Chapter (1)

Introduction

A *fluid* is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be. Because the fluid motion continues under the application of a shear stress, we can also define a fluid as any substance that cannot sustain a shear stress when at rest.

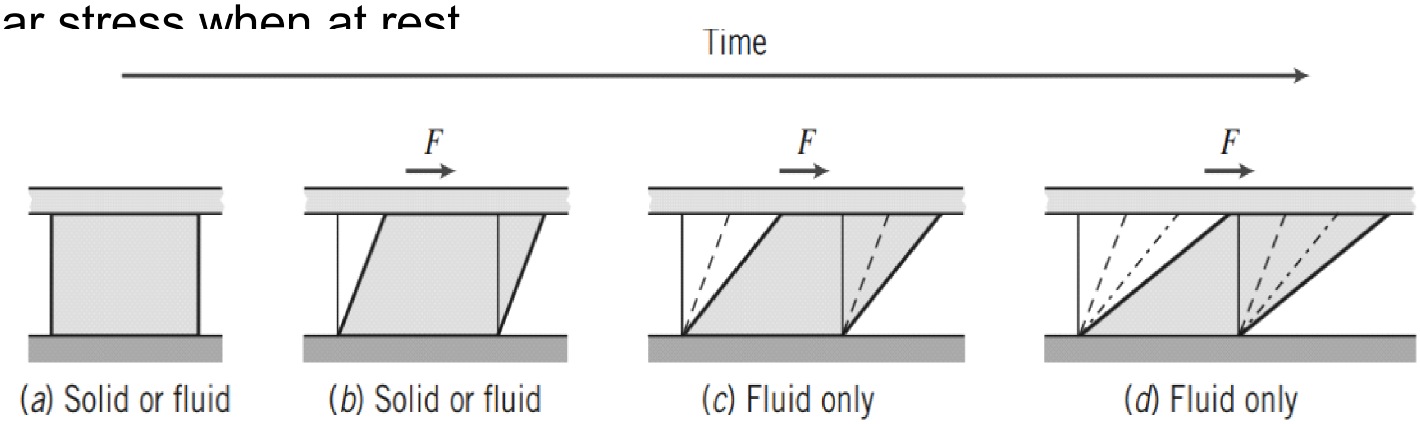


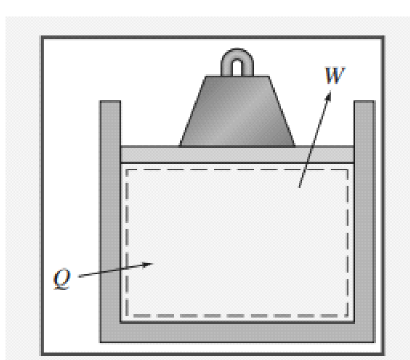
Fig. 1.1 Difference in behavior of a solid and a fluid due to a shear force.

1.2 *Basic Equations*

Analysis of any problem in fluid mechanics necessarily includes statement of the basic laws governing the fluid motion. The basic laws, which are applicable to any fluid, are:

1. The conservation of mass
2. Newton's second law of motion (also termed the principle of linear momentum)
3. The principle of angular momentum
4. The first law of thermodynamics
5. The second law of thermodynamics

Example 1.1



Example 1.1 FIRST LAW APPLICATION TO CLOSED SYSTEM

A piston-cylinder device contains 0.95 kg of oxygen initially at a temperature of 27°C and a pressure due to the weight of 150 kPa (abs). Heat is added to the gas until it reaches a temperature of 627°C. Determine the amount of heat added during the process.

Solution

Givens

$m = 0.95$ kg

$T_1 = 27\ °C$

$P = 150$ kPa

$T_2 = 627\ °C$

Required

$Q_{1\to2}$

Assumptions

1) $E = U$

2) Ideal gas

with C_v = Constant

Steps

$\Delta E = \Delta U = m C_v \Delta T$

from 1st law of Thermodynamics

$\Delta U = Q - W$

$W = \int_{V_1}^{V_2} P\,dV = P \Delta V$

$\therefore m C_v \Delta T = Q - P \Delta V$

but $P = \frac{mRT}{V}$;

$m C_v \Delta T = Q - m R \Delta T$

$Q = m \Delta T (C_v - R)$

but $R = C_v - C_p$

$Q = m C_p \Delta T$

$= 0.95 * 909.4 * (627 - 27)$

$Q = 518$ KJ #

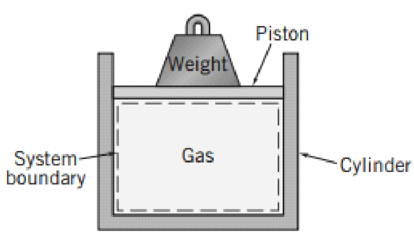
1.3 *Methods of Analysis*

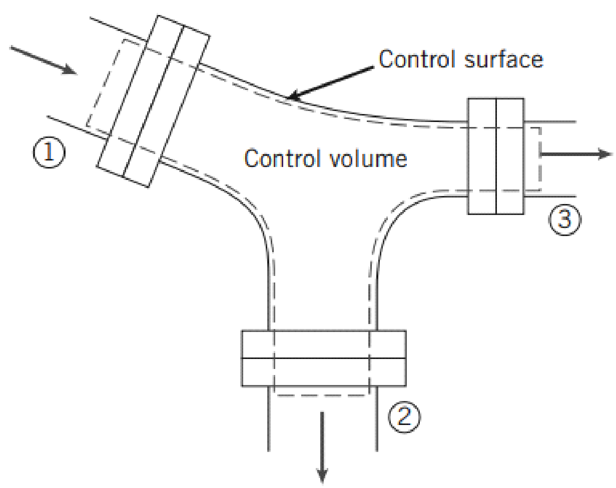
In basic mechanics, we made extensive use of the *free-body diagram*. In fluid mechanics, we will use a *system* or a *control volume*, depending on the problem being studied. In thermodynamics we applied the conservation of mass and the first and second laws of thermodynamics in most problems. In our study of fluid mechanics, we will usually apply conservation of mass and Newton's second law of motion. In thermodynamics our focus was energy; in fluid mechanics it will mainly be forces and motion.

System and Control Volume

A *system* is defined as a fixed, identifiable quantity of mass with the system boundaries separating the system from the surroundings. The boundaries of the system may be fixed or movable; however, no mass crosses the system boundaries.

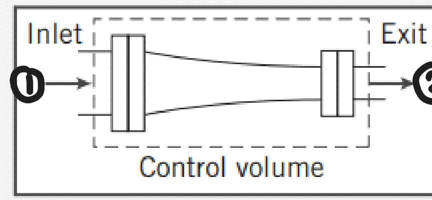
A *control volume* is an arbitrary volume in space through which fluid flows. The geometric boundary of the control volume is called the control surface. The control surface may be real or imaginary; it may be at rest or in motion.





Example 1.2 MASS CONSERVATION APPLIED TO CONTROL VOLUME

A reducing water pipe section has an inlet diameter of 50 mm and exit diameter of 30 mm. If the steady inlet speed (averaged across the inlet area) is 2.5 m/s, find the exit speed.



Solution

Givens: $d_1 = 50\,mm$, $V_1 = 2.7\,m/s$

$d_2 = 30\,mm$, $V_2 = ?$

Assumption: Incompressible flow ($\rho = \text{Constant}$)

Steps

* Conservation of mass:

$$\dot{m}_1 = \dot{m}_2$$

$$\cancel{\rho} A_1 V_1 = \cancel{\rho} A_2 V_2$$

$$\cancel{\frac{\pi}{4}} d_1^2 V_1 = \cancel{\frac{\pi}{4}} d_2^2 V_2$$

$$V_2 = \frac{d_1^2}{d_2^2} V_1 = \left(\frac{50 \times \cancel{10^{-3}}}{30 \times \cancel{10^{-3}}}\right)^2 \times 2.7 = 7.5\ m/s$$

\#

Proof

$$m = \rho \forall = \rho A S$$

$$\frac{dm}{dt} = \dot{m} = \rho A \frac{dS}{dt}$$

$$\boxed{\dot{m} = \rho A V}$$

Differential versus Integral Approach

In the case of *infinitesimal* systems the resulting equations are differential equations. Solution of the differential equations of motion provides a means of determining the detailed behavior of the flow.

Frequently the information sought does not require a detailed knowledge of the flow. We often are interested in the gross behavior of a device; in such cases it is more appropriate to use integral formulations of the basic laws. An example might be the overall lift a wing produces. Integral formulations, using *finite* systems or control volumes, usually are easier to treat analytically.

Methods of Description

We use a method of description that follows the particle when we want to keep track of it. This sometime is referred to as the *Lagrangian* method of description. Consider, for example, the application of

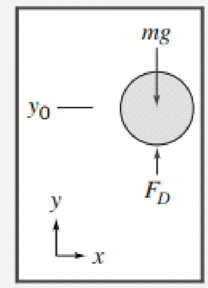
Newton's second law to a particle of fixed mass. Mathematically, we can write Newton's second law for a system of mass m as

$$\Sigma\vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = m\frac{d^2\vec{r}}{dt^2} \tag{1.2}$$

In Eq. 1.2, $\Sigma\vec{F}$ is the sum of all external forces acting on the system, $\vec{a}$ is the acceleration of the center of mass of the system, $\vec{V}$ is the velocity of the center of mass of the system, and $\vec{r}$ is the position vector of the center of mass of the system relative to a fixed coordinate system. In Example 1.3, we show how Newton's second law is applied to a falling object to determine its speed.

Example 1.3 FREE FALL OF BALL IN AIR

The air resistance (drag force) on a 200 g ball in free flight is given by $F_D = 2 \times 10^{-4} V^2$, where F_D is in newtons and V is in meters per second. If the ball is dropped from rest 500 m above the ground, determine the speed at which it hits the ground. What percentage of the terminal speed is the result? (The *terminal speed* is the steady speed a falling body eventually attains.)



Givens

$m = 200g$

$F_D = 2\times10^{-4} V^2$

$y_0 = 500m$

Required

1) V

2) V/V_t %

Assumptions

Ignore Buoyancy Force

Steps

$$\Sigma F_y = ma_y = m\frac{dV}{dt}$$

$$F_D - mg = m\frac{dV}{dt}$$

Solution

$$KV^2 - mg = m\frac{dV}{dt} = mV\frac{dV}{dy}$$

$$\int_0^V \frac{mV}{KV^2 - mg}\,dV = \int_{y_0}^{y} dy$$

$$\frac{m}{2K}\int \frac{2KV}{KV^2 - mg}\,dV = (y - y_0)$$

$$\frac{m}{2K}\left(\ln|KV^2 - mg|\right)\Big|_0^V = y - y_0$$

$$\ln|KV^2 - mg| - \ln|-mg| = \frac{2K}{m}(y - y_0)$$

$$\ln\left|\frac{KV^2 - mg}{-mg}\right| = \frac{2K}{m}(y - y_0)$$

$$\frac{KV^2 - mg}{-mg} = e^{\frac{2K}{m}(y - y_0)}$$

$$KV^2 - mg = -mg\, e^{\frac{2K}{m}(y - y_0)}$$

$$kv^2 - mg = -mg\, e^{\frac{2k}{m}(y-y_0)}$$

$$kv^2 = mg - mg\, e^{\frac{2k}{m}(y-y_0)}$$

$$v^2 = \frac{mg}{k}\left[1 - e^{\frac{2k}{m}(y-y_0)}\right]$$

$$v = \left\{\frac{mg}{k}\left[1 - e^{\frac{2k}{m}(y-y_0)}\right]\right\}^{1/2}$$

$$v = \left\{\frac{0.2 * 9.81}{2 \times 10^{-4}}\left[1 - e^{\frac{2*2*10^{-4}}{0.2}(0-500)}\right]\right\}^{1/2}$$

$$\boxed{v = 78.7 \text{ m/s}} \quad \#$$

at terminal speed $\Rightarrow a_y = 0$

$$\sum F_y = ma_y = 0$$

$$F_D - mg = 0$$

$$F_D = mg$$

$$kv_t^2 = mg \Rightarrow v = \sqrt{\frac{mg}{k}}$$

$$v_t = \sqrt{\frac{0.2*9.81}{2\times 10^{-4}}} = 99 \text{ m/s}$$

$$\frac{v}{v_t} = \frac{78.7}{99} \times 100 = 79.5\%$$

1.4 *Dimensions and Units*

System of Dimensions

(a) Mass $[M]$, length $[L]$, time $[t]$, temperature $[T]$
(b) Force $[F]$, length $[L]$, time $[t]$, temperature $[T]$
(c) Force $[F]$, mass $[M]$, length $[L]$, time $[t]$, temperature $[T]$

System of Units

Table 1.1
Common Unit Systems

System of Dimensions	Unit System	Force F	Mass M	Length L	Time t	Temperature T
a. MLtT	Système International d'Unités (SI)	(N)	kg	m	s	°K
b. FLtT	British Gravitational (BG)	lbf	(slug)	ft	s	°R
c. FMLtT	English Engineering (EE)	lbf	lbm	ft	s	°R

Dimensional Consistency and "Engineering" Equations

In engineering, we strive to make equations and formulas have consistent dimensions. That is, each term in an equation, should be reducible to the same dimensions. For example, a very important equation we will derive later on is the Bernoulli equation

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Example 1.4 USE OF UNITS

The label on a jar of peanut butter states its net weight is 510 g. Express its mass and weight in SI, BG, and EE units.

Solution

$$m_{SI} = 0.51\,kg$$

$$\boxed{W = mg}$$

$$m_{EE} = \frac{0.51}{0.454} = 1.12\ lbm$$

$$m_{BG} = \frac{1.12}{32.2} = 0.0347\ slug$$

$$W_{SI} = 0.51 * 9.81 = 5\ N$$

$$W_{EE} = 1.12 * 32.2 = 36.1\ N$$

$$W_{BG} = 0.0347 * 32.2 = 1.12\ N$$