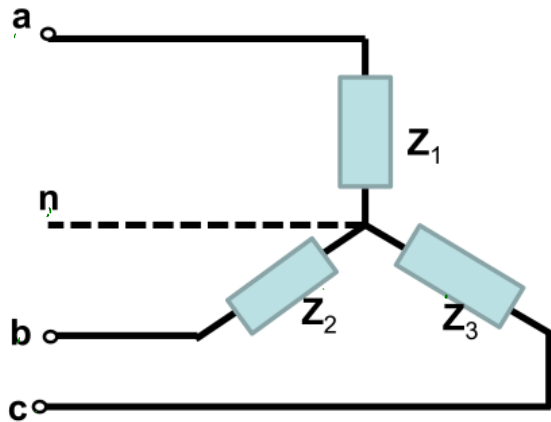
$$V_{ab} = \sqrt{2} V_p \cos(\omega t) \rightarrow V_{ab} = V_p \angle 0^\circ$$

$$V_{ca} = \sqrt{2} V_p \cos(\omega t - 120^\circ) \rightarrow V_{ca} = V_p \angle -120^\circ$$

$$V_{bc} = \sqrt{2} V_p \cos(\omega t + 120^\circ) \rightarrow V_{bc} = V_p \angle 120^\circ$$

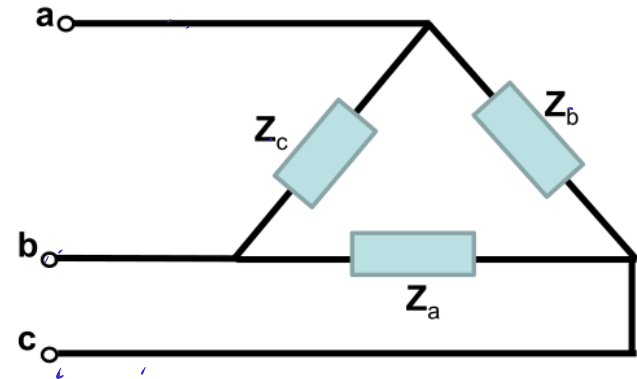
load connection

star :



$$Z_Y = Z_1 = Z_2 = Z_3$$

delta :

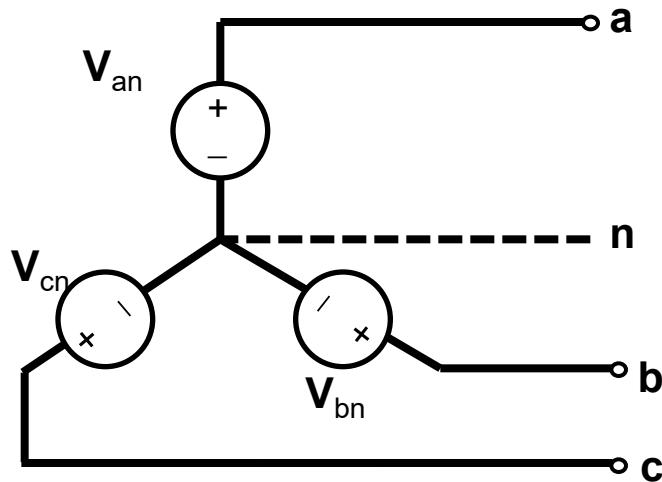


$$Z_{\Delta} = Z_a = Z_b = Z_c$$

$$Z_Y = \frac{Z_{\Delta}}{3}$$

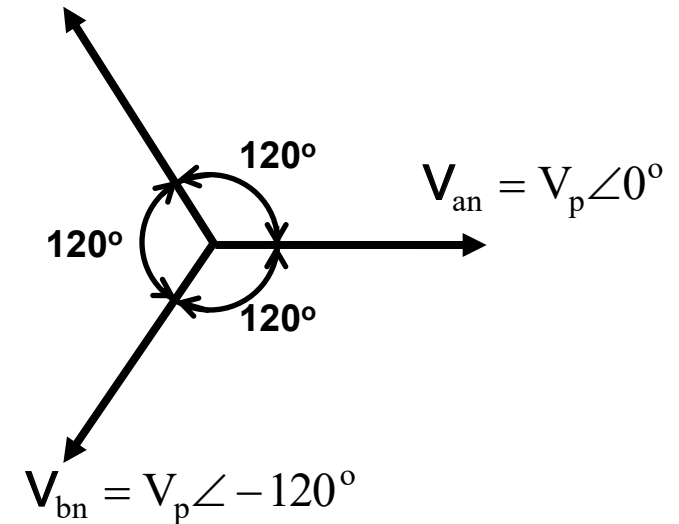
Balanced 3-phase systems

Source : Y connection



SOURCE CONNECTIONS

$$V_{cn} = V_p \angle 120^\circ = V_p \angle -240^\circ$$



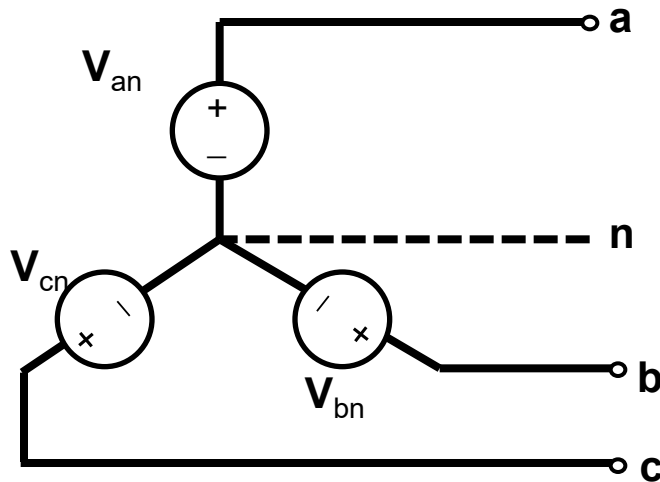
Phase sequence : V_{an} leads V_{bn} by 120° and V_{bn} leads V_{cn} by 120°

→ This is known as **abc sequence** or **positive sequence**

Balanced 3-phase systems

SOURCE CONNECTIONS

Source : Y connection

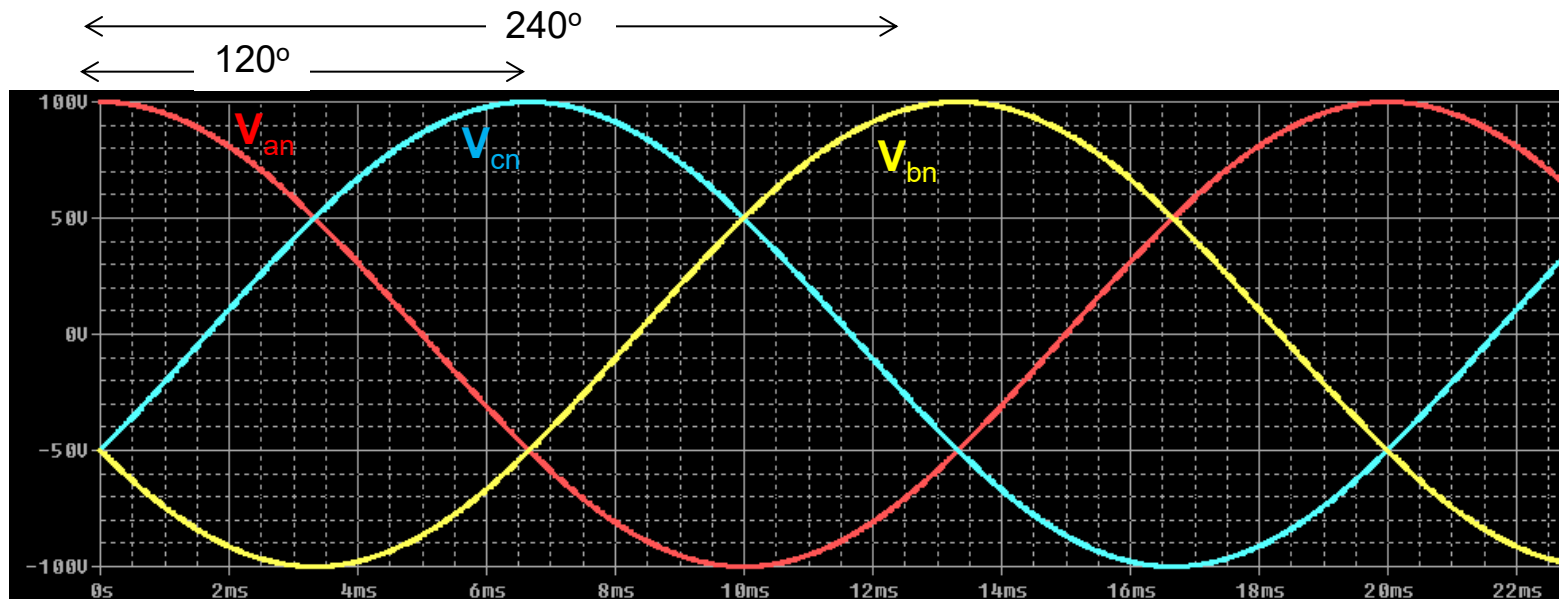


$$v_{an}(t) = \sqrt{2}V_p \cos(\omega t) \Rightarrow V_{an} = V_p \angle 0^\circ$$

$$v_{cn}(t) = \sqrt{2}V_p \cos(\omega t - 120^\circ) \Rightarrow V_{cn} = V_p \angle -120^\circ$$

$$v_{bn}(t) = \sqrt{2}V_p \cos(\omega t + 120^\circ) \Rightarrow V_{bn} = V_p \angle 120^\circ$$

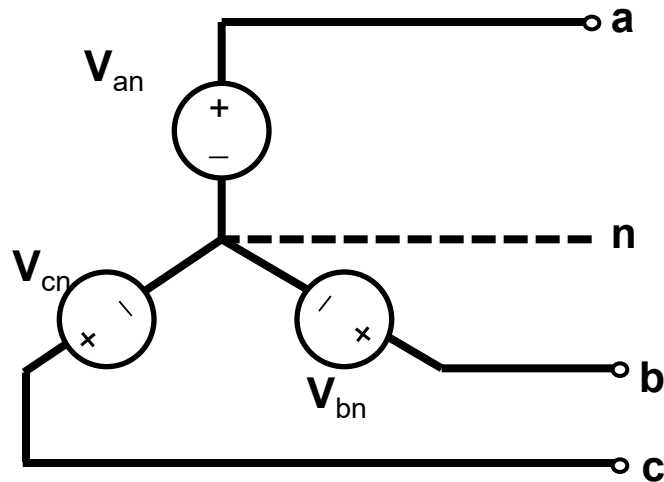
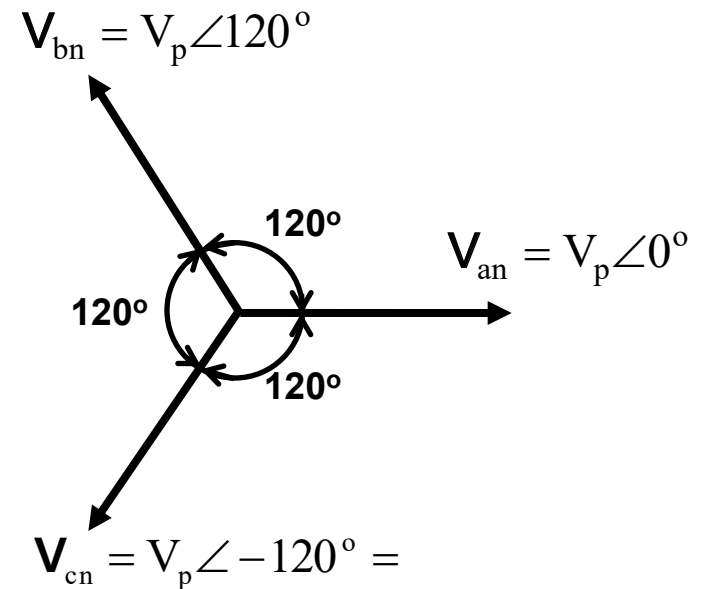
RMS phasors !



Balanced 3-phase systems

SOURCE CONNECTIONS

Source : **Y** connection



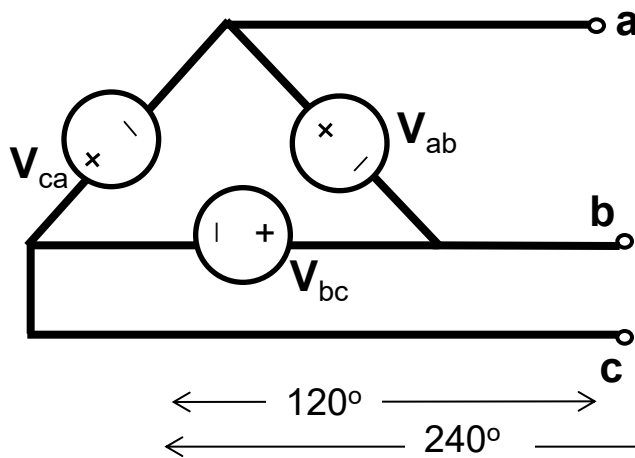
Phase sequence : V_{an} leads V_{cn} by 120° and V_{cn} leads V_{bn} by 120°

→ This is known as **acb sequence** or **negative sequence**

Balanced 3-phase systems

SOURCE CONNECTIONS

Source : Δ connection

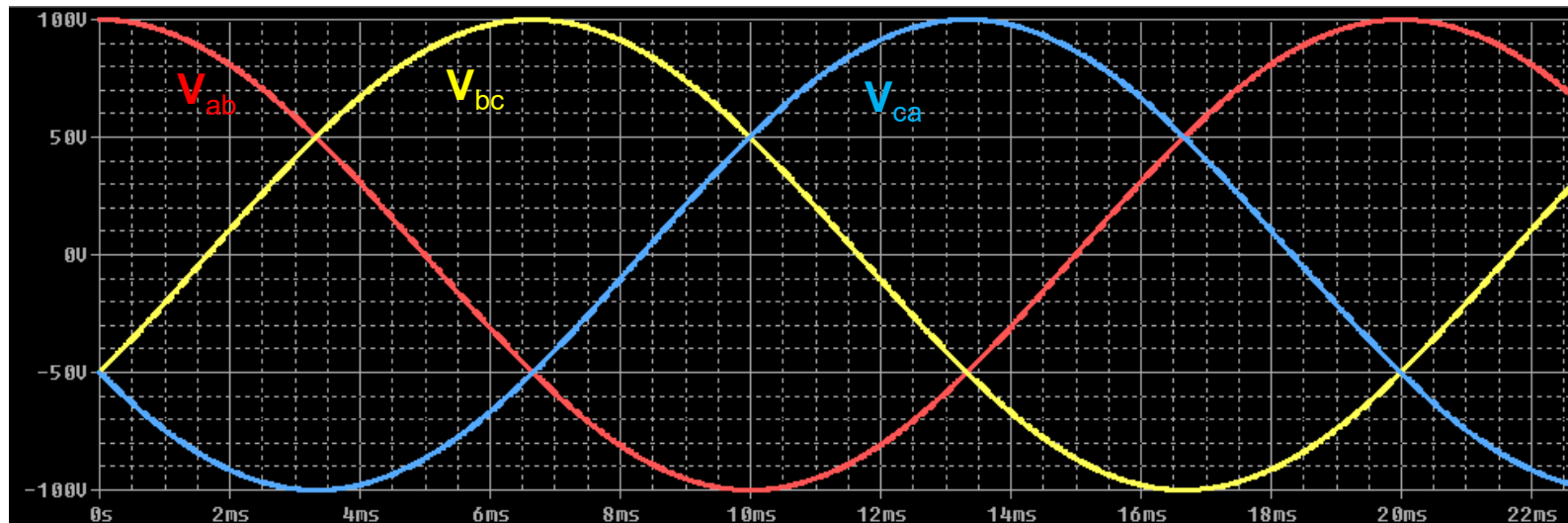


$$v_{ab}(t) = \sqrt{2}V_p \cos(\omega t) \Rightarrow V_{ab} = V_p \angle 0^\circ$$

$$v_{bc}(t) = \sqrt{2}V_p \cos(\omega t - 120^\circ) \Rightarrow V_{bc} = V_p \angle -120^\circ$$

$$v_{ca}(t) = \sqrt{2}V_p \cos(\omega t + 120^\circ) \Rightarrow V_{ca} = V_p \angle 120^\circ$$

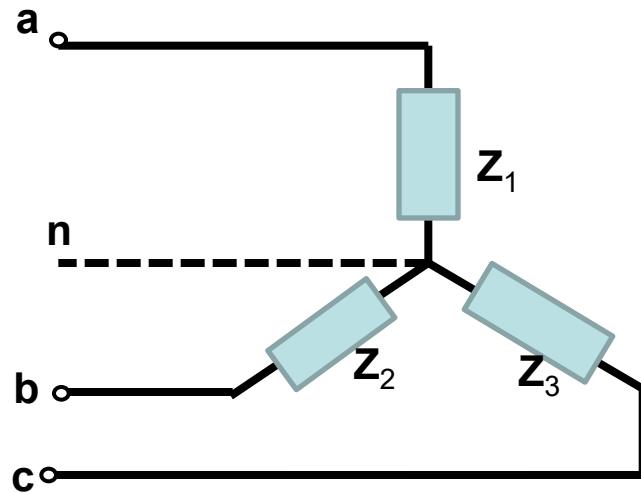
RMS phasors !



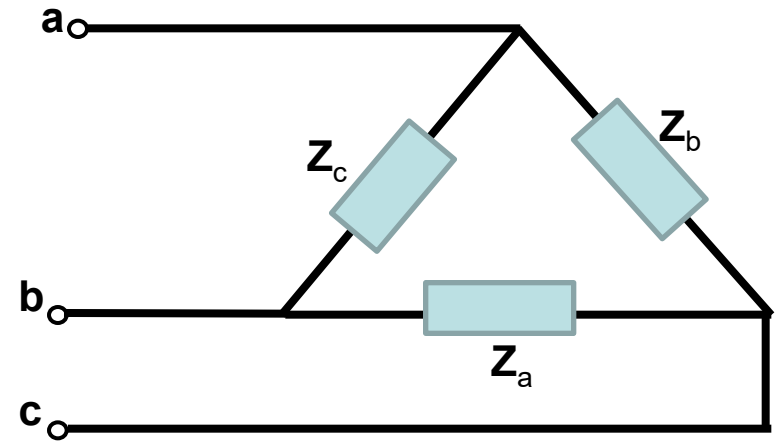
Balanced 3-phase systems

LOAD CONNECTIONS

Y connection



Δ connection

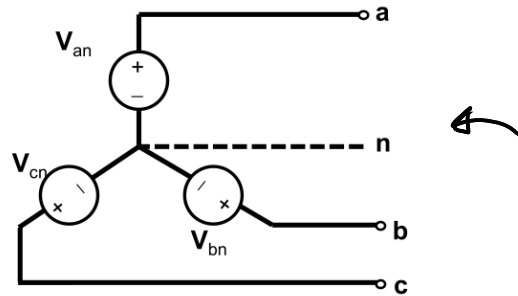


Balanced load:

$$Z_1 = Z_2 = Z_3 = Z_Y \qquad Z_a = Z_b = Z_c = Z_{\Delta} \qquad Z_Y = \frac{Z_{\Delta}}{3}$$

Unbalanced load: each phase load may not be the same.

Y-connected



① Positive sequence

$$v_{an}(t) = \sqrt{2}V_p \cos(\omega t) \Rightarrow V_{an} = V_p \angle 0^\circ$$

$$v_{bn}(t) = \sqrt{2}V_p \cos(\omega t - 120^\circ) \Rightarrow V_{bn} = V_p \angle -120^\circ$$

$$v_{cn}(t) = \sqrt{2}V_p \cos(\omega t + 120^\circ) \Rightarrow V_{cn} = V_p \angle 120^\circ$$

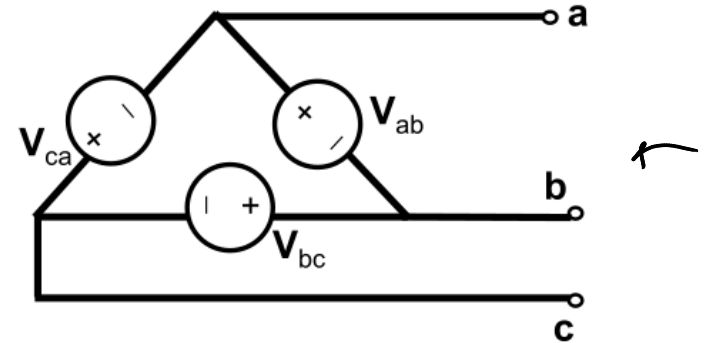
② negative sequence

$$v_{an}(t) = \sqrt{2}V_p \cos(\omega t) \Rightarrow V_{an} = V_p \angle 0^\circ$$

$$v_{cn}(t) = \sqrt{2}V_p \cos(\omega t - 120^\circ) \Rightarrow V_{cn} = V_p \angle -120^\circ$$

$$v_{bn}(t) = \sqrt{2}V_p \cos(\omega t + 120^\circ) \Rightarrow V_{bn} = V_p \angle 120^\circ$$

Δ-connected



① Positive sequence

$$v_{ab}(t) = \sqrt{2}V_p \cos(\omega t) \Rightarrow V_{ab} = V_p \angle 0^\circ$$

$$v_{bc}(t) = \sqrt{2}V_p \cos(\omega t - 120^\circ) \Rightarrow V_{bc} = V_p \angle -120^\circ$$

$$v_{ca}(t) = \sqrt{2}V_p \cos(\omega t + 120^\circ) \Rightarrow V_{ca} = V_p \angle 120^\circ$$

② negative sequence

$$v_{ab}(t) = \sqrt{2}V_p \cos(\omega t) \Rightarrow V_{ab} = V_p \angle 0^\circ$$

$$v_{ca}(t) = \sqrt{2}V_p \cos(\omega t - 120^\circ) \Rightarrow V_{ca} = V_p \angle -120^\circ$$

$$v_{bc}(t) = \sqrt{2}V_p \cos(\omega t + 120^\circ) \Rightarrow V_{bc} = V_p \angle 120^\circ$$

Balanced 3-phase systems Example

Determine the phase sequence of the set of voltages

$$\begin{aligned}v_{an} &= 200 \cos(\omega t + 10^\circ) \\v_{bn} &= 200 \cos(\omega t - 230^\circ), \quad v_{cn} = 200 \cos(\omega t - 110^\circ)\end{aligned}$$

Solution:

The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200 \angle 10^\circ \text{ V}, \quad \mathbf{V}_{bn} = 200 \angle -230^\circ \text{ V}, \quad \mathbf{V}_{cn} = 200 \angle -110^\circ \text{ V}$$

We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120° and $\mathbf{V}_{cn}$ in turn leads $\mathbf{V}_{bn}$ by 120° . Hence, we have an *acb* sequence.

Determine the phase sequence of the set of voltages

$$v_{an} = 200 \cos(\omega t + 10^\circ)$$

$$v_{bn} = 200 \cos(\omega t - 230^\circ), \quad v_{cn} = 200 \cos(\omega t - 110^\circ)$$

$$V_{an} = \frac{200}{\sqrt{2}} \angle 10$$

$$\theta_{bn} - \theta_{an} = -230 - 10 = -240 + 360 = 120$$

$$V_{bn} = \bigcirc \angle -230$$

$$\theta_{cn} - \theta_{an} = -110 - 10 = -120$$

$$V_{cn} = \bigcirc \angle -110$$

-ve sequence

Balanced 3-phase systems

$$270^\circ - 360^\circ$$

Example

Given that $V_{bn} = 110 \angle 30^\circ \text{ V}$, find V_{an} and V_{cn} , assuming a positive (abc) sequence.

Solution:

$$V_{an} = 110 \angle 150^\circ \text{ V}$$

$$V_{cn} = 110 \angle -90^\circ \text{ V}$$

$$V_{an} = 110 \angle 30 + 120 = 110 \angle 150$$

$$V_{bn} = 110 \angle 30 = 110 \angle 30$$

$$V_{cn} = 110 \angle 150 + 120 = 110 \angle -90$$

Example

Given that $V_{bn} = 110 \angle 30^\circ \text{ V}$, find V_{an} and V_{cn} , assuming a negative (acb) sequence.

Solution:

$$-210 + 360$$

$$V_{cn} = 110 \angle 150^\circ \text{ V}$$

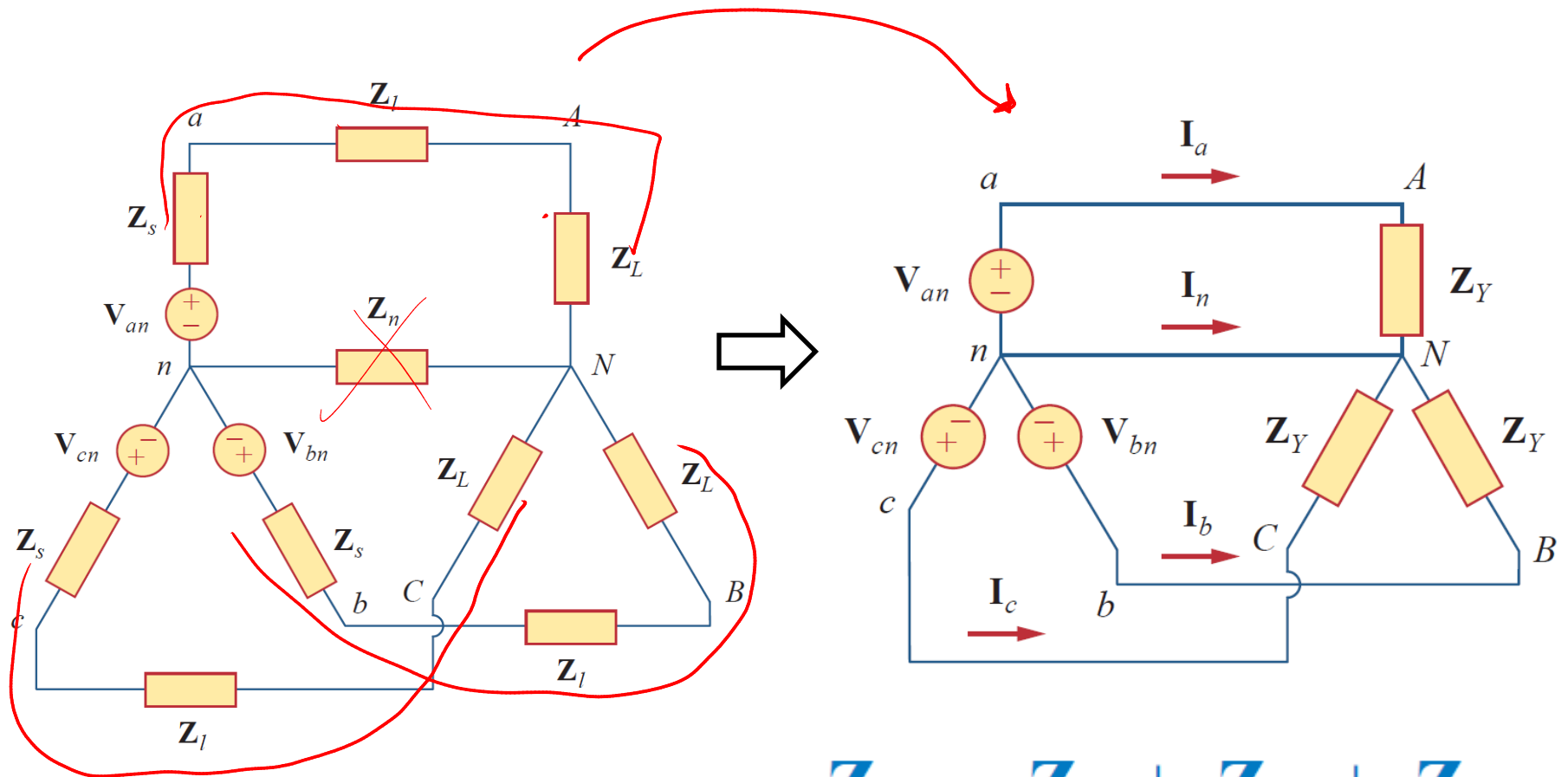
$$V_{an} = 110 \angle -90^\circ \text{ V}$$

$$V_{an} = 110 \angle 30 - 120 = 110 \angle -90$$

$$V_{bn} = 110 \angle 30 = 110 \angle 30$$

$$V_{cn} = 110 \angle -90 + 120 = 110 \angle 150$$

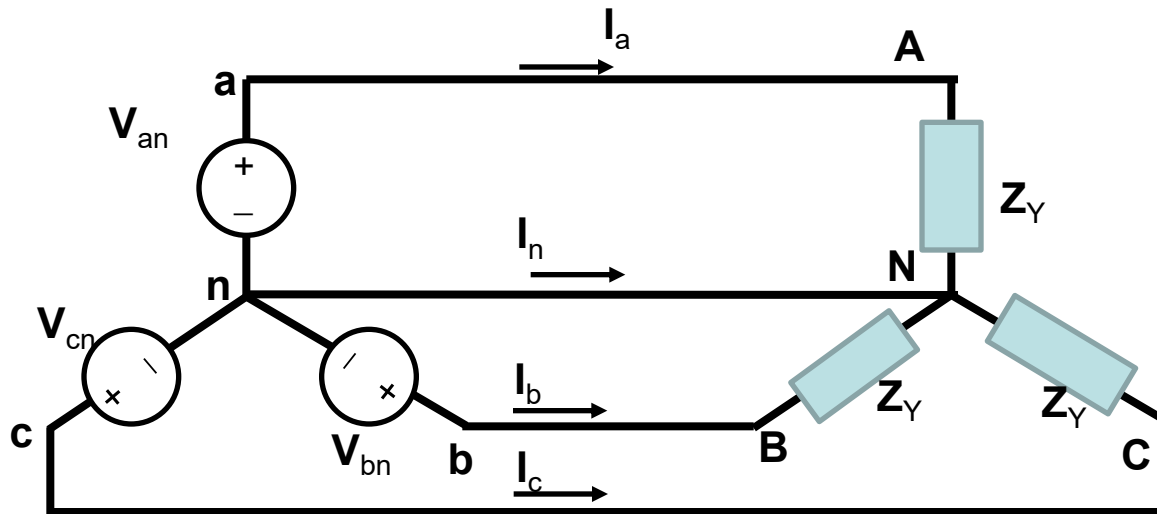
Balanced 3-phase systems



$$Z_Y = Z_s + Z_\ell + Z_L$$

Balanced 3-phase systems

Balanced Y-Y Connection



$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle 120^\circ$$

**Phase
voltages**

$$I_a = \frac{V_p \angle 0^\circ}{Z_Y}$$

$$I_b = \frac{V_p \angle -120^\circ}{Z_Y}$$

$$I_c = \frac{V_p \angle 120^\circ}{Z_Y}$$

**line
currents**

$$\therefore I_a + I_b + I_c = -I_n = 0$$

The wire connecting n and N can be removed !

$$V_{ab} = V_{an} - V_{bn}$$

$$= V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$= V_p (1 \angle 0^\circ - 1 \angle -120^\circ)$$

$$= \sqrt{3} V_p \angle 30^\circ$$

$$V_{bc} = V_{bn} - V_{cn}$$

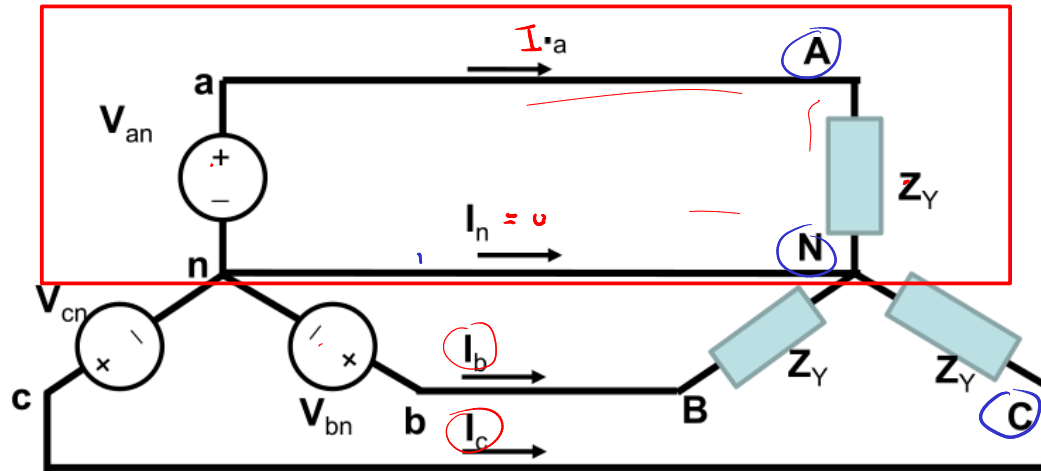
$$= \sqrt{3} V_p \angle -90^\circ$$

$$V_{ca} = V_{cn} - V_{na}$$

$$= \sqrt{3} V_p \angle 150^\circ$$

**line-line
voltages
OR
Line
voltages**

Y-Y connection



Phase $\rightarrow$ $\begin{matrix} V \\ I \end{matrix}$

Line $\rightarrow$ $\begin{matrix} V \\ I \end{matrix}$

① Phase voltage

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle 120^\circ$$

Line voltage:

$$V_{ab} = \sqrt{3} V_p \angle 30^\circ$$

$$V_{bc} = \sqrt{3} V_p \angle -90^\circ$$

$$V_{ca} = \sqrt{3} V_p \angle 150^\circ$$

② Phase current / line current

$$I_a = \frac{V_p \angle 0^\circ}{Z_Y} = \square \angle \square$$

$$I_b = \frac{V_p \angle -120^\circ}{Z_Y} = \square \angle \square - 120^\circ$$

$$I_c = \frac{V_p \angle 120^\circ}{Z_Y} = \square \angle \square + 120^\circ$$

$$I_n = 0$$

Y - V_{an}, V_{bn}, V_{cn} phase
 V_{ub}, V_{bc}, V_{ca} Line

$$V_{an} = V_p \angle 0$$

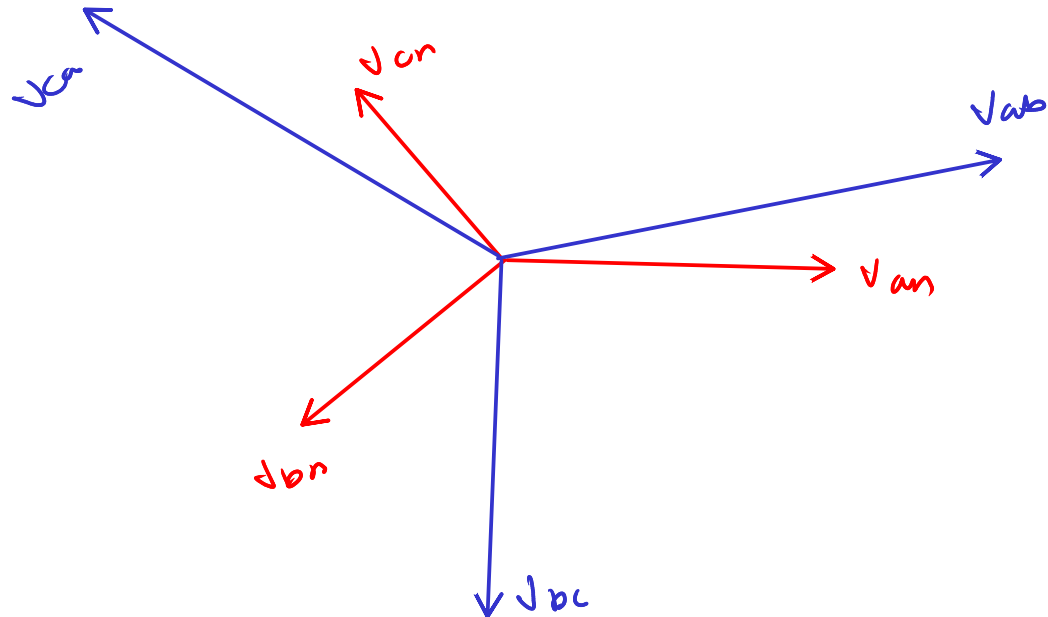
$$V_{bn} = V_p \angle -120$$

$$V_{cn} = V_p \angle +120$$

$$V_{ub} = \sqrt{3} V_p \angle 30$$

$$V_{bc} = \sqrt{3} V_p \angle -90$$

$$V_{ca} = \sqrt{3} V_p \angle 150$$



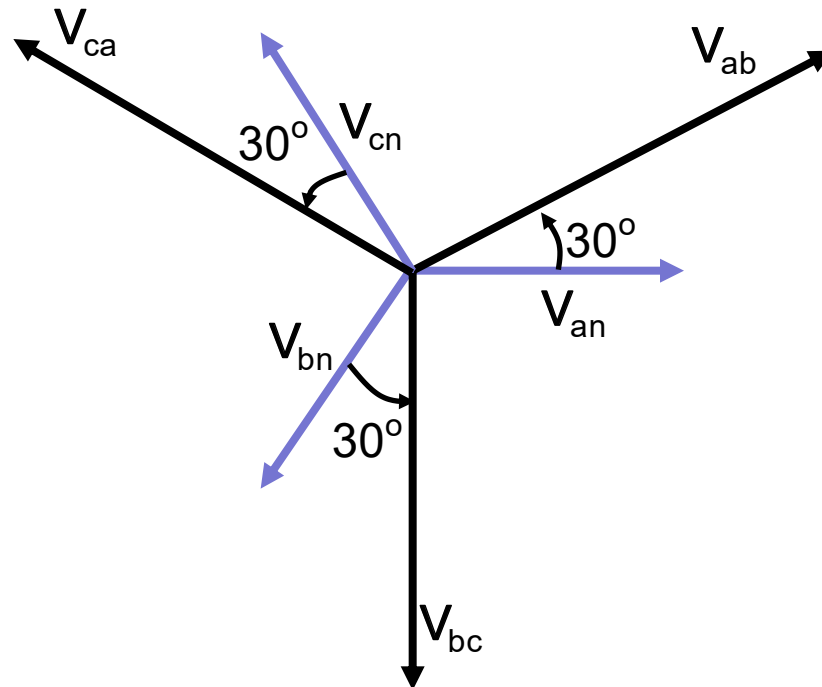
Balanced 3-phase systems

Balanced Y-Y Connection

$$\begin{aligned}\mathbf{V}_{ab} &= \mathbf{V}_{an} - \mathbf{V}_{bn} \\ &= V_p \angle 0^\circ - V_p \angle -120^\circ \\ &= \sqrt{3}V_p \angle 30^\circ\end{aligned}$$

$$\begin{aligned}\mathbf{V}_{bc} &= \mathbf{V}_{bn} - \mathbf{V}_{cn} \\ &= \sqrt{3}V_p \angle -90^\circ\end{aligned}$$

$$\begin{aligned}\mathbf{V}_{ca} &= \mathbf{V}_{cn} - \mathbf{V}_{an} \\ &= \sqrt{3}V_p \angle 150^\circ\end{aligned}$$



$$V_L = \sqrt{3}V_p$$

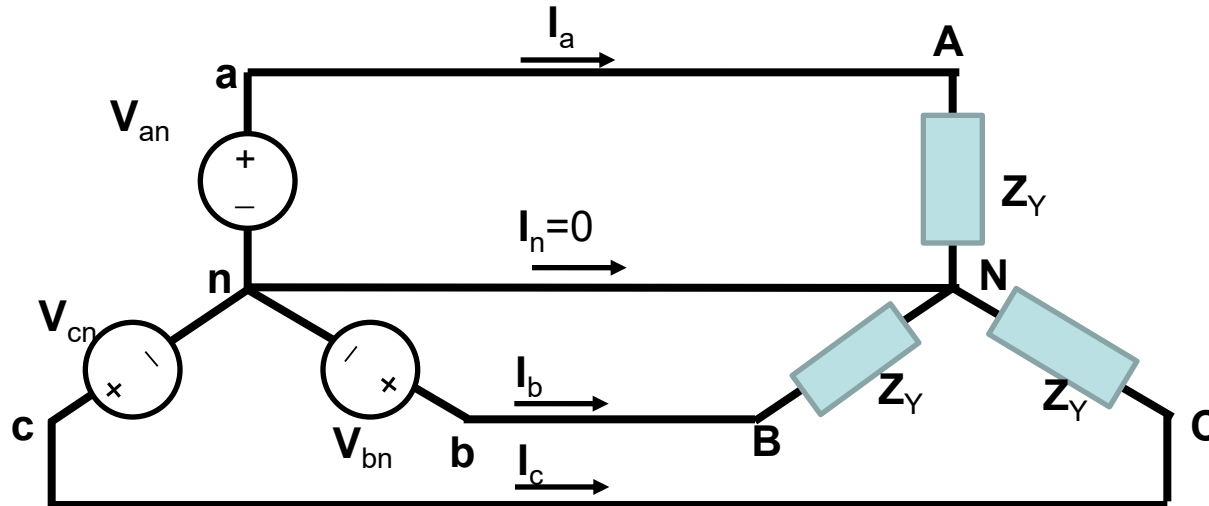
where $V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$ and $V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$

Line voltage LEADS phase voltage by 30°

Balanced 3-phase systems

Balanced Y-Y Connection

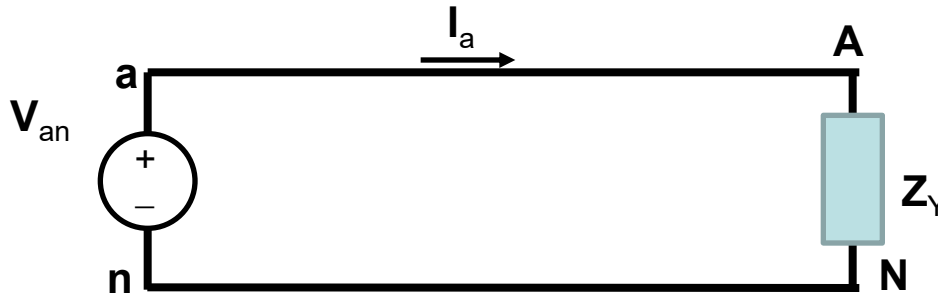
For a **balanced Y-Y** connection, analysis can be performed using an equivalent per-phase circuit: e.g. for phase A:



Balanced 3-phase systems

Balanced Y-Y Connection

For a **balanced Y-Y** connection, analysis can be performed using an equivalent per-phase circuit: e.g. for phase A:



$$I_a = \frac{V_{an}}{Z_Y}$$

Based on the sequence, the other line currents can be obtained from:

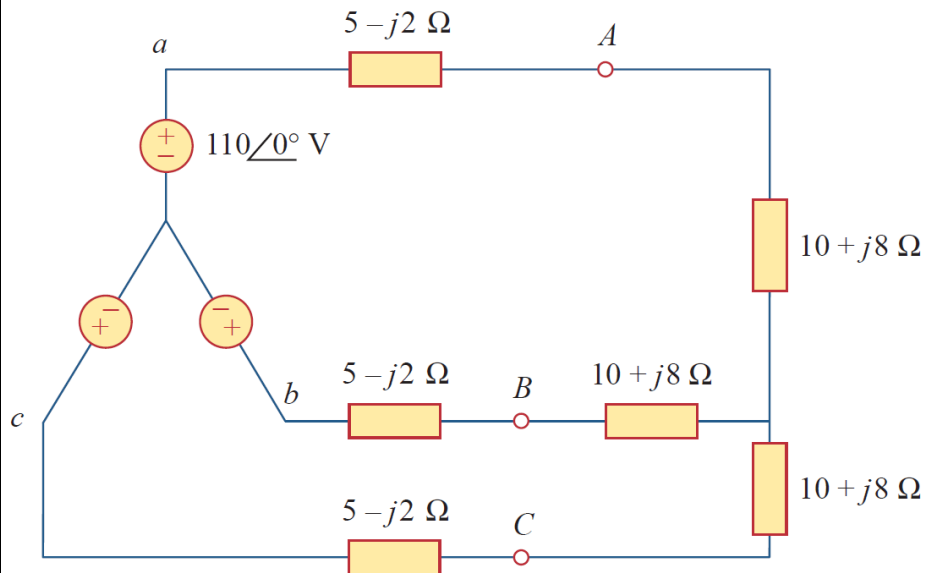
$$I_b = I_a \angle -120^\circ$$

$$I_c = I_a \angle 120^\circ$$

Balanced 3-phase systems

Balanced Y-Y Connection

Example: In a Y-Y three-phase system below, suppose a positive sequence Connection. Find V_{bn} , V_{cn} , I_a , I_b , I_c .



Solution:

Since abc (+ve):

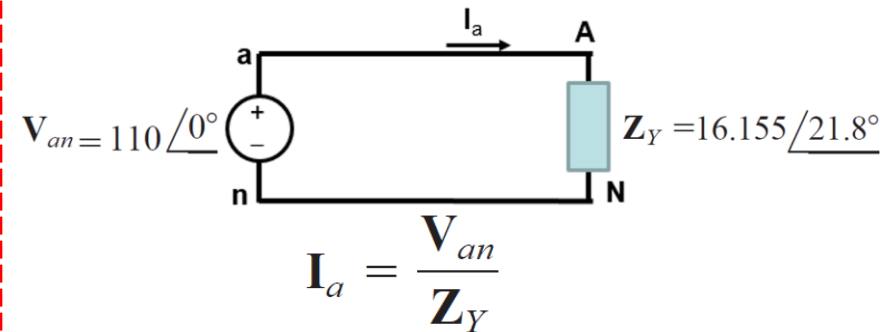
$$V_{bn} = 110 \angle -120^\circ \text{ V}$$

$$V_{cn} = 110 \angle -240^\circ \text{ V}$$

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$

$$= 0 + (5 - j2) + (10 + j8) = 15 + j6$$

$$= 16.155 \angle 21.8^\circ$$

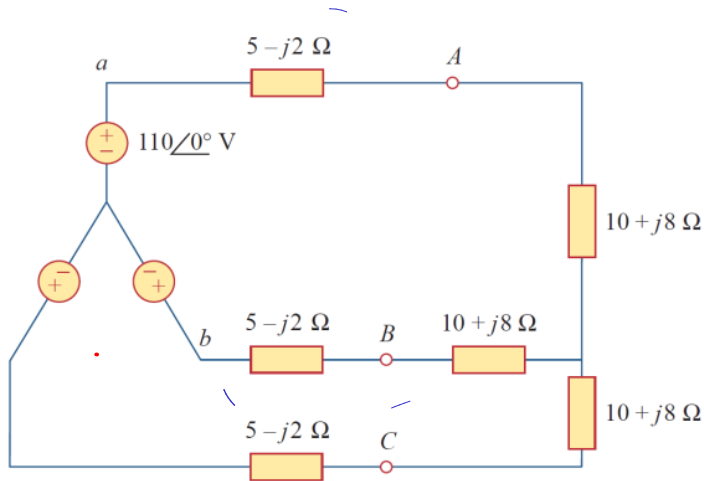
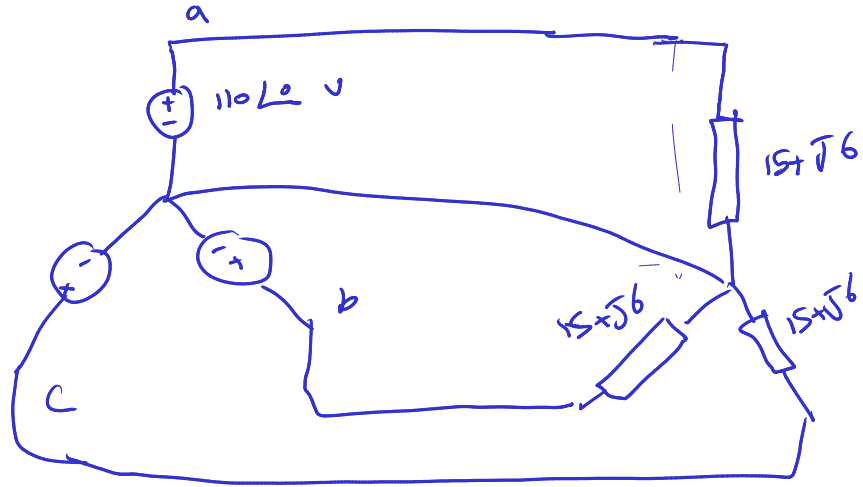


$$\mathbf{I}_a = \frac{110 \angle 0^\circ}{16.155 \angle 21.8^\circ} = 6.81 \angle -21.8^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = 6.81 \angle -141.8^\circ \text{ A}$$

$$\begin{aligned} \mathbf{I}_c &= \mathbf{I}_a \angle -240^\circ = 6.81 \angle -261.8^\circ \text{ A} \\ &= 6.81 \angle 98.2^\circ \text{ A} \end{aligned}$$

Example: In a Y-Y three-phase system below, suppose a positive sequence Connection. Find $\underline{V}_{bn}$, $\underline{V}_{cn}$, $\underline{I}_a$, $\underline{I}_b$, $\underline{I}_c$.



$$\underline{Z}_Y = \underline{Z}_S + \underline{Z}_L$$

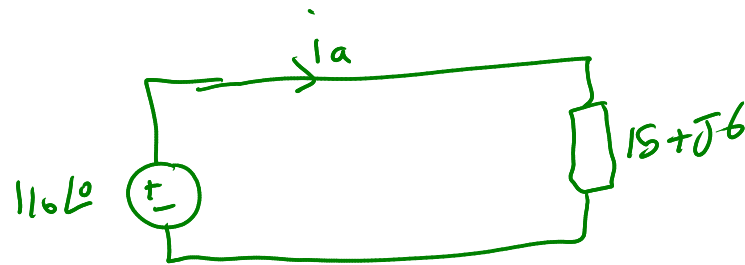
$$= 0 + 5 - j2 + 10 + j8$$

$$\underline{Z}_Y = 15 + j6$$

$$\underline{V}_{an} = 110 \angle 0^\circ$$

$$\underline{V}_{bn} = 110 \angle -120^\circ$$

$$\underline{V}_{cn} = 110 \angle 120^\circ$$



$$\underline{I}_a = \frac{110 \angle 0^\circ}{15 + j6} = 6.8 \angle -21.8^\circ \text{ A}$$

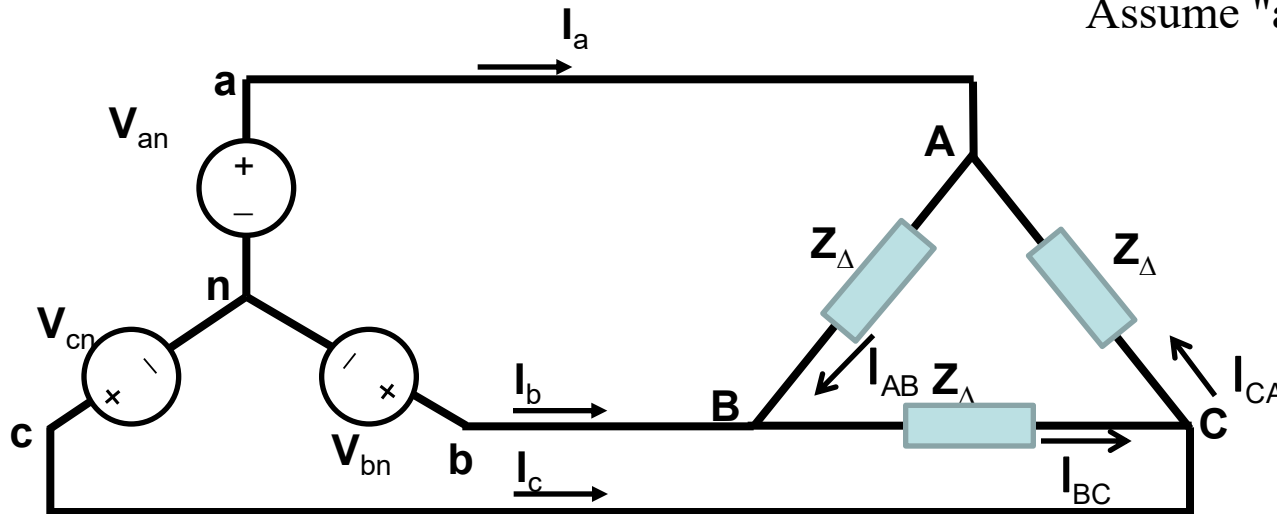
$$\underline{I}_b = 6.8 \angle -141.8^\circ \text{ A}$$

$$\underline{I}_c = 6.8 \angle 98.2^\circ \text{ A}$$

Balanced 3-phase systems

Balanced Y-Δ Connection

Assume "abc"



$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle 120^\circ$$

Phase Voltages

$$V_{ab} = \sqrt{3}V_p \angle 30^\circ$$

$$= V_{AB}$$

$$V_{bc} = \sqrt{3}V_p \angle -90^\circ$$

$$= V_{BC}$$

$$V_{ca} = \sqrt{3}V_p \angle 150^\circ$$

$$= V_{CA}$$

Line Voltages

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

Phase Currents

Using KCL,

$$I_a = I_{AB} - I_{CA}$$

$$= I_{AB} \angle 0^\circ - I_{AB} \angle 120^\circ$$

$$= I_{AB} (1 - 1 \angle 120^\circ)$$

$$= I_{AB} \sqrt{3} \angle -30^\circ$$

$$I_b = I_{BC} - I_{AB}$$

$$= I_{BC} (1 - 1 \angle 120^\circ)$$

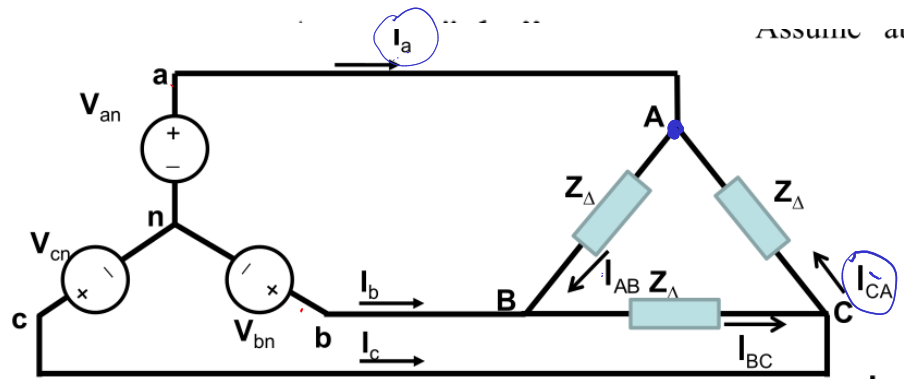
$$= I_{BC} \sqrt{3} \angle -30^\circ$$

$$I_c = I_{CA} \sqrt{3} \angle -30^\circ$$

Line Currents

دسته ۱۵۸
۱۳۹۷

Balanced Y-Δ Connection



① Phase voltage

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle 120^\circ$$

② Line voltage

$$V_{ab} = \sqrt{3} V_p \angle 30^\circ$$

$$V_{bc} = \sqrt{3} V_p \angle -90^\circ$$

$$V_{ca} = \sqrt{3} V_p \angle 150^\circ$$

③ Phase current

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

④ Line currents :-

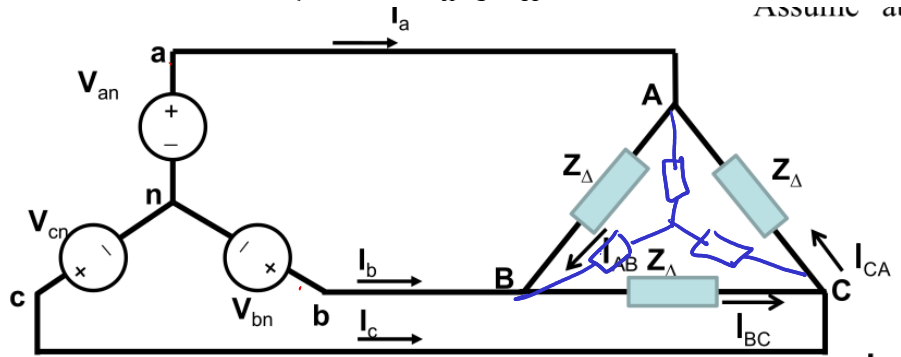
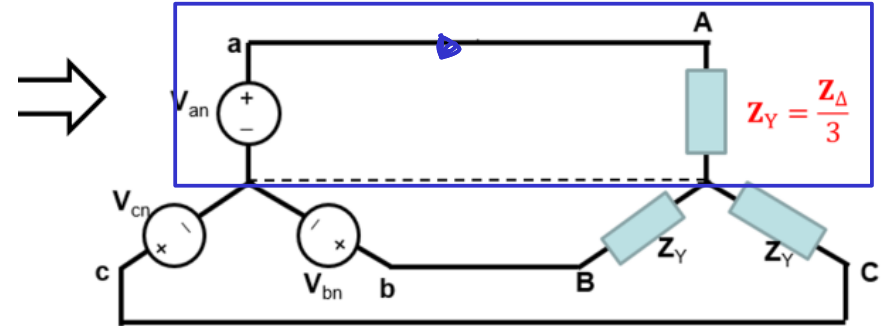
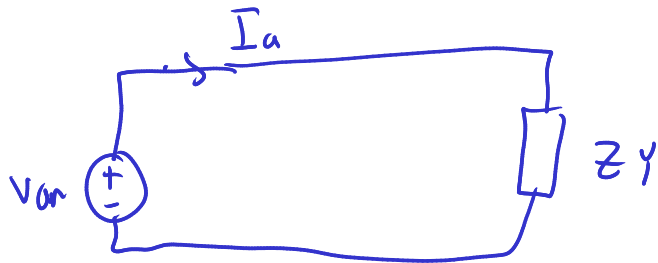
$$I_a = \sqrt{3} I_{AB} \angle -30^\circ$$

$$I_b = \sqrt{3} I_{BC} \angle -30^\circ$$

$$I_c = \sqrt{3} I_{CA} \angle -30^\circ$$

طريقة اكل الناسة

Balanced Y-Δ Connection



$$I_a = \frac{V_{an}}{Z_Y}$$

$$I_b = \angle -120^\circ$$

$$I_c = \angle +120^\circ$$

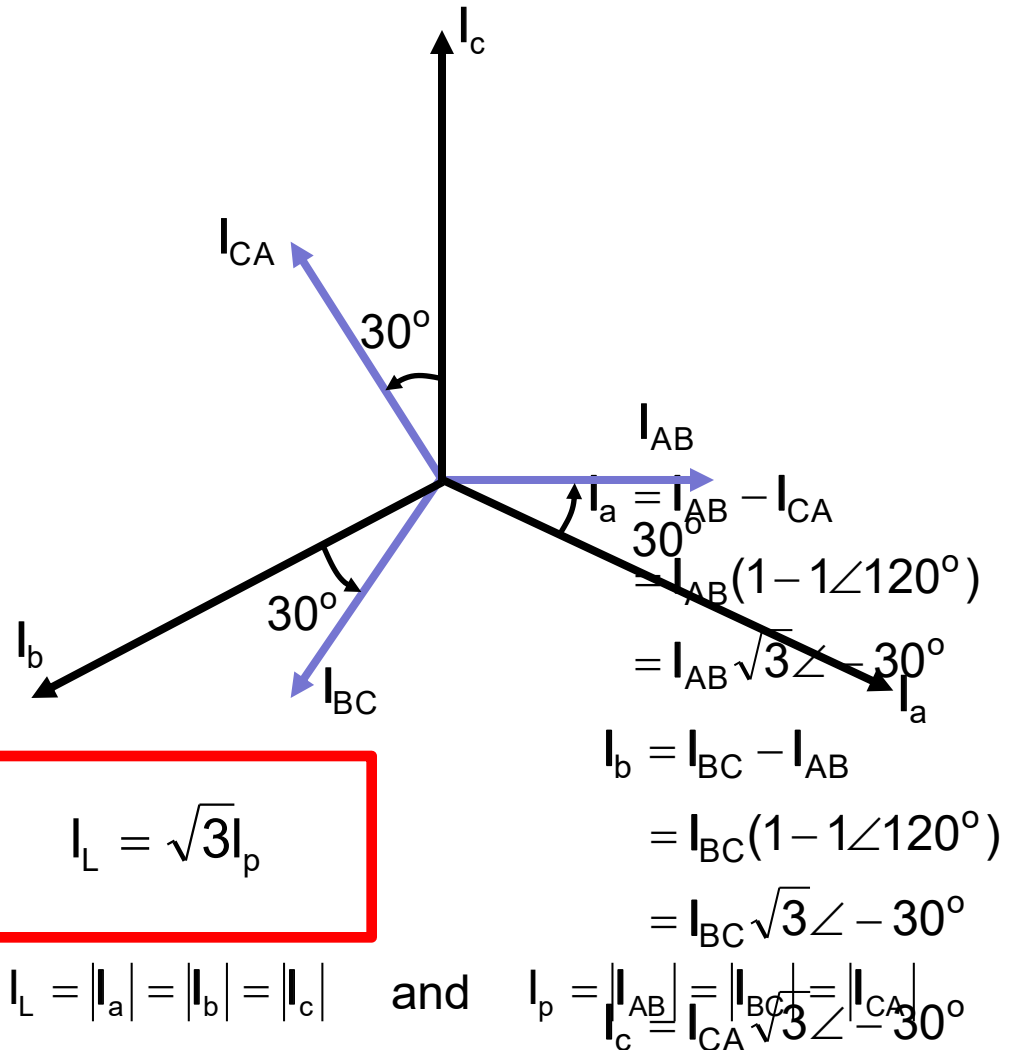
$$I_{AB} = \frac{I_a}{\sqrt{3} \angle -30^\circ}$$

$$I_{BC} = \frac{I_b}{\sqrt{3} \angle 30^\circ}$$

$$I_{CA} = \frac{I_c}{\sqrt{3} \angle -30^\circ}$$

Balanced 3-phase systems

Balanced Y-Δ Connection

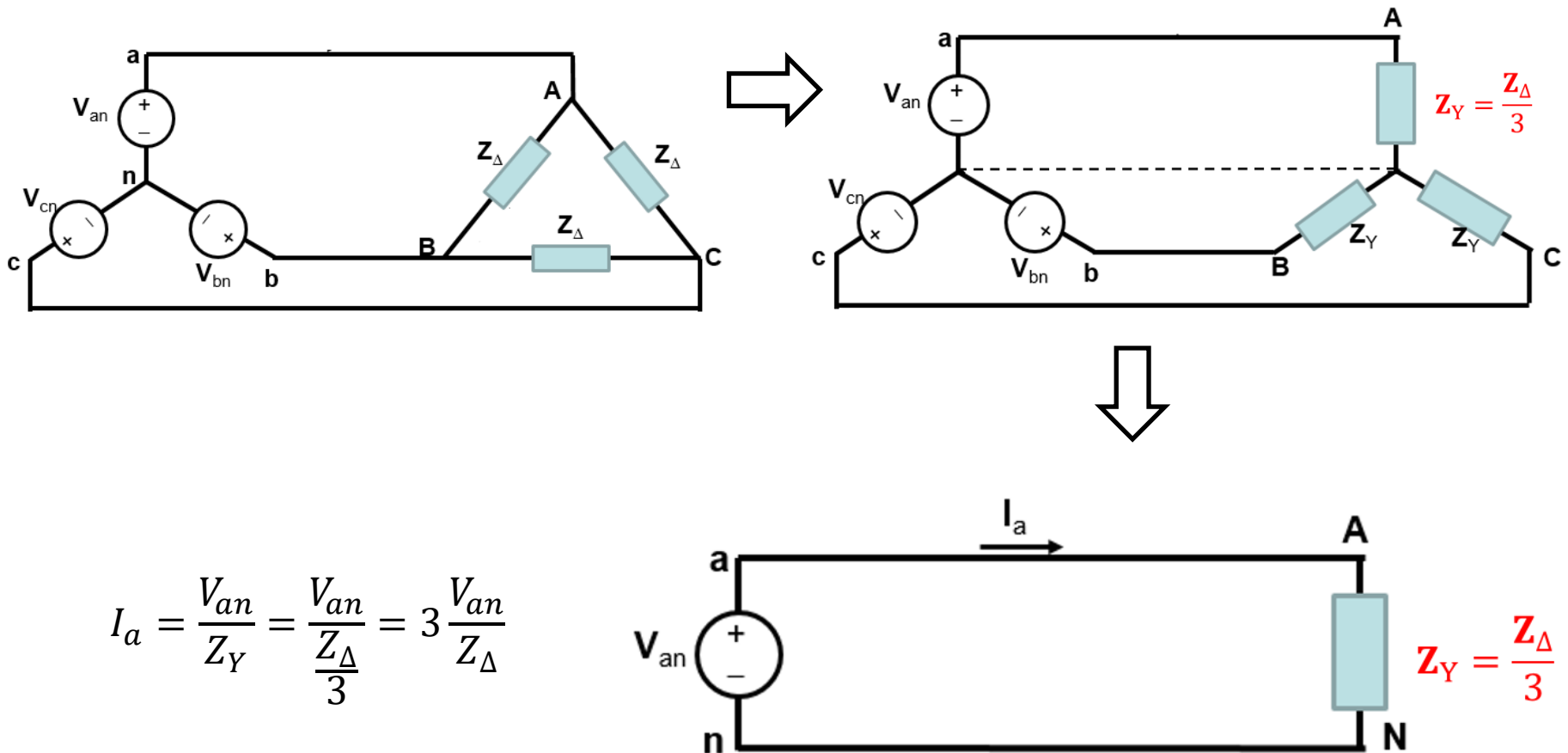


Phase current LEADS line current by 30°

Balanced 3-phase systems

Balanced Y-Δ Connection

Another way to solve Y-Δ connection by converting it into Y-Y connection:



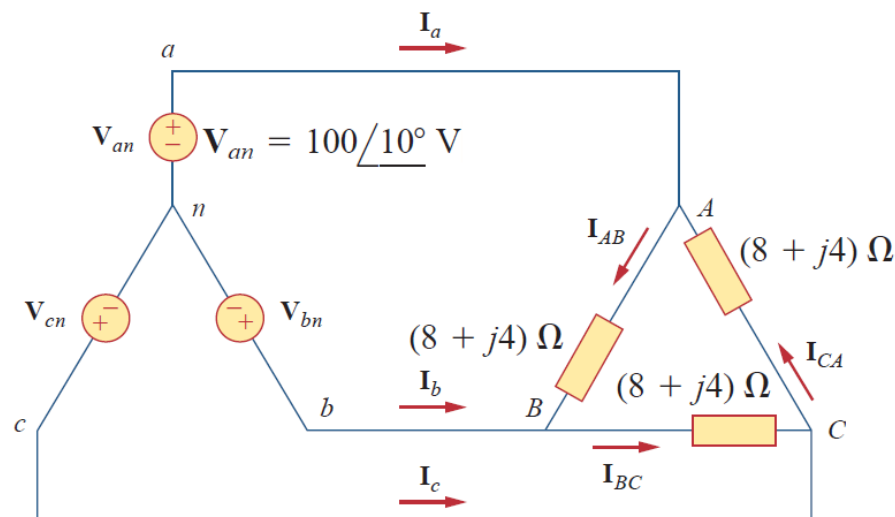
$$I_a = \frac{V_{an}}{Z_Y} = \frac{V_{an}}{\frac{Z_{\Delta}}{3}} = 3 \frac{V_{an}}{Z_{\Delta}}$$

Balanced 3-phase systems

Balanced Y-Δ Connection

Example: A balanced *abc*-sequence Y-connected source with $V_{an} = 100\angle 10^\circ$ V is connected to a Δ-connected balanced load $(8 + j4)\Omega$ per phase. Calculate the phase and line currents.

Solution:



The load impedance is

$$Z_{\Delta} = 8 + j4 = 8.944\angle 26.57^\circ \Omega$$

$$\text{the line voltage is } V_{ab} = V_{an} \sqrt{3}\angle 30^\circ$$

$$V_{ab} = 100\sqrt{3}\angle 10^\circ + 30^\circ = V_{AB} = 173.2\angle 40^\circ \text{ V}$$

The phase currents are

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{173.2\angle 40^\circ}{8.944\angle 26.57^\circ} = 19.36\angle 13.43^\circ \text{ A}$$

$$I_{BC} = I_{AB}\angle -120^\circ = 19.36\angle -106.57^\circ \text{ A}$$

$$I_{CA} = I_{AB}\angle +120^\circ = 19.36\angle 133.43^\circ \text{ A}$$

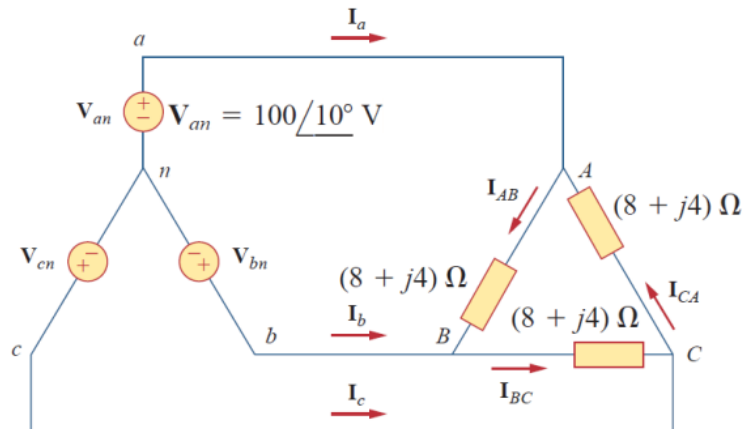
The line currents are

$$I_a = I_{AB}\sqrt{3}\angle -30^\circ = \sqrt{3}(19.36)\angle 13.43^\circ - 30^\circ = 33.53\angle -16.57^\circ \text{ A}$$

$$I_b = I_a\angle -120^\circ = 33.53\angle -136.57^\circ \text{ A}$$

$$I_c = I_a\angle +120^\circ = 33.53\angle 103.43^\circ \text{ A}$$

Example: A balanced *abc*-sequence Y-connected source with $V_{an} = 100 \angle 10^\circ$ V is connected to a Δ -connected balanced load $(8 + j4) \Omega$ per phase. Calculate the phase and line currents.



الطريق الاول

Phase voltages

$$① V_{an} = 100 \angle 10^\circ \text{ V}$$

$$V_{bn} = 100 \angle -110^\circ \text{ V}$$

$$V_{cn} = 100 \angle 130^\circ \text{ V}$$

② Line voltage

$$V_{ab} = 173.2 \angle 40^\circ$$

$$V_{bc} = 173.2 \angle -80^\circ$$

$$V_{ca} = 173.2 \angle 160^\circ$$

③ phase current

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = 19.36 \angle 13.43^\circ$$

$$I_{BC} = \frac{V_{BC}}{Z_{\Delta}} = 19.36 \angle -66.57^\circ$$

$$I_{CA} = \frac{V_{CA}}{Z_{\Delta}} = 19.36 \angle 133.43^\circ$$

(u) Line current

$$I_a = \sqrt{3} I_{AB} \angle -30^\circ = 33.53 \angle -16.57^\circ \text{ A}$$

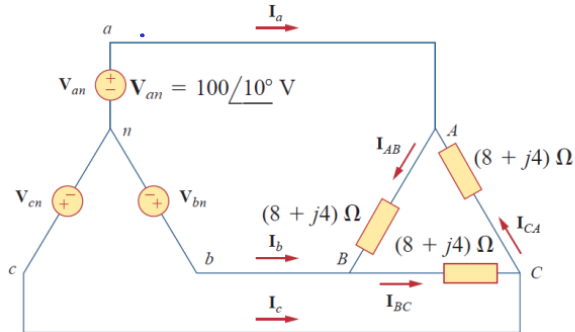
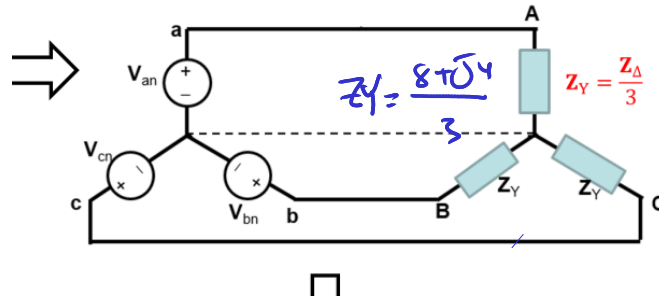
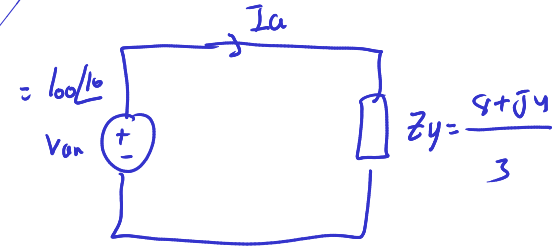
$$I_b = \sqrt{3} I_{BC} \angle -30^\circ = 33.53 \angle -96.57^\circ \text{ A}$$

$$I_c = \sqrt{3} I_{CA} \angle -30^\circ = 33.53 \angle 103.43^\circ \text{ A}$$

الطريقة الثانية

Example: A balanced *abc*-sequence Y-connected source with $V_{an} = 100 \angle 10^\circ$ V is connected to a Δ -connected balanced load $(8 + j4) \Omega$ per phase. Calculate the phase and line currents.

! The phase currents are



Line

$$I_a = \frac{V_{an}}{Z_Y} = \frac{100 \angle 10^\circ}{8 + j4 / 3} = 33.54 \angle -16.56^\circ$$

$$I_b = 33.54 \angle -16.56^\circ - 120^\circ = 33.54 \angle -136.56^\circ$$

$$I_c = 33.54 \angle -16.56^\circ + 120^\circ = 33.54 \angle 103.44^\circ$$

Phase

$$I_{AB} = \frac{I_a}{\sqrt{3} \angle -30^\circ} = 19.36 \angle 13.44^\circ$$

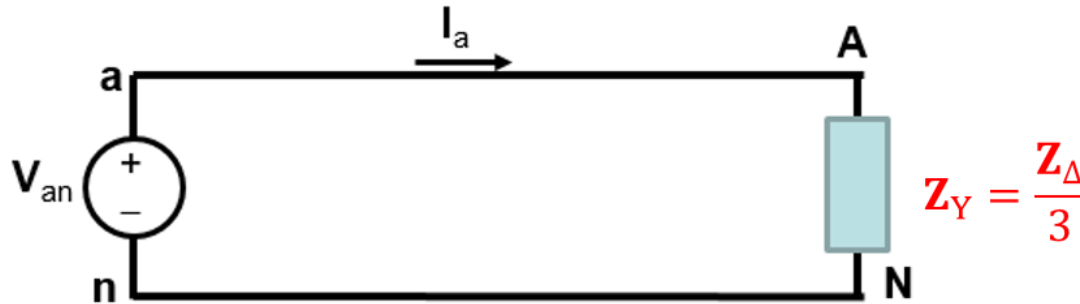
$$I_{BC} = \frac{I_b}{\sqrt{3} \angle -30^\circ} = 19.36 \angle -106.56^\circ$$

$$I_{CA} = \frac{I_c}{\sqrt{3} \angle -30^\circ} = 19.36 \angle 133.44^\circ$$

Balanced 3-phase systems

Balanced Y-Δ Connection

Alternatively, the example can be solved using single-phase analysis by converting Δ-load into Y-load connection :



$$Z_Y = \frac{Z_{\Delta}}{3} = \frac{8.944 \angle 26.57^\circ}{3} = 2.981 \angle 26.57^\circ$$

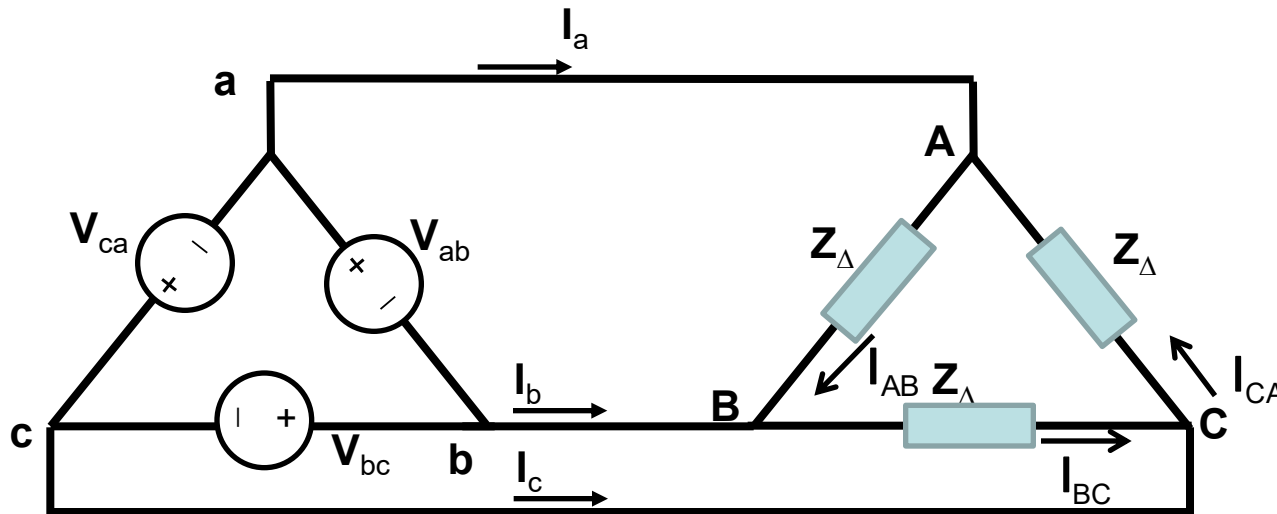
$$I_a = \frac{V_{an}}{Z_Y} = \frac{100 \angle 10^\circ}{2.981 \angle 26.57^\circ} = 33.53 \angle -16.57^\circ \text{ A}$$

how to find I_b and I_c ? and $I_{AB} = \frac{I_a}{(\sqrt{3} \angle -30^\circ)} = 19.36 \angle 13.43^\circ \text{ A}$

$$V_{ab}? \quad V_{ab} = \sqrt{3} V_{an} \angle 30^\circ = 173.2 \angle 40^\circ = V_{AB}$$

Balanced 3-phase systems

Balanced Δ - Δ Connection



$$V_{ab} = V_p \angle 0^\circ$$

$$V_{bc} = V_p \angle -120^\circ$$

$$V_{ca} = V_p \angle 120^\circ$$

$$V_{ab} = V_{AB}$$

$$V_{bc} = V_{BC}$$

$$V_{ca} = V_{CA}$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

Using KCL, $I_a = I_{AB} - I_{CA}$

$$= I_{AB}(1 - 1 \angle 120^\circ)$$

$$= I_{AB} \sqrt{3} \angle -30^\circ$$

$$I_b = I_{BC} - I_{AB}$$

$$= I_{BC}(1 - 1 \angle 120^\circ)$$

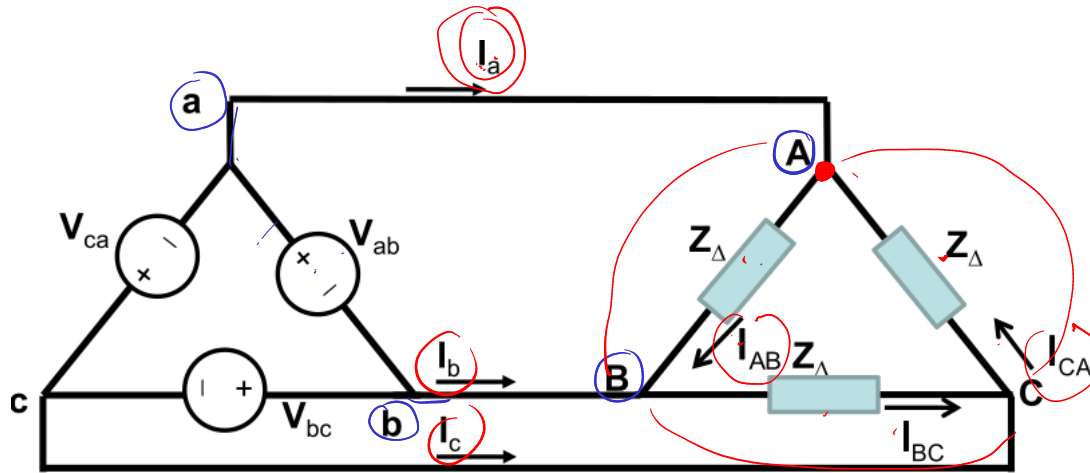
$$= I_{BC} \sqrt{3} \angle -30^\circ$$

$$I_c = I_{CA} \sqrt{3} \angle -30^\circ$$

Phase currents

line currents

Balanced Δ - Δ Connection



① Phase / line voltage

$$V_{ob} = V_p \angle 0^\circ = V_{AB}$$

$$V_{bc} = V_p \angle -120^\circ = V_{BC}$$

$$V_{ca} = V_p \angle 120^\circ = V_{CA}$$

② Phase current

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}}$$

$$I_{BC} = \frac{V_{BC}}{Z_{\Delta}}$$

$$I_{CA} = \frac{V_{CA}}{Z_{\Delta}}$$

③ Line current

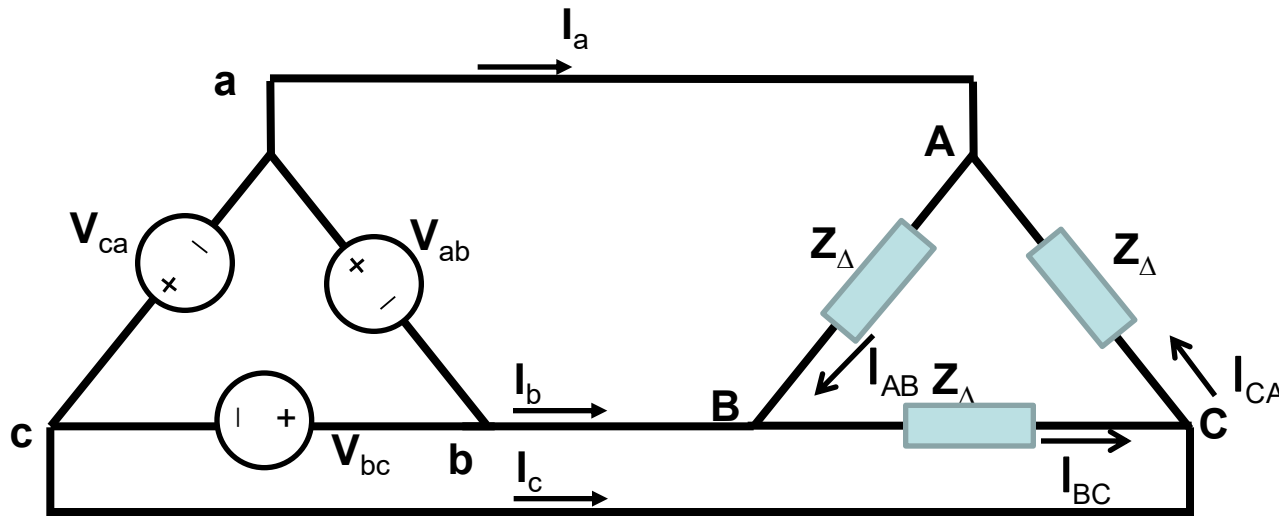
$$I_a = \sqrt{3} I_{AB} \angle -30^\circ$$

$$I_b = \sqrt{3} I_{BC} \angle -30^\circ$$

$$I_c = \sqrt{3} I_{CA} \angle -30^\circ$$

Balanced 3-phase systems

Balanced Δ - Δ Connection



$$V_{ab} = V_p \angle 0^\circ$$

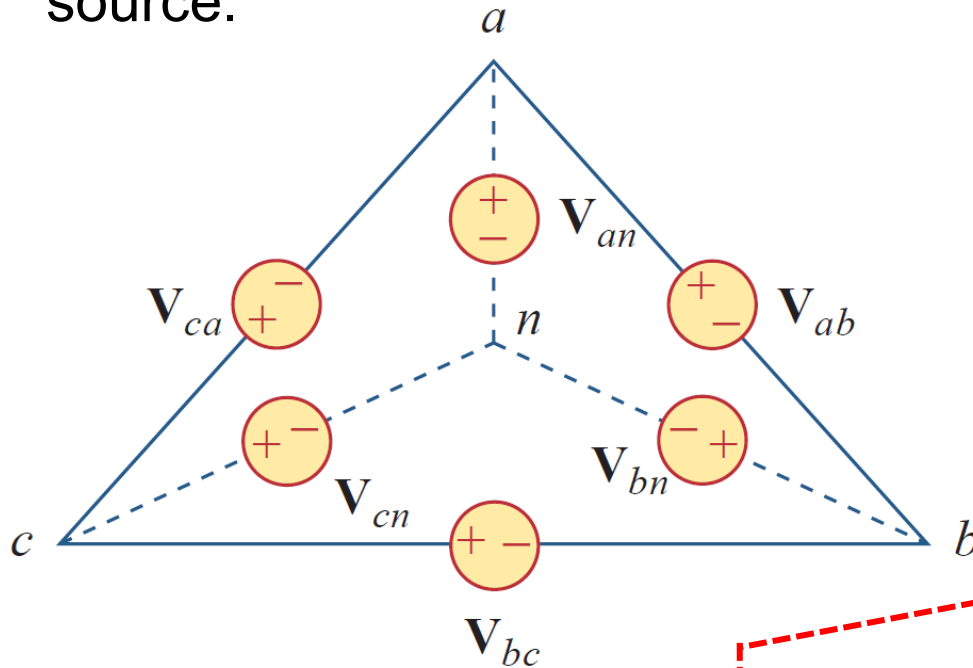
$$V_{bc} = V_p \angle -120^\circ$$

$$V_{ca} = V_p \angle 120^\circ$$

Alternatively, by transforming the Δ connections to the equivalent Y connections per phase equivalent circuit analysis can be performed.

Balanced 3-phase systems Balanced Δ - Δ Connection

Transforming a Δ -connected source to an equivalent Y -connected source.



$$V_{ab} = V_p \angle 0^\circ$$

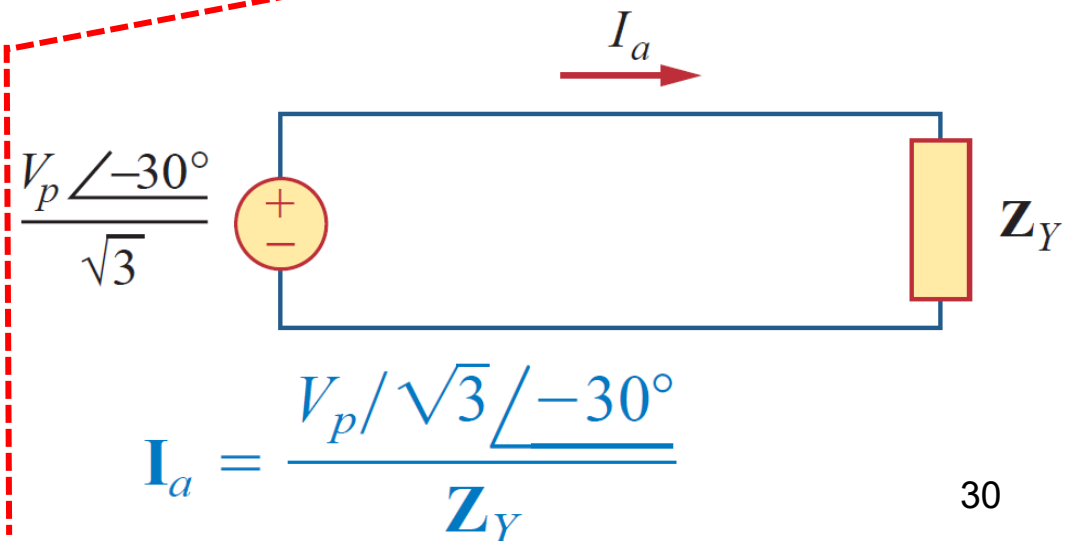
$$V_{bc} = V_p \angle -120^\circ$$

$$V_{ca} = V_p \angle +120^\circ$$

$$V_{an} = \frac{V_p}{\sqrt{3}} \angle -30^\circ$$

$$V_{bn} = \frac{V_p}{\sqrt{3}} \angle -150^\circ$$

$$V_{cn} = \frac{V_p}{\sqrt{3}} \angle +90^\circ$$



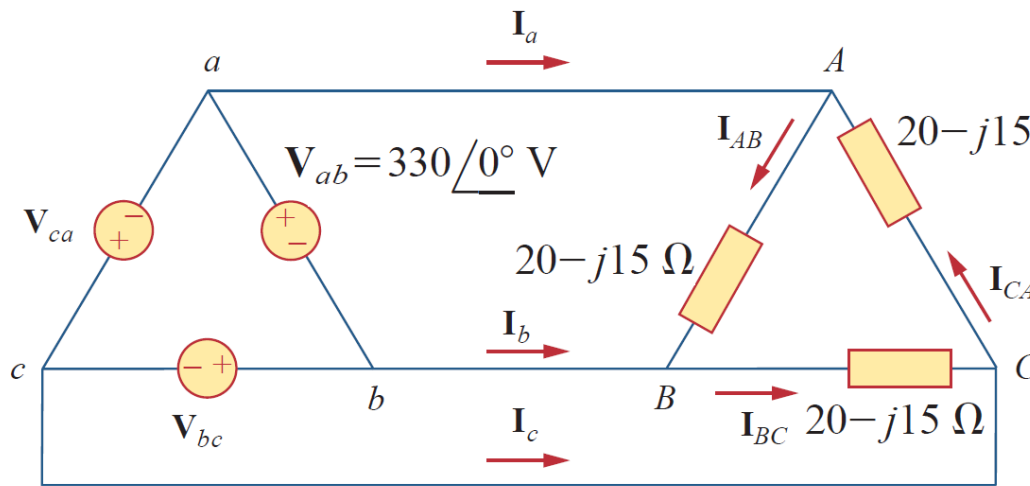
How to get V_{an} , V_{bn} , V_{cn} ?

Balanced 3-phase systems

Balanced Δ - Δ Connection

Example A balanced Δ -connected load having an impedance $20 - j15 \Omega$ is connected to a Δ -connected, positive-sequence generator having $V_{ab} = 330 \angle 0^\circ$ V. Calculate the phase currents of the load and the line currents.

Solution:



The load impedance per phase is
 $\mathbf{Z}_{\Delta} = 20 - j15 = 25 \angle -36.87^\circ \Omega$

Since $V_{AB} = V_{ab}$:

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{330 \angle 0^\circ}{25 \angle -36.87^\circ} = 13.2 \angle 36.87^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB} \angle -120^\circ = 13.2 \angle -83.13^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB} \angle +120^\circ = 13.2 \angle 156.87^\circ \text{ A}$$

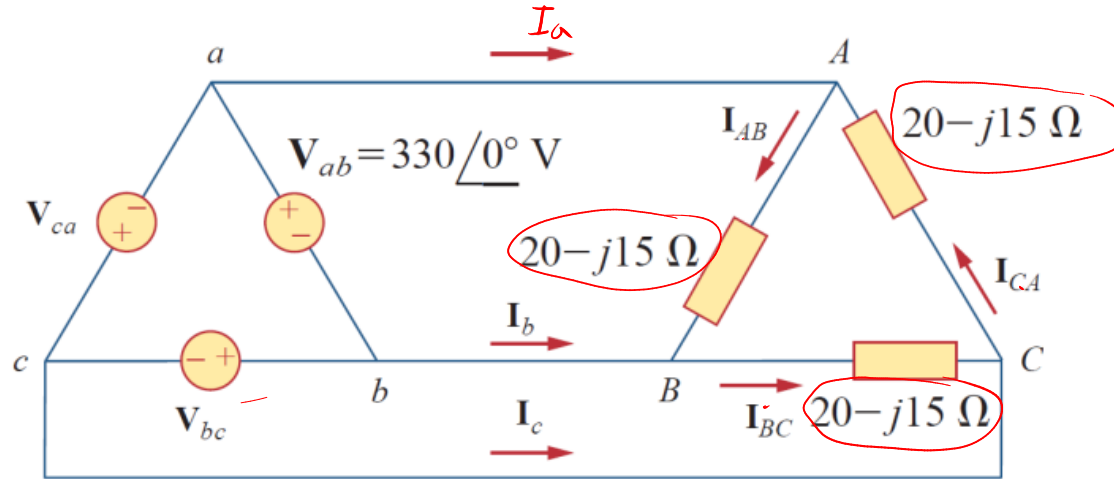
The line currents:

$$\begin{aligned} \mathbf{I}_a &= \mathbf{I}_{AB} \sqrt{3} \angle -30^\circ \\ &= (13.2 \angle 36.87^\circ)(\sqrt{3} \angle -30^\circ) \\ &= 22.86 \angle 6.87^\circ \text{ A} \end{aligned}$$

$$\begin{aligned} \mathbf{I}_b &= \mathbf{I}_a \angle -120^\circ \\ &= 22.86 \angle -113.13^\circ \text{ A} \end{aligned}$$

$$\begin{aligned} \mathbf{I}_c &= \mathbf{I}_a \angle +120^\circ \\ &= 22.86 \angle 126.87^\circ \text{ A} \end{aligned}$$

Example A balanced Δ -connected load having an impedance $20 - j15 \Omega$ is connected to a Δ -connected, positive-sequence generator having $V_{ab} = 330 \angle 0^\circ$ V. Calculate the phase currents of the load and the line currents.



① Phase/line voltage

$$V_{ab} = V_{AB} = 330 \angle 0^\circ \text{ V}$$

$$V_{bc} = V_{BC} = 330 \angle -120^\circ \text{ V}$$

$$V_{ca} = V_{CA} = 330 \angle +120^\circ \text{ V}$$

② Phase current

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{330 \angle 0^\circ}{20 - j15}$$

$$I_{AB} = 13.2 \angle 36.87^\circ \text{ A}$$

$$I_{BC} = I_{AB} \angle -120^\circ = 13.2 \angle -83.13^\circ \text{ A}$$

$$I_{CA} = I_{AB} \angle 120^\circ = 13.2 \angle 156.87^\circ \text{ A}$$

③ line current

$$I_a = \sqrt{3} I_{AB} \angle -30^\circ$$

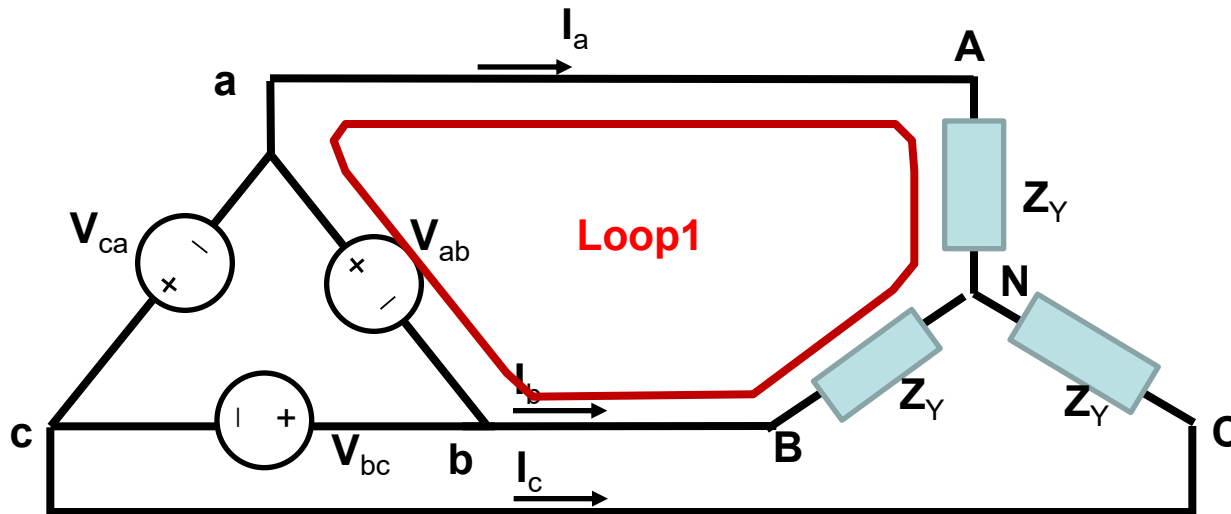
$$I_a = 22.86 \angle 6.87^\circ \text{ A}$$

$$I_b = 22.86 \angle -113.13^\circ \text{ A}$$

$$I_c = 22.86 \angle 126.87^\circ \text{ A}$$

Balanced 3-phase systems

Balanced Δ -Y Connection



$$V_{ab} = V_p \angle 0^\circ$$

$$V_{bc} = V_p \angle -120^\circ$$

$$V_{ca} = V_p \angle 120^\circ$$

How to find I_a ? Use KVL at the loop1: b-a-A-B-b.

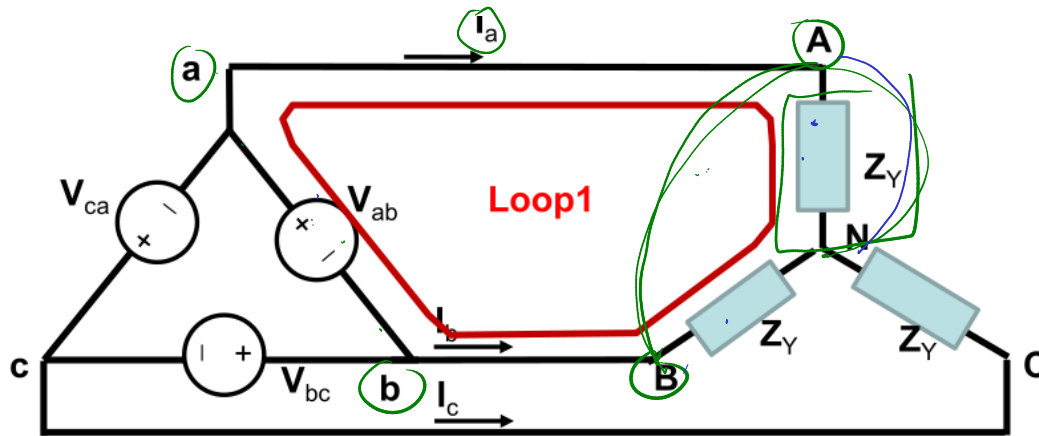
$$\text{Loop1} \quad -V_{ab} + Z_Y I_a - Z_Y I_b = 0 \quad \Rightarrow I_a - I_b = \frac{V_{ab}}{Z_Y}$$

$$\text{Since circuit is balanced, } I_b = I_a \angle -120^\circ \quad \Rightarrow I_a - I_b = I_a (1 - 1 \angle (-120^\circ))$$

$$= I_a \sqrt{3} \angle 30^\circ$$

$$\text{Therefore } I_a = \frac{V_p / \sqrt{3}}{Z_Y} \angle -30^\circ$$

Balanced Δ -Y Connection



Line voltage

$$V_{ab} = V_p \angle 0^\circ = V_{AB}$$

$$V_{bc} = V_p \angle -120^\circ = V_{BC}$$

$$V_{ca} = V_p \angle 120^\circ = V_{CA}$$

Phase load

$$V_{AB} = \sqrt{3} V_{AN} \angle 30^\circ$$

$$V_{AN} = \frac{V_{AB}}{\sqrt{3}} \angle -30^\circ$$

$$V_{BN} = V_{AN} \angle -120^\circ$$

$$V_{CN} = V_{AN} \angle 120^\circ$$

Line / Phase load

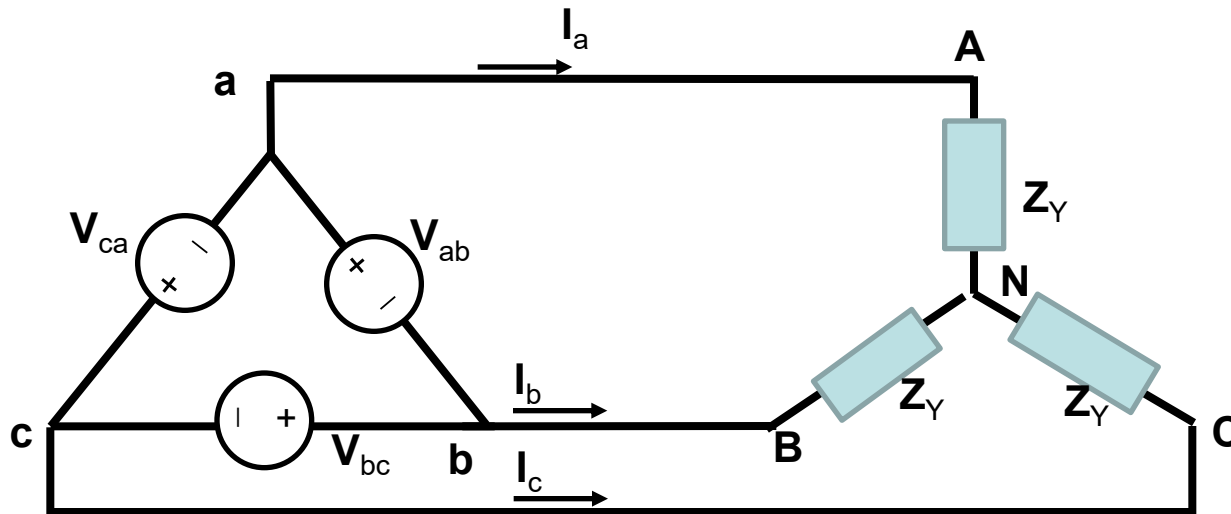
$$I_a = \frac{V_{AN}}{Z_Y} = \frac{\frac{V_{AB}}{\sqrt{3}} \angle -30^\circ}{Z_Y}$$

$$I_b = I_a \angle -120^\circ$$

$$I_c = I_a \angle 120^\circ$$

Balanced 3-phase systems

Balanced Δ -Y Connection



$$V_{ab} = V_p \angle 0^\circ$$

$$V_{bc} = V_p \angle -120^\circ$$

$$V_{ca} = V_p \angle 120^\circ$$

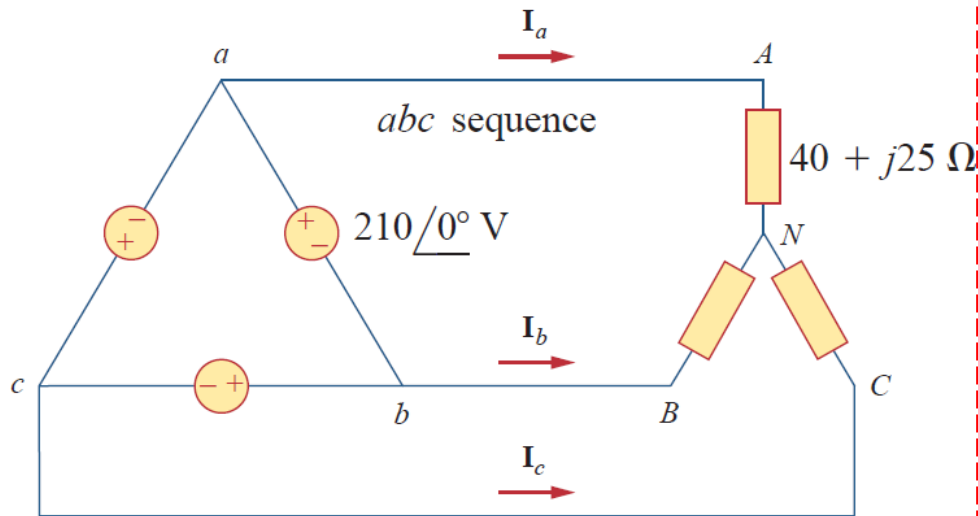
How to find I_a ? (Alternative)

Transform the delta source connection to an equivalent Y and then perform the per phase circuit analysis

Balanced 3-phase systems

Balanced Δ -Y Connection

Example Calculate the phase currents of the load and the line currents.



Solution:

The load impedance is

$$Z_Y = 40 + j25 = 47.17 \angle 32^\circ \Omega$$

and the source voltage is

$$V_{ab} = 210 \angle 0^\circ \text{ V}$$

When the Δ -connected source is transformed to a Y-connected source,

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle -30^\circ = 121.2 \angle -30^\circ \text{ V}$$

The line currents are

$$I_a = \frac{V_{an}}{Z_Y} = \frac{121.2 \angle -30^\circ}{47.12 \angle 32^\circ} = 2.57 \angle -62^\circ \text{ A}$$

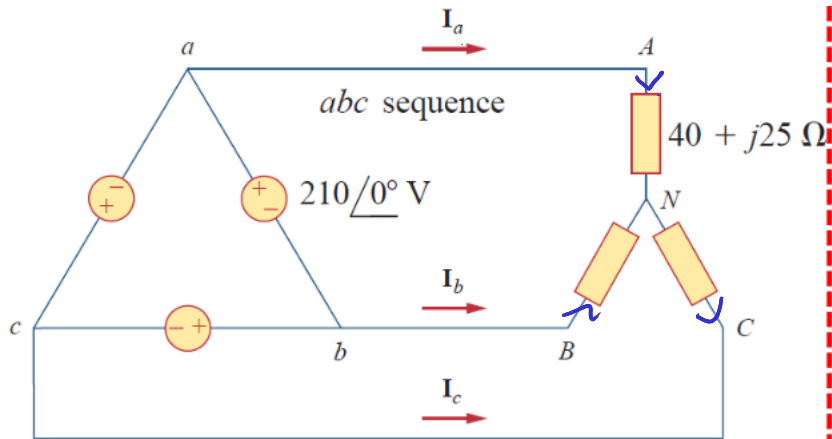
$$I_b = I_a \angle -120^\circ = 2.57 \angle -178^\circ \text{ A}$$

$$I_c = I_a \angle 120^\circ = 2.57 \angle 58^\circ \text{ A}$$

What are the phase currents?

They are the same as the phase currents.

Example Calculate the phase currents of the load and the line currents.



$$V_{ab} = 210 \angle 0^\circ = V_{AB}$$

$$V_{bc} = 210 \angle -120^\circ = V_{BC}$$

$$V_{ca} = 210 \angle 120^\circ = V_{CA}$$

$$I_a = \frac{\frac{210 \angle 0^\circ}{\sqrt{3}}}{40 + j25} \approx 1 \angle -30^\circ = 2.57 \angle -62^\circ \text{ A}$$

$$I_b = 2.57 \angle -182^\circ \text{ A}$$

$$I_c = 2.57 \angle 58^\circ \text{ A}$$

phase load
Line current

Balanced 3-phase systems Summary

Summary of phase and line voltages/currents for balanced three-phase systems.¹

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$V_{an} = V_p/0^\circ$ $V_{bn} = V_p/-120^\circ$ $V_{cn} = V_p/+120^\circ$ Same as line currents	$V_{ab} = \sqrt{3}V_p/30^\circ$ $V_{bc} = V_{ab}/-120^\circ$ $V_{ca} = V_{ab}/+120^\circ$ $I_a = V_{an}/Z_Y$ $I_b = I_a/-120^\circ$ $I_c = I_a/+120^\circ$
Y-Δ	$V_{an} = V_p/0^\circ$ $V_{bn} = V_p/-120^\circ$ $V_{cn} = V_p/+120^\circ$ $I_{AB} = V_{AB}/Z_\Delta$ $I_{BC} = V_{BC}/Z_\Delta$ $I_{CA} = V_{CA}/Z_\Delta$	$V_{ab} = V_{AB} = \sqrt{3}V_p/30^\circ$ $V_{bc} = V_{BC} = V_{ab}/-120^\circ$ $V_{ca} = V_{CA} = V_{ab}/+120^\circ$ $I_a = I_{AB}\sqrt{3}/-30^\circ$ $I_b = I_a/-120^\circ$ $I_c = I_a/+120^\circ$
Δ-Δ	$V_{ab} = V_p/0^\circ$ $V_{bc} = V_p/-120^\circ$ $V_{ca} = V_p/+120^\circ$ $I_{AB} = V_{ab}/Z_\Delta$ $I_{BC} = V_{bc}/Z_\Delta$ $I_{CA} = V_{ca}/Z_\Delta$	Same as phase voltages $I_a = I_{AB}\sqrt{3}/-30^\circ$ $I_b = I_a/-120^\circ$ $I_c = I_a/+120^\circ$
Δ-Y	$V_{ab} = V_p/0^\circ$ $V_{bc} = V_p/-120^\circ$ $V_{ca} = V_p/+120^\circ$ Same as line currents	Same as phase voltages $I_a = \frac{V_p/-30^\circ}{\sqrt{3}Z_Y}$ $I_b = I_a/-120^\circ$ $I_c = I_a/+120^\circ$

¹ Positive or abc sequence is assumed.

Balanced 3-phase systems Power in a Balanced System

Assume we have an “abc” sequence and Y-Y balanced connection:

$$V_{an} = V_p \angle 0^\circ, \quad V_{bn} = V_p \angle -120^\circ, \quad V_{cn} = V_p \angle +120^\circ$$

$$\text{Let } \mathbf{Z}_Y = \mathbf{Z} \angle \theta,$$

$$\mathbf{I}_a = \frac{V_{an}}{\mathbf{Z} \angle \theta} = \frac{V_p \angle (-\theta)}{\mathbf{Z}}, \quad \text{Let } \mathbf{I}_p = \frac{V_p}{\mathbf{Z}}$$

$$\Rightarrow \mathbf{I}_a = \mathbf{I}_p \angle (-\theta), \quad \mathbf{I}_b = \mathbf{I}_p \angle -(\theta + 120^\circ), \quad \mathbf{I}_c = \mathbf{I}_p \angle -(\theta - 120^\circ)$$

We know that: **The complex power** $S = VI^*$

$$\Rightarrow \text{The complex power per phase: } S_p = V_p \angle \theta V_p (I_p \angle -\theta_{I_p})^*$$

$$S_a = V_p \angle 0^\circ (I_p \angle -\theta)^* = V_p \angle 0^\circ (I_p \angle +\theta) = V_p I_p \angle +\theta$$

$$S_b = V_p \angle -120^\circ (I_p \angle -(\theta + 120^\circ))^* = V_p \angle -120^\circ (I_p \angle +(\theta + 120^\circ)) = V_p I_p \angle +\theta$$

$$S_c = V_p \angle +120^\circ (I_p \angle -(\theta - 120^\circ))^* = V_p \angle +120^\circ (I_p \angle +(\theta - 120^\circ)) = V_p I_p \angle +\theta$$

$$\text{The total complex power: } S = S_a + S_b + S_c = 3V_p I_p \angle \theta$$

Balanced 3-phase systems

Power in a Balanced System

∴ The complex power (VA) $S = P + jQ$

The **total active** (real/average) power (W): $P = 3V_p I_p \cos\theta$ (1)

The **total reactive** power (VAR): $Q = 3V_p I_p \sin\theta$ (2)

P is constant in time! It is not sinusoidal anymore.

The **apparent power** per phase (VA): $S_p = V_p I_p$

In terms of Line parameters:

Since $V_L = \sqrt{3} V_p$ in **Y-Y** connection, while $I_L = I_p$

$$\Rightarrow \text{from (1)} \quad P = 3V_p I_p \cos\theta = 3 \frac{V_L}{\sqrt{3}} I_L \cos\theta \Rightarrow P = \sqrt{3} V_L I_L \cos\theta$$

$$\text{and } Q = \sqrt{3} V_L I_L \sin\theta$$

These formulas are the same for all other connections:

Y-Δ, Δ-Δ and Δ-Y

$$\text{The total complex power } S = 3S_p = 3V_p I_p^* = 3I_p^2 Z_p = \frac{3V_p^2}{Z_p^*}$$

$$Z_p = Z_p \angle \theta$$

and could be Z_Y or Z_Δ

Or

$$S = P + jQ = \sqrt{3} V_L I_L \angle \theta$$

θ is the angle of load impedance or angle between V_p and I_p

Balanced 3-phase systems

Notes

- Δ -connected loads are most common to allow easy addition and removal of loads in each phase
- Y-connected sources are most common to avoid circulating currents when there is a small imbalance.

power in 3 phase
القدرة الفعلية

القدرة الظاهرة
apparent power S

active power P

القدرة غير الفعلية
reactive power Q

$$S = 3 V_p \times I_p^*$$

$$P = S \cos \theta = S \cdot PF$$

$$Q = S \sin \theta$$

مقابل $\cos \theta$ ال phase

$$P = 3 V_p I_p \cos \theta$$

$$Q = 3 V_p I_p \sin \theta$$

$$S = \sqrt{3} V_L \times I_L^*$$

مقابل $\cos \theta$ ال line

$$P = \sqrt{3} V_L I_L \cos \theta$$

$$Q = \sqrt{3} V_L I_L \sin \theta$$

$$S = \boxed{} + \boxed{} j$$

$\downarrow$
 P

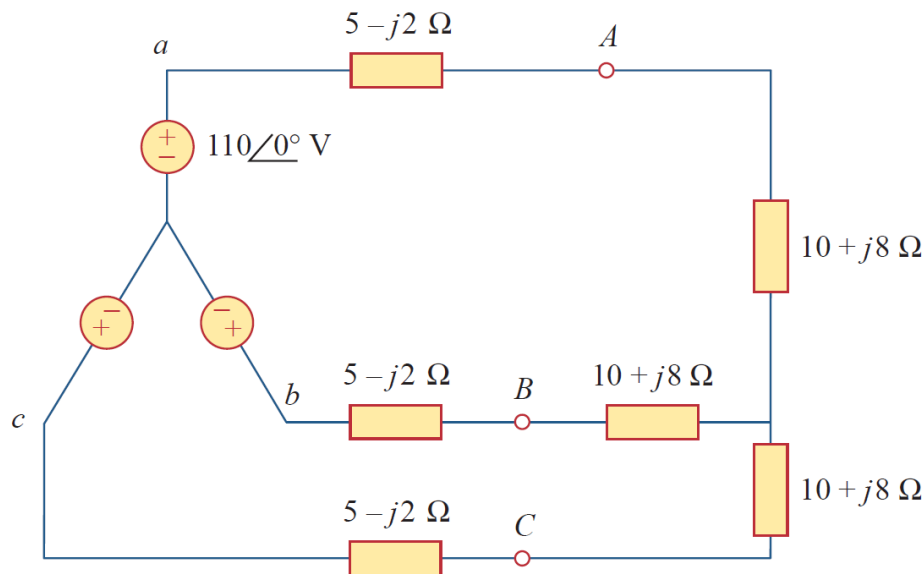
$\downarrow$
 Q

$$I_p = 5 \angle 10^\circ \rightarrow \bar{I}_p^* = 5 \angle -10^\circ$$

Balanced 3-phase systems

Power in a Balanced System

Example: In a Y-Y three-phase system below, suppose a positive sequence Connection. Find P , Q , and S at the source and at the load



Solution:

We found that in the previous example:

$$I_a = 6.81 \angle -21.8^\circ \text{ A}$$

$$I_b = 6.81 \angle -141.8^\circ \text{ A}$$

$$I_c = 6.81 \angle 98.2^\circ \text{ A}$$

For phase "a":

$$V_p = 110 \angle 0^\circ, \quad I_p = 6.81 \angle -21.8^\circ$$

We take one phase:

At the source:

$$\begin{aligned} S_s &= 3V_p I_p^* = 3 \times 110 \angle 0^\circ \times 6.81 \angle +21.8^\circ \\ &= 2247 \angle 21.8^\circ \\ &= \underset{P}{(2087)} + j \underset{Q}{834.6} \text{ VA} \end{aligned}$$

$$\Rightarrow P = 2087 \text{ W}, \quad Q = 834.6 \text{ VAR}$$

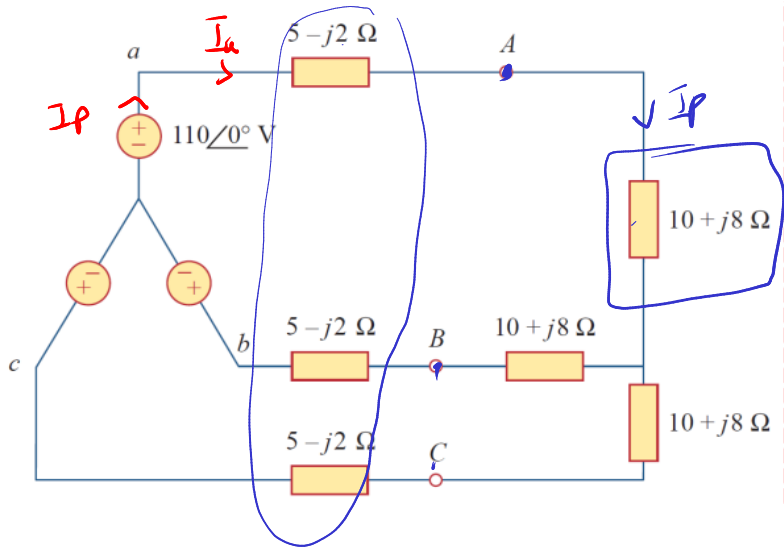
At the load:

$$Z_p = 12.81 \angle 38.66^\circ \Omega,$$

$$I_p = I_a = 6.81 \angle -21.8^\circ \text{ A}$$

$$\begin{aligned} S_L &= 3V_p I_p^* = 3(I_p Z_p) I_p^* \\ &= 3 \times 6.81 \angle -21.8^\circ \times 6.81 \angle 21.8^\circ \times 12.81 \angle 38.66^\circ \\ \Rightarrow S_L &= 1782 \angle 38.66^\circ \end{aligned}$$

Example: In a Y-Y three-phase system below, suppose a positive sequence Connection.
Find P , Q , and S at the source and at the load



$$\begin{aligned} I_a &= 6.81 \angle -21.8^\circ \text{ A} \\ I_b &= 6.81 \angle -141.8^\circ \text{ A} \\ I_c &= 6.81 \angle 98.2^\circ \text{ A} \end{aligned}$$

losses in lines

$$P_L = P_s - P_L = 2086.57 - 1391.28$$

$$P_L = 695.3 \text{ W} \quad \text{S}_L = 695.3 - 279j$$

$$Q_L = -279 \text{ VAR}$$

Source

$$S = 3 V_p I_p^* = 3 \times 110 \angle 0^\circ \times 6.81 \angle +21.8^\circ$$

$$S = 2247.3 \angle 21.8^\circ \text{ VA}$$

$$S = 2086.58 + 834.57j \text{ VA}$$

$$P = 2086.57 \text{ W}$$

$$Q = 834.57 \text{ VAR}$$

load

$$S = 3 V_p I_p^* = 3 (I_p Z_p) \times I_p^*$$

$$S = 3 \times 6.81 \angle -21.8^\circ (10 + j8) \times 6.81 \angle +21.8^\circ$$

$$S = 1781.71 \angle 38.65^\circ \text{ VA}$$

$$S = 1391.28 + 1113.02j \text{ VA}$$

$$P = 1391.28 \text{ W}$$

$$Q = 1113.02 \text{ VAR}$$

Balanced 3-phase systems

$$\Rightarrow S_L = (1392 + j 1113) \text{ VA}$$

$$\Rightarrow P_L = 1392 \text{ W}, \quad Q_L = 1113 \text{ VAR}$$

The difference between the two complex powers is absorbed by the line impedance $(5 - j2)$

So, the power loss S_l :

$$\begin{aligned} S_S - S_L &= (2087 - 1392) + j(834.6 - 1113) \\ &= (695.6 - j 278.3) \text{ VA} \end{aligned}$$

Another way to calculate the loss:

$$\begin{aligned} S_l &= 3|I_p|^2 Z_l = 3(6.81)^2(5 - j2) \\ &= 695.6 - j278.3 \text{ VA} \end{aligned}$$

which is the difference between and that is,

$$S_S + S_l + S_L = 0$$

as expected.

Balanced 3-phase systems

Example:

A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

Solution:

The apparent power is

$$S = \sqrt{3}V_L I_L = \sqrt{3}(220)(18.2) = 6935.13 \text{ VA}$$

Since the real power is

$$P = S \cos \theta = 5600 \text{ W}$$

the power factor is

$$\text{pf} = \cos \theta = \frac{P}{S} = \frac{5600}{6935.13} = 0.8075$$

Example:

A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

Solution:

$$P = 5.6 \text{ kW} = 5.6 \times 10^3 = 5600 \text{ W} \quad V_L = 220 \text{ V} \quad I_L = 18.2 \text{ A}$$

$$S = \sqrt{3} V_L I_L = \sqrt{3} \times 220 \times 18.2 = 6935.13 \text{ VA}$$

$$P = S \cos \theta = S \cdot \text{PF}$$

$$5600 = 6935.13 \times \text{PF}$$

$$\text{PF} = 0.8075$$