

# CH1/ Introduction

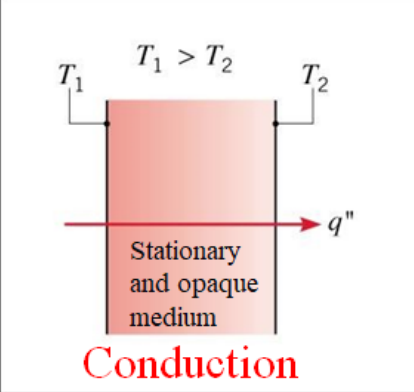
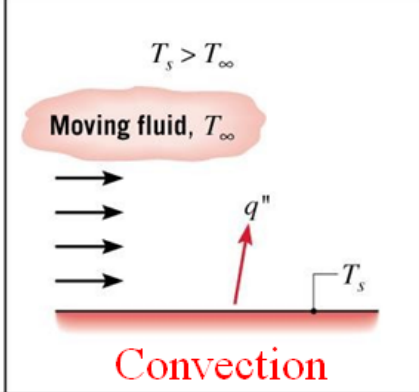
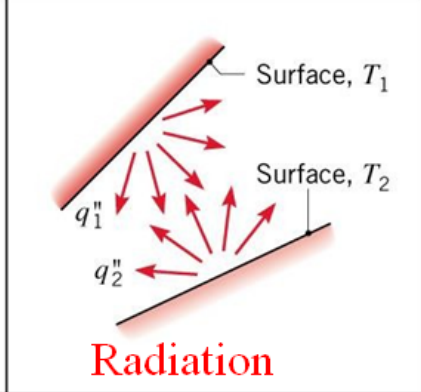
Heat  $\rightarrow \phi \rightarrow J$

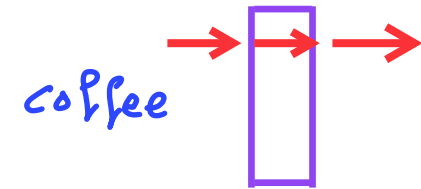
Heat Rate  $\rightarrow q \rightarrow J/s = W$

Heat Flux  $\rightarrow q'' \rightarrow W/m^2$

$q' \rightarrow W/m$

$\phi = q \Delta t$

| Conduction through a solid or a stationary fluid                                                     | Convection from a surface to a moving fluid                                                           | Net radiation heat exchange between two surfaces                                                     |
|------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|
|  <p>Conduction</p> |  <p>Convection</p> |  <p>Radiation</p> |



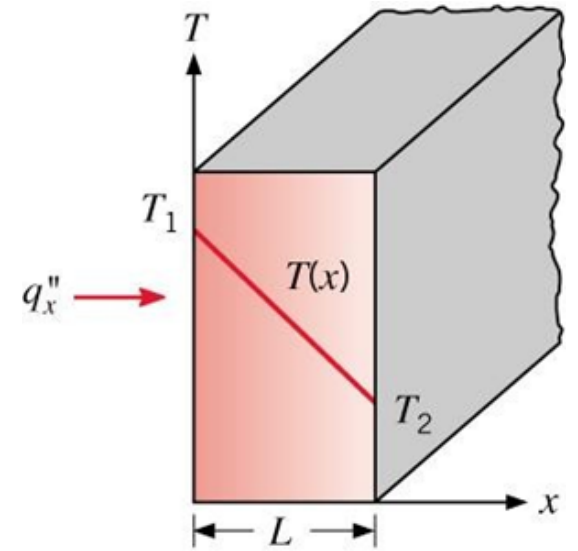
# Conduction      التوصيل

$$q'' = -k \frac{dT}{dx}$$

heat flux  
 $W/m^2$

thermal  
conductivity  
 $W/m^{\circ}C$  or  $W/m^{\circ}K$

Temperature  
gradient  $^{\circ}C/m$   
 $K/m$



$$q''_x = -k \frac{dT}{dx} = -k \cdot \frac{T_2 - T_1}{x_2 - x_1} = k \frac{T_1 - T_2}{L}$$

$$q_x = A q''_x = k A \frac{T_1 - T_2}{L}$$

# Convection جاذ

## Newton's law

Cooling

$$q'' = h (T_s - T_\infty)$$

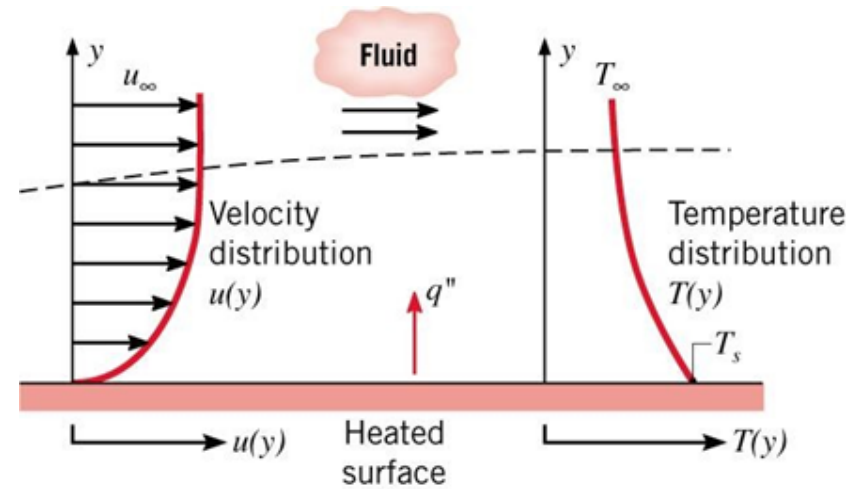
convection  
coefficient  
«fluid»

surface  
temp

fluid  
temp

Heating

$$q'' = h (T_\infty - T_s)$$



# Radiation الإشعاع

$$G_{abs} = \alpha G = \alpha \sigma T_{surr}^4$$

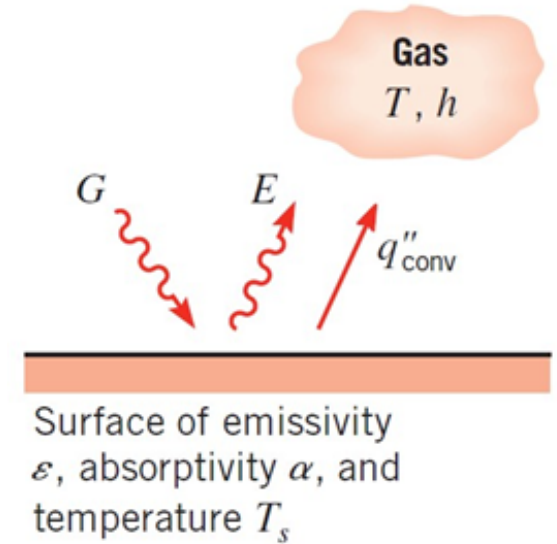
$$G = \sigma T_{surr}^4$$

$$E_b = \sigma T_s^4 \quad \text{W/m}^2$$

$$E = \varepsilon E_b = \varepsilon \sigma T_s^4 \quad 0 \leq \varepsilon \leq 1$$

$$0 \leq \alpha \leq 1$$

- |                                             |                                                                                                                                                                                                                                                                 |
|---------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $E_b \rightarrow$ Blackbody emissive power  | $\left\{ \begin{array}{l} \alpha \rightarrow \text{surface absorptivity} \\ \varepsilon \rightarrow \text{surface emissivity} \\ \sigma \rightarrow \text{Stefan-Boltzmann constant} \\ 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \end{array} \right.$ |
| $E \rightarrow$ Real surface emissive power |                                                                                                                                                                                                                                                                 |
| $G \rightarrow$ Irradiation                 |                                                                                                                                                                                                                                                                 |
| $G_{abs} \rightarrow$ Absorbed irradiation  |                                                                                                                                                                                                                                                                 |





$$q''_{\text{net}} = q''_{\text{emitted}} - q''_{\text{absorb}}$$

$$q''_{\text{rad}} = E - G_{\text{abs}}$$

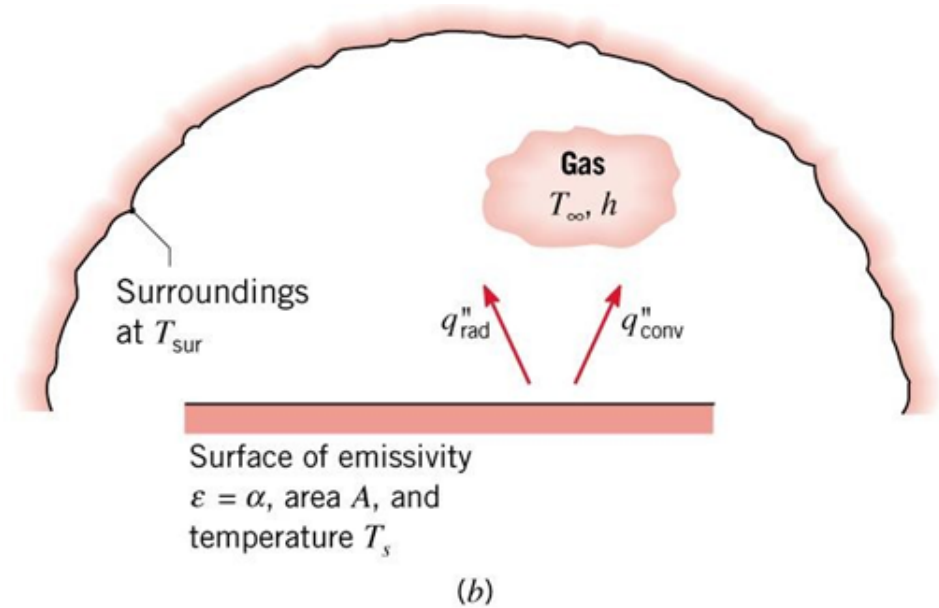
$$q''_{\text{rad}} = \epsilon \sigma T_s^4 - \alpha \sigma T_{\text{sur}}^4$$

$$\epsilon = \alpha$$

$$q''_{\text{rad}} = \epsilon \sigma (T_s^4 - T_{\text{sur}}^4)$$

$$q''_{\text{rad}} = h_r (T_s - T_{\text{sur}})$$

$$h_r = \epsilon \sigma (T_s + T_{\text{sur}})(T_s^2 + T_{\text{sur}}^2)$$

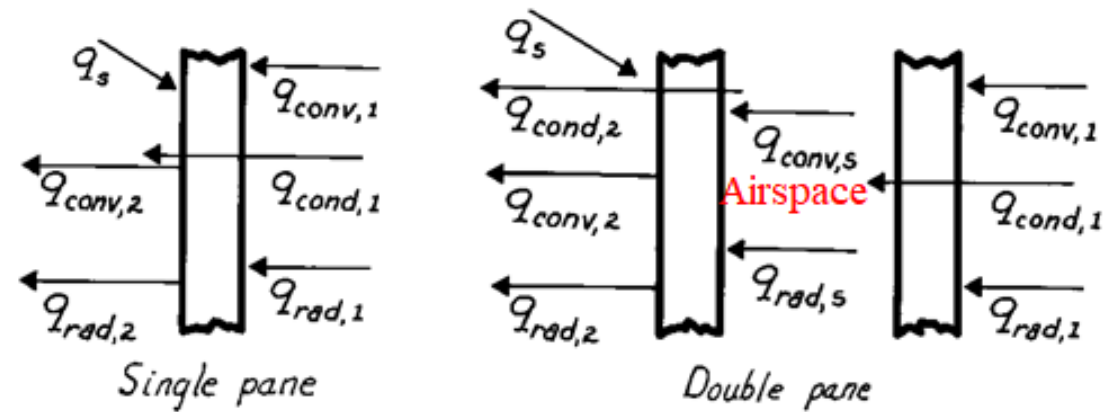
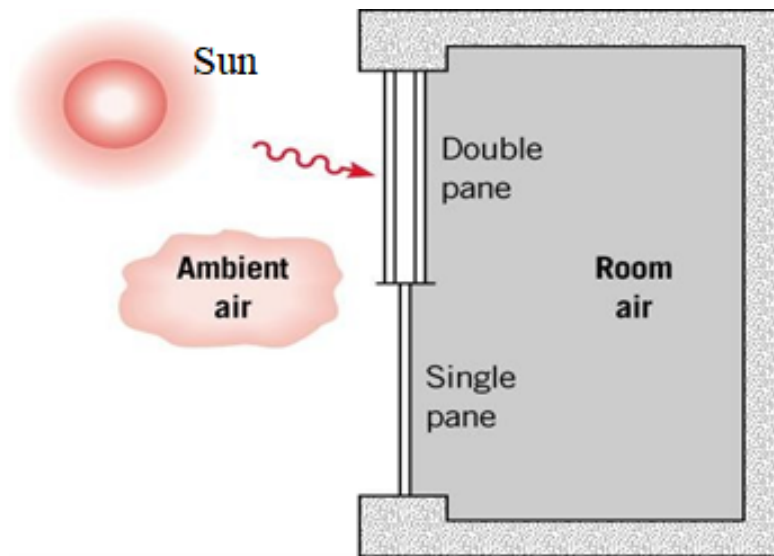


$$q'' = q''_{\text{conv}} + q''_{\text{rad}} = h_c (\underline{T_s - T_{\infty}}) + h_r (\underline{T_s - T_{\text{sur}}})$$

$$T_{\infty} = T_{\text{sur}}$$

$$q'' = (h_c + h_r)(T_s - T_{\infty}) = h_{\text{comb}} (T_s - T_{\infty})$$

$$h_{\text{comb}} = h_c + h_r \rightarrow \text{combined heat transfer coefficient}$$



$q_{conv,1}$  Convection from room air to inner surface of first pane

$q_{rad,1}$  Net radiation exchange between room walls and inner surface of first pane

$q_{cond,1}$  Conduction through first pane

$q_{conv,s}$  Convection across airspace between panes

$q_{rad,s}$  Net rad. exch. between outer surface of 1<sup>st</sup> pane and inner surface of 2<sup>nd</sup> pane (across airspace)

$q_{cond,2}$  Conduction through a second pane

$q_{conv,2}$  Convection from outer surface of single (or second) pane to ambient air

$q_{rad,2}$  Net radiation exchange between outer surface of single (or second) pane and surroundings

$q_s$  Incident solar radiation during day; fraction transmitted to room is smaller for double pane

# Relationship to thermodynamics

At instant time

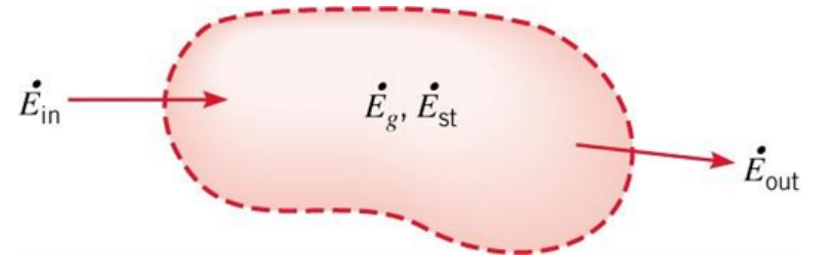
$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} = \frac{dE_{st}}{dt}$$

Rate of energy transfer in and out of system

Rate of energy generation

Rate of energy stored in the system

\*  $\dot{E}_{st} = 0$  for steady process



over time interval

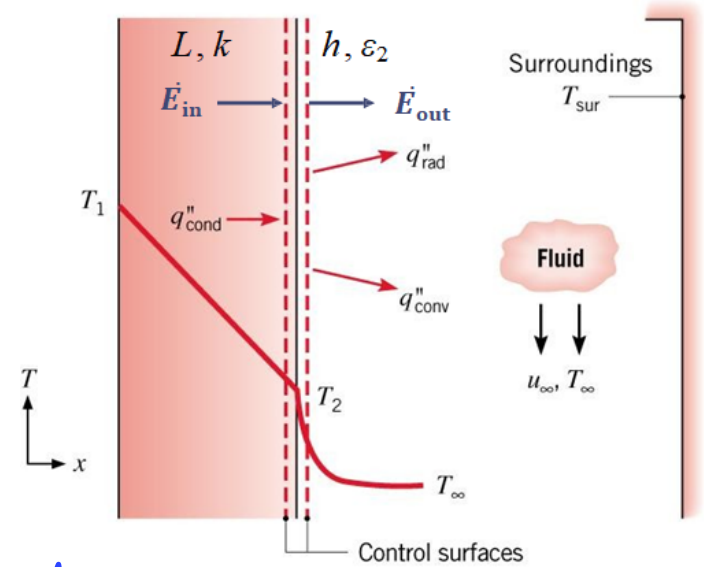
$$E_{in} - E_{out} + E_g = \Delta E_{st}$$

## Surface energy balance

$$\dot{E}_{in} = \dot{E}_{out}$$

$$q''_{cond} = q''_{conv} + q''_{rad}$$

$$\frac{k(T_1 - T_2)}{L} = h(T_2 - T_\infty) + \epsilon \sigma (T_2^4 - T_{sur}^4)$$



$$\dot{E}_g = \dot{q} \cdot V \longrightarrow \text{uniform heat generation}$$

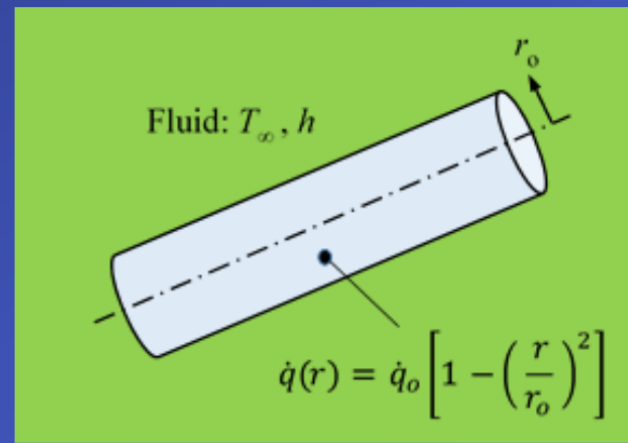
$$\dot{E}_g = \int_V \dot{q} dV \longrightarrow \text{non uniform heat generation}$$

(Incropera 1.44 7<sup>th</sup> Ed. / 1.41 Global)

Radioactive wastes are packed in a long, thin-walled cylindrical container. The wastes generate thermal energy non-uniformly according to the relation:

$$\dot{q}(r) = \dot{q}_o \left[ 1 - \left( \frac{r}{r_o} \right)^2 \right]$$

where  $\dot{q}$  is local rate of energy generation per unit volume,  $\dot{q}_o$  is constant, and  $r_o$  is radius of container. Container is submerged in liquid at  $T_\infty$  with convection coefficient  $h$ .



(4.1) Specify the assumptions relevant to the analysis of this problem.

(4.2) Obtain an expression for the total rate at which energy is generated in the container ( $\dot{E}_g$ ).

(4.3) Use the result in (4.2) to obtain an expression for the temp.  $T_s$  of the container wall.

4.1) \* steady state  
\* 1D in radial direction  
\* constant properties  
\* negligible radiation

$$A_s = 2\pi r_o L$$

$$\underline{4.2)} \quad \dot{E}_g = \int_V \dot{q} dV$$

$$V = \pi r^2 \cdot L \rightarrow dV = 2\pi r L dr$$

$$\dot{E}_g = \int_0^{r_0} \dot{q} (2\pi r L) dr = \dot{q}_0 (2\pi L) \int_0^{r_0} \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right] r dr$$

$$\dot{E}_g = 2\pi L \dot{q}_0 \int_0^{r_0} r - \frac{r^3}{r_0^2} dr = 2\pi L \dot{q}_0 \left[ \frac{r^2}{2} - \frac{r^4}{4r_0^2} \right]_0^{r_0}$$

$$\dot{E}_g = 2\pi L \dot{q}_0 \left[ \frac{r_0^2}{2} - \frac{r_0^4}{4r_0^2} \right] = \frac{2\pi L \dot{q}_0 r_0^2}{4} = \frac{\pi L \dot{q}_0 r_0^2}{2}$$

#

$$\underline{4.3)} \quad \cancel{\dot{E}_{in}} - \dot{E}_{out} + \dot{E}_g = \cancel{\dot{E}_{sf}}$$

$$\dot{E}_g = \dot{E}_{out} = \dot{q}_{conv}$$

$$\frac{\cancel{\pi} \cancel{L} \dot{q}_0 \cancel{r_0}}{2} = h (2 \cancel{\pi} \cancel{r_0}) (T_s - T_\infty)$$

$$T_s - T_\infty = \frac{\dot{q}_0 r_0}{4h}$$

$$\therefore T_s = T_\infty + \frac{\dot{q}_0 r_0}{4h} \quad \neq$$



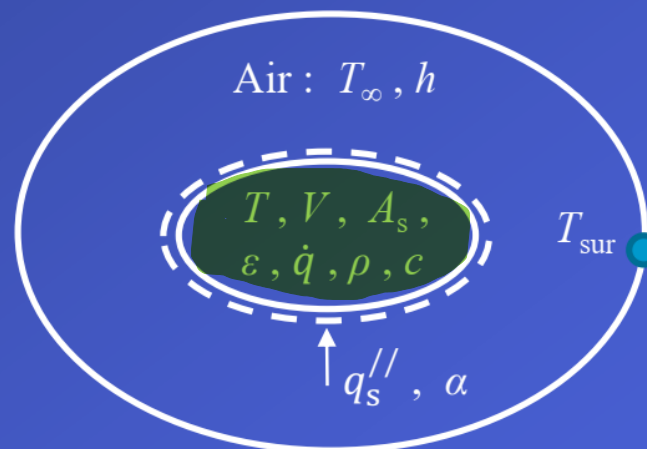
Solid body of temp.  $T$ , volume  $V$ , surface area  $A_s$  and emissivity  $\varepsilon$  has density  $\rho$  and specific heat  $c$ . The body is placed in air at temp.  $T_\infty$  with convection coefficient  $h$ , and is enclosed by blackbody surroundings at temp.  $T_{\text{sur}}$ . Heat is generated uniformly inside the body at rate per unit volume  $\dot{q}$ . The body is subjected to uniform heat flux  $q_s''$  with absorptivity  $\alpha$ .

(1) Assume body temp. to be uniform at  $T$ , develop expression for rate of change of body temp. with time ( $dT/dt$ ) in terms of:  $T, V, A_s, \varepsilon, \rho, c, T_\infty, h, T_{\text{sur}}, \dot{q}, q_s'', \alpha$ , and  $\sigma$ .

(2) Obtain expression for convection coefficient  $h$  under steady-state conditions.

(3) Under s.s., determine  $h$  for body of cubical shape with side length  $L = 3$  cm when:

$$\varepsilon = 0.9, \dot{q} = 5000 \text{ W/m}^3, q_s'' = 125 \text{ W/m}^2, T = 50^\circ\text{C}, T_\infty = 63^\circ\text{C}, T_{\text{sur}} = 10^\circ\text{C}, \text{ and } \alpha = 0.2.$$



$$1) \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$\alpha A_s q_{rs}'' - (q_{conv} + q_{rad}) + \dot{q}V = \rho V C \frac{dT}{dt}$$

$$\alpha A_s q_{rs}'' - [h A_s (T - T_{\infty}) + \epsilon \sigma A_s (T^4 - T_{surr}^4)] + \dot{q}V = \rho V C \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{\alpha A_s q_{rs}'' - [h A_s (T - T_{\infty}) + \epsilon \sigma A_s (T^4 - T_{surr}^4)] + \dot{q}V}{\rho V C} \quad \textcircled{1}$$

$$2) \frac{dT}{dt} = 0$$

$$\alpha A_s q_{rs}'' - [h A_s (T - T_{\infty}) + \epsilon \sigma A_s (T^4 - T_{surr}^4)] + \dot{q}V = 0$$

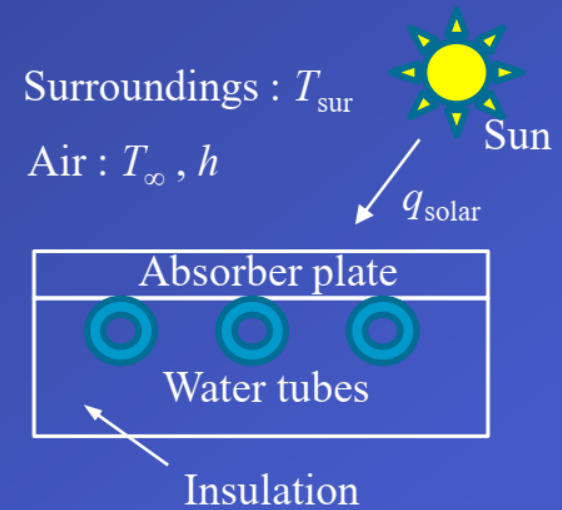
$$h = \frac{\alpha q_{rs}'' A_s - \epsilon \sigma A_s (T^4 - T_{surr}^4) + \dot{q}V}{A_s (T - T_{\infty})} \quad \textcircled{2}$$

$$3) A_s = 6L^2 = 6(0.03)^2 = 5.4 \times 10^{-3} \text{ m}^2 \quad \text{and} \quad V = L^3 = (0.03)^3 = 2.7 \times 10^{-5} \text{ m}^3$$

$$h = \frac{(0.2)(125)(5.4 \times 10^{-3}) - 0.9 \times 5.67 \times 10^{-8} \times 5.4 \times 10^{-3} \times (323^4 - 283^4) + 5000 \times 2.7 \times 10^{-5}}{5.4 \times 10^{-3} \times (50 - 63)}$$

$$h = 13.7 \text{ W/m}^2 \cdot \text{K} \quad \neq$$

Cross-section in flat-plate solar collector is as shown. Collector is used to heat water by having water flow through tubes attached at back of thin solar absorber plate. Absorber plate has surface area of  $4 \text{ m}^2$  with emissivity and absorptivity of 0.9. Surface temp. of absorber is  $35^\circ\text{C}$ , and solar radiation is incident on absorber at  $500 \text{ W/m}^2$  with surroundings temp. ( $T_{\text{sur}}$ ) of  $10^\circ\text{C}$ . Convection h.t. coefficient at absorber surface is  $15 \text{ W/m}^2\cdot\text{K}$ , while air temp. ( $T_\infty$ ) is  $25^\circ\text{C}$ . Net heat rate absorbed by solar collector heats water from inlet temp. ( $T_{\text{in}}$ ) to outlet temp. ( $T_{\text{out}}$ ). If water flow rate is  $10 \text{ g/s}$  with specific heat of  $4.2 \text{ kJ/kg}\cdot\text{K}$ , determine:



- (1) rate of heat transfer to water ( $q_{\text{water}}$ ),
- (2) temp. rise of water ( $\Delta T = T_{\text{out}} - T_{\text{in}}$ ), and
- (3) efficiency of solar collector ( $\eta$ ), which is defined as ratio of usable heat collected by absorber to incident solar radiation on absorber.

1) Energy balance over absorber plate

$$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_g} = \cancel{\dot{E}_{st}}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\alpha q''_{solar} A_s = q_{conv} + q_{rad} + q_{water}$$

$$q_{water} = \alpha q''_{solar} A_s - q_{conv} - q_{rad}$$

$$q_{water} = \alpha q''_{solar} A_s - h A_s (T - T_{\infty}) - \epsilon \sigma A_s (T^4 - T_{sur}^4)$$

$$q_{water} = 0.9 \times 5000 \times 4 - 15 \times 4 (35 - 25) - 0.9 \times (5.67 \times 10^{-8}) \times 4 \times (308^4 - 283^4)$$

$$q_{water} = 672.36 \text{ W} \quad \text{--- ①}$$

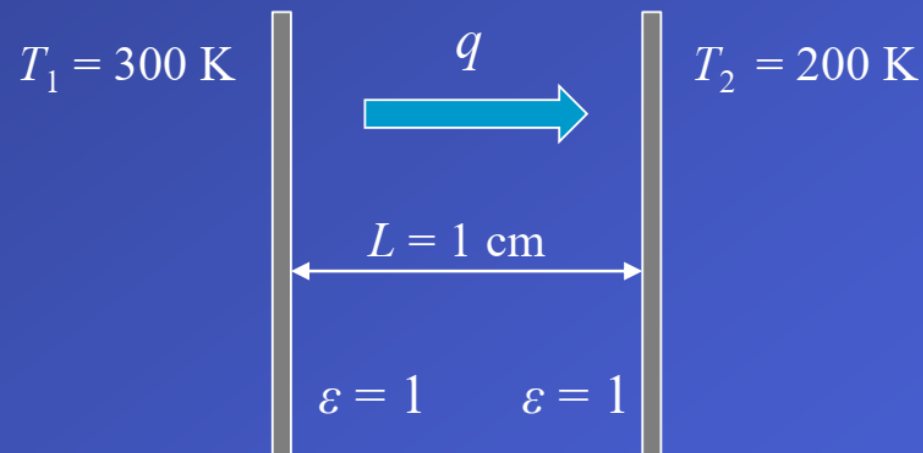
$$2) \dot{q}_{\text{water}} = \dot{m} c_p \Delta T$$

$$T_{\text{out}} - T_{\text{in}} = \frac{\dot{q}_{\text{water}}}{\dot{m} c_p} = \frac{672.36}{201 \times 4.2 \times 1000} = 16^\circ \text{C} \quad \textcircled{2} \quad \#$$

$$3) h = \frac{\dot{q}_{\text{water}}}{\dot{q}_{\text{collector}}'' \times A_s} = \frac{672.36}{500 \times 4} = 0.3362 = 33.6\% \quad \textcircled{3} \quad \#$$

Consider steady heat transfer between two large black parallel plates at  $T_1 = 300$  K and  $T_2 = 200$  K that are  $L = 1$  cm apart. Determine the rate of heat transfer between the plates per unit surface area assuming the gap between the plates is filled with:  
(a) still air, (b) evacuated, (c) urethane insulation, and (d) superinsulation.

---



$$a) q_{\text{cond}}'' = k \frac{T_1 - T_2}{L} = 0.0219 * \frac{300 - 200}{0.01} = 219 \text{ W}$$

$$q_{\text{rad}}'' = \epsilon \sigma (T_1^4 - T_2^4) = 1 * 5.67 * 10^{-8} * (300^4 - 200^4) = 369 \text{ W}$$

$$q_{\text{tot}}'' = q_{\text{cond}}'' + q_{\text{rad}}'' = 219 + 369 = 588 \text{ W} \quad \#$$

$$b) q_{\text{rad}}'' = 369 \text{ W} = q_{\text{tot}}'' \quad \#$$

$$c) q_{\text{cond}}'' = k \frac{T_1 - T_2}{L} = 0.026 * \frac{300 - 200}{0.01} = 260 \text{ W} \quad \#$$

$$d) q_{\text{cond}}'' = k \frac{T_1 - T_2}{L} = 0.00002 * \frac{300 - 200}{0.01} = 0.2 \text{ W} \quad \#$$

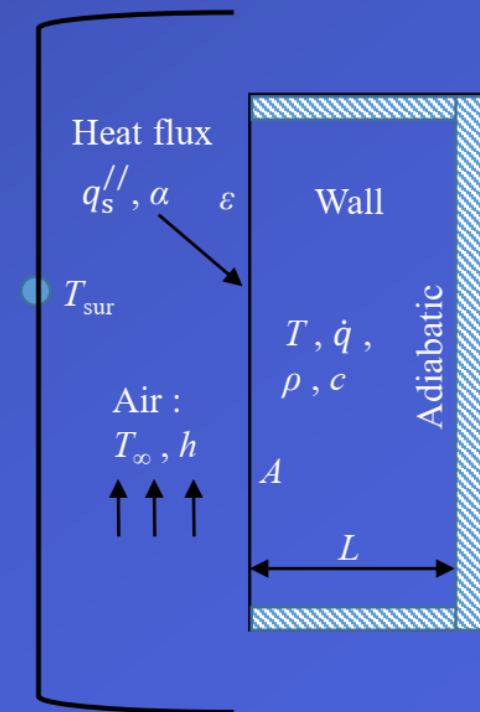


**Question 1:** A wall of temperature  $T$ , thickness  $L$ , surface area  $A$ , and surface emissivity  $\varepsilon$  has a density  $\rho$  and specific heat  $c$ . All surfaces of the wall are adiabatic except its left surface which is exposed to air at temperature  $T_\infty$  and convection coefficient  $h$ , and is enclosed by blackbody surroundings at temperature  $T_{\text{sur}}$ . Heat is generated uniformly inside the wall at a rate per unit volume  $\dot{q}$ . The wall is subjected to uniform heat flux  $q_s''$  with surface absorptivity  $\alpha$ .

(1.1) Assuming the wall temperature to be uniform at  $T$ , develop an expression for the rate of change of wall temperature with time ( $dT/dt$ ) in terms of:  $T$ ,  $L$ ,  $\varepsilon$ ,  $\rho$ ,  $c$ ,  $T_\infty$ ,  $h$ ,  $T_{\text{sur}}$ ,  $\dot{q}$ ,  $q_s''$ ,  $\alpha$ , and  $\sigma$ .

(1.2) Obtain an expression for the surface emissivity  $\varepsilon$  under steady-state conditions.

(1.3) Under steady-state conditions, determine  $\varepsilon$  of the surface for:  
 $L = 10 \text{ cm}$ ,  $h = 10 \text{ W/m}^2\cdot\text{K}$ ,  $\dot{q} = 10^3 \text{ W/m}^3$ ,  $q_s'' = 250 \text{ W/m}^2$ ,  
 $T = 60^\circ\text{C}$ ,  $T_\infty = 70^\circ\text{C}$ ,  $T_{\text{sur}} = 10^\circ\text{C}$ , and  $\alpha = 0.4$ .



## 1.1 Energy balance

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$\alpha A q_s'' - (q_{conv} + q_{rad}) + \dot{q} V = \rho V C \frac{dT}{dt}$$

$$\alpha \cancel{A} q_s'' - (h \cancel{A} (T - T_\infty) + \epsilon \cancel{A} \sigma (T^4 - T_{sur}^4)) + \dot{q} \cancel{A} L = \rho \cancel{A} L C \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{\alpha q_s'' - [h(T - T_\infty) + \epsilon \sigma (T^4 - T_{sur}^4)] + \dot{q} L}{\rho L C} \quad \textcircled{1}$$

## 1.2 $\frac{dT}{dt} = 0$

$$\alpha q_s'' - [h(T - T_\infty) + \epsilon \sigma (T^4 - T_{sur}^4)] + \dot{q} L = 0$$

$$\Sigma = \frac{\alpha q_{rs}'' - h(T - T_{\infty}) + \dot{q}L}{\sigma(T^4 - T_{sur}^4)} \quad \text{②}$$

1.3

$$\Sigma = \frac{0.4 \times 250 - 10(60 - 70) + 10^3 \times 0.1}{5.67 \times 10^{-8} (333^4 - 283^4)} = 0.9 \quad \text{#}$$

**1.1** The thermal conductivity of a sheet of rigid, extruded insulation is reported to be  $k = 0.029 \text{ W/m}\cdot\text{K}$ . The measured temperature difference across a 20-mm-thick sheet of the material is  $T_1 - T_2 = 10^\circ\text{C}$ .

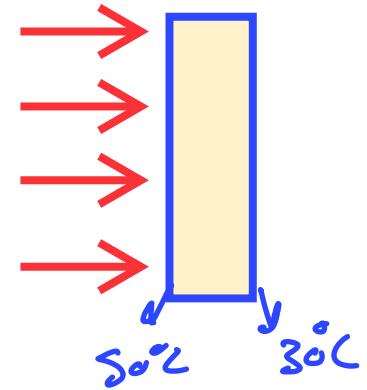
- (a) What is the heat flux through a  $2 \text{ m} \times 2 \text{ m}$  sheet of the insulation?
- (b) What is the rate of heat transfer through the sheet of insulation?

a)

$$q''_{\text{cond}} = k \frac{T_1 - T_2}{L} = 0.029 * \frac{10}{0.02} = 14.5 \text{ W/m}^2 \quad \textcircled{a} \quad \#$$

$$\text{b) } Q = A q''_{\text{cond}} = 2 * 2 * 14.5 = 58 \text{ W} \quad \textcircled{b} \quad \#$$

**1.2** The heat flux that is applied to the left face of a plane wall is  $q'' = 20 \text{ W/m}^2$ . The wall is of thickness  $L = 10 \text{ mm}$  and of thermal conductivity  $k = 12 \text{ W/m}\cdot\text{K}$ . If the surface temperatures of the wall are measured to be  $50^\circ\text{C}$  on the left side and  $30^\circ\text{C}$  on the right side, do steady-state conditions exist?

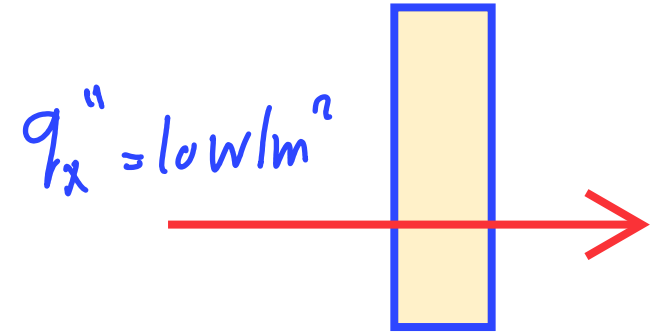


$$q''_{\text{in}} = q''_{\text{out}} \rightarrow \text{steady state.}$$

$$q''_{\text{out}} = k \frac{T_1 - T_2}{L} = 12 * \frac{50 - 30}{0.01} = 24000 \text{ W/m}^2$$

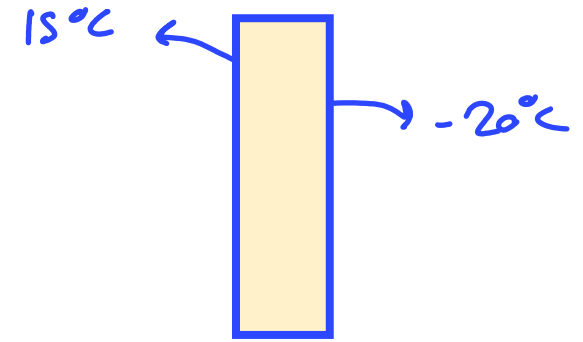
$$q''_{\text{in}} \neq q''_{\text{out}} \rightarrow \text{not steady state}$$

**1.5** Consider Figure 1.3. The heat flux in the  $x$ -direction is  $q_x'' = 10 \text{ W/m}^2$ , the thermal conductivity and wall thickness are  $k = 2.3 \text{ W/m}\cdot\text{K}$  and  $L = 20 \text{ mm}$ , respectively, and steady-state conditions exist. Determine the value of the temperature gradient in units of  $\text{K/m}$ . What is the value of the temperature gradient in units of  $^\circ\text{C/m}$ ?



$$q_x'' = -k \frac{dT}{dx} \quad \therefore \frac{dT}{dx} = \frac{-q_x''}{k} = \frac{-10}{2.3} = -4.35 \frac{^\circ\text{C}}{\text{m}}$$

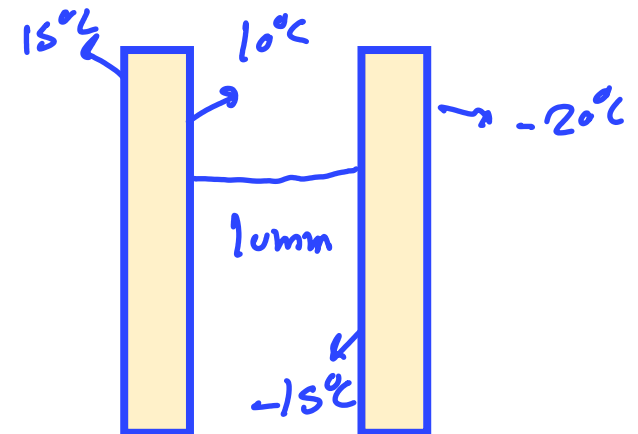
**1.9** A glass window of width  $W = 1$  m and height  $H = 2$  m is 5 mm thick and has a thermal conductivity of  $k_g = 1.4$  W/m·K. If the inner and outer surface temperatures of the glass are  $15^\circ\text{C}$  and  $-20^\circ\text{C}$ , respectively, on a cold winter day, what is the rate of heat loss through the glass? To reduce heat loss through windows, it is customary to use a double pane construction in which adjoining panes are separated by an air space. If the spacing is 10 mm and the glass surfaces in contact with the air have temperatures of  $10^\circ\text{C}$  and  $-15^\circ\text{C}$ , what is the rate of heat loss from a  $1\text{ m} \times 2\text{ m}$  window? The thermal conductivity of air is  $k_a = 0.024$  W/m·K.



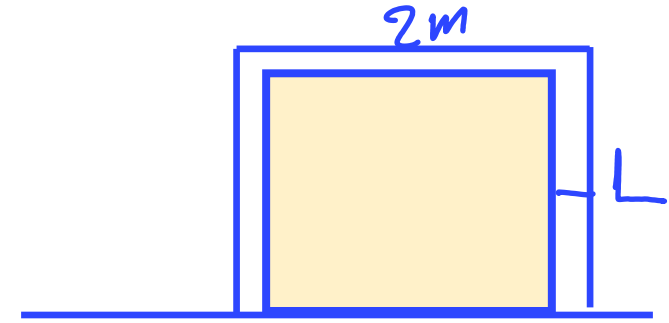
$$q = KA \frac{T_1 - T_2}{L} = 1.4 * 2 * 1 * \frac{15 - (-20)}{0.005} = 19600 \text{ W} \quad \#$$

$$q = KA \frac{T_1 - T_2}{L} = 0.024 * 2 * 1 * \frac{10 - (-15)}{0.01}$$

$$q = 120 \text{ W} \quad \#$$



**1.10** A freezer compartment consists of a cubical cavity that is 2 m on a side. Assume the bottom to be perfectly insulated. What is the minimum thickness of styrofoam insulation ( $k = 0.030 \text{ W/m} \cdot \text{K}$ ) that must be applied to the top and side walls to ensure a heat load of less than 500 W, when the inner and outer surfaces are  $-10$  and  $35^\circ\text{C}$ ?



$$q = 500 \text{ W} \quad T_1 = 35^\circ\text{C} \quad (T_2 = -10^\circ\text{C}) \quad (k = 0.03)$$

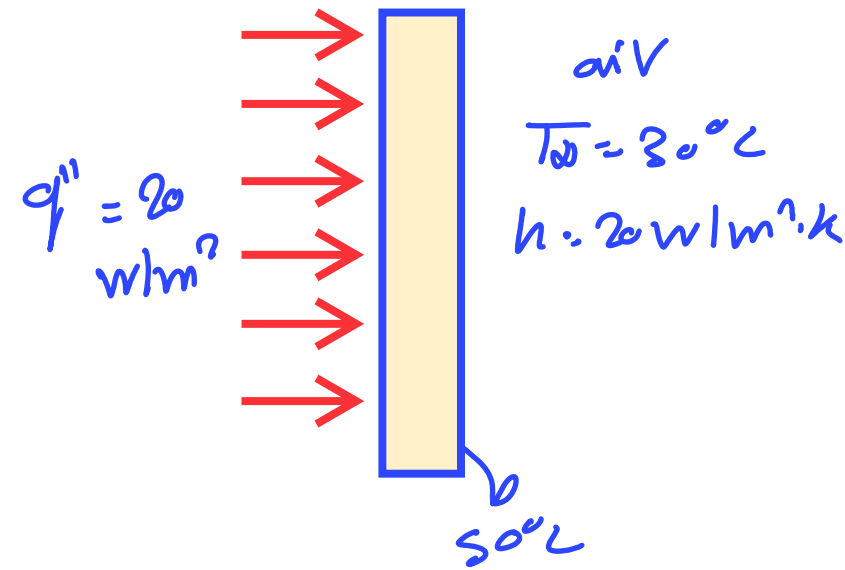
$$q = k A \frac{T_1 - T_2}{L} \quad \therefore L = \frac{(T_1 - T_2) k A}{q}$$

$$A = 3 \times 2 \times 2 = 12 \text{ m}^2$$

$$L = \frac{[35 - (-10)] \times 0.03 \times 12}{500} = 0.054 \text{ m} = 54 \text{ mm}$$



**1.11** The heat flux that is applied to one face of a plane wall is  $q'' = 20 \text{ W/m}^2$ . The opposite face is exposed to air at temperature  $30^\circ\text{C}$ , with a convection heat transfer coefficient of  $20 \text{ W/m}^2 \cdot \text{K}$ . The surface temperature of the wall exposed to air is measured and found to be  $50^\circ\text{C}$ . Do steady-state conditions exist? If not, is the temperature of the wall increasing or decreasing with time?



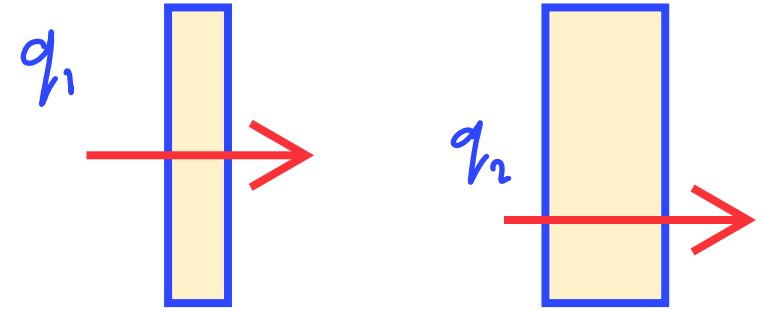
$$q''_{in} = 20 \text{ W/m}^2$$

$$q''_{out} = h(T - T_\infty) = 20 * (50 - 30) = 400 \text{ W/m}^2$$

not steady state

$q''_{out} > q''_{in} \rightarrow$  wall temperature is decreasing

**1.13** What is the thickness required of a masonry wall having thermal conductivity  $0.75 \text{ W/m} \cdot \text{K}$  if the heat rate is to be 80% of the heat rate through a composite structural wall having a thermal conductivity of  $0.25 \text{ W/m} \cdot \text{K}$  and a thickness of  $100 \text{ mm}$ ? Both walls are subjected to the same surface temperature difference.



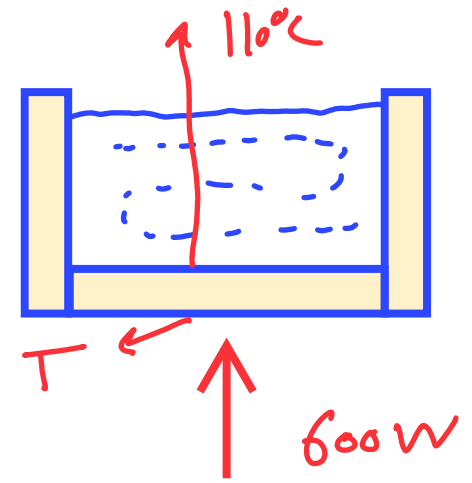
$$q_1'' = 0.8 q_2''$$

$$\frac{K_1 (\cancel{T_1 - T_2})}{L_1} = 0.8 * \frac{K_2 (\cancel{T_1 - T_2})}{L_2}$$

$$\frac{0.75}{L_1} = \frac{0.8 * 0.25}{100}$$

$$L_1 = \frac{0.75 * 100}{0.8 * 0.25} = 375 \text{ mm} \neq$$

**1.15** The 5-mm-thick bottom of a 200-mm-diameter pan may be made from aluminum ( $k = 240 \text{ W/m} \cdot \text{K}$ ) or copper ( $k = 390 \text{ W/m} \cdot \text{K}$ ). When used to boil water, the surface of the bottom exposed to the water is nominally at  $110^\circ\text{C}$ . If heat is transferred from the stove to the pan at a rate of  $600 \text{ W}$ , what is the temperature of the surface in contact with the stove for each of the two materials?



$$A = \frac{\pi D^2}{4} = \frac{\pi \times 0.2^2}{4} = 0.0314 \text{ m}^2$$

$$q = kA \frac{T_1 - T_2}{L} \quad T_1 = T_2 + \frac{qL}{kA}$$

Aluminum:  $T_1 = 110 + \frac{600 \times 0.005}{240 \times 0.0314} = 110.4^\circ\text{C}$

Copper:  $T_1 = 110 + \frac{600 \times 0.005}{390 \times 0.0314} = 110.2^\circ\text{C}$

**1.18** You've experienced convection cooling if you've ever extended your hand out the window of a moving vehicle or into a flowing water stream. With the surface of your hand at a temperature of  $30^{\circ}\text{C}$ , determine the convection heat flux for (a) a vehicle speed of 35 km/h in air at  $-5^{\circ}\text{C}$  with a convection coefficient of  $40 \text{ W/m}^2 \cdot \text{K}$  and (b) a velocity of 0.2 m/s in a water stream at  $10^{\circ}\text{C}$  with a convection coefficient of  $900 \text{ W/m}^2 \cdot \text{K}$ . Which condition would *feel* colder? Contrast these results with a heat loss of approximately  $30 \text{ W/m}^2$  under normal room conditions.

air  $q'' = h(T_s - T_{\infty}) = 40 * (30 - (-5)) = 1400 \text{ W/m}^2$

water  $q'' = h(T_s - T_{\infty}) = 900(30 - 10) = 18000 \text{ W/m}^2$  ~~#~~

**1.20** A wall has inner and outer surface temperatures of 16 and 6°C, respectively. The interior and exterior air temperatures are 20 and 5°C, respectively. The inner and outer convection heat transfer coefficients are 5 and 20 W/m<sup>2</sup>·K, respectively. Calculate the heat flux from the interior air to the wall, from the wall to the exterior air, and from the wall to the interior air. Is the wall under steady-state conditions?

$$h_i = 5 \text{ W/m}^2 \cdot \text{K}$$

$$T_{\infty,i} = 20^\circ\text{C}$$

$$T_i = 16^\circ\text{C}$$

$$T_{\infty,o} = 5^\circ\text{C}$$

$$h_o = 20 \text{ W/m}^2 \cdot \text{K}$$

$$T_o = 6^\circ\text{C}$$

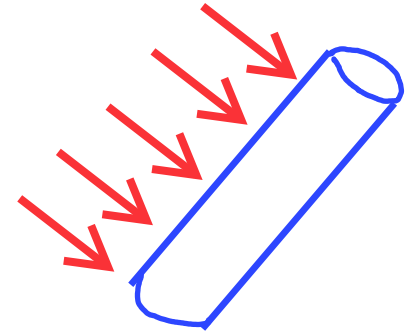
$$1) q'' = h(T_o - T_{s,i}) = 5 * (20 - 16) = 20 \text{ W/m}^2$$

$$2) q'' = h(T_{s,o} - T_o) = 20 * (6 - 5) = 20 \text{ W/m}^2$$

$$3) q'' = h(T_{s,i} - T_i) = 5 * (16 - 20) = -20 \text{ W/m}^2$$

$$q''_{in} = q''_{out} \rightarrow \text{steady state.}$$

**1.21** An electric resistance heater is embedded in a long cylinder of diameter 30 mm. When water with a temperature of 25°C and velocity of 1 m/s flows crosswise over the cylinder, the power per unit length required to maintain the surface at a uniform temperature of 90°C is 28 kW/m. When air, also at 25°C, but with a velocity of 10 m/s is flowing, the power per unit length required to maintain the same surface temperature is 400 W/m. Calculate and compare the convection coefficients for the flows of water and air.



$$T_s = 90^\circ\text{C} \quad T_\infty = 25^\circ\text{C} \quad D = 30\text{ mm} \quad q' = 28\text{ kW/m}$$

water  $q = h A (T_s - T_\infty) = h (\pi D L) (T_s - T_\infty)$

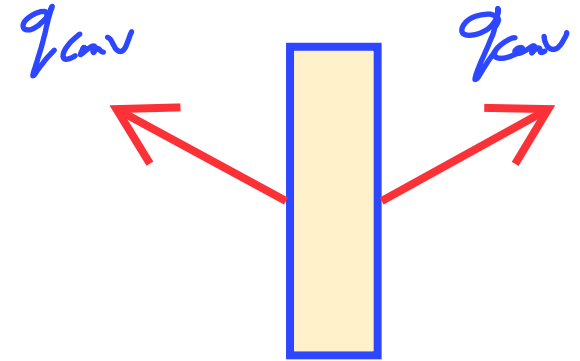
$$q' = h \pi D (T_s - T_\infty) \quad \therefore h = \frac{q'}{\pi D (T_s - T_\infty)}$$

$$h = \frac{28 \times 10^3}{\pi \times 30 \times 10^{-3} \times (90 - 25)} = 4570\text{ W/m}^2 \cdot \text{K}$$

cnr  $q' = 400 \text{ W/m}$

$$h = \frac{400}{\pi \times 30 \times 10^{-3} (90 - 25)} = 65 \text{ W/m}^2 \cdot \text{K} \quad \neq$$

**1.22** The free convection heat transfer coefficient on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temperature with time as it cools. Assuming the plate is isothermal and radiation exchange with its surroundings is negligible, evaluate the convection coefficient at the instant of time when the plate temperature is  $225^\circ\text{C}$  and the change in plate temperature with time ( $dT/dt$ ) is  $-0.022\text{ K/s}$ . The ambient air temperature is  $25^\circ\text{C}$  and the plate measures  $0.3 \times 0.3\text{ m}$  with a mass of  $3.75\text{ kg}$  and a specific heat of  $2770\text{ J/kg} \cdot \text{K}$ .



$$\cancel{\dot{E}_{\text{in}}} - \dot{E}_{\text{out}} + \cancel{\dot{E}_g} = \dot{E}_{\text{st}}$$

$$-2q_{\text{conv}} = mc_p \frac{dT}{dt} = -2hA_s(T_s - T_\infty)$$

$$h = \frac{mc_p \frac{dT}{dt}}{-2A_s(T_s - T_\infty)} = \frac{3.75 \times 2770 \times (-0.022)}{-2 \times 0.3 \times 0.3 \times (225 - 25)} = 6.3\text{ W/m}^2 \cdot \text{K}$$



**1.32** Consider the conditions of Problem 1.22. However, now the plate is in a vacuum with a surrounding temperature of 25°C. What is the emissivity of the plate? What is the rate at which radiation is emitted by the surface?

$$-2 q_{\text{rad}} = m c p \frac{dT}{dt}$$

$$-2 * A \epsilon \sigma (T^4 - T_{\text{sur}}^4) = m c p \frac{dT}{dt}$$

$$-2 * 0.3 * 0.3 * \epsilon * 5.67 * 10^{-8} * (498^4 - 298^4) = 3.75 * 2770 * (-0.022)$$

$$\epsilon = 0.42$$

$$q_{\text{rad}} = \epsilon \sigma A (T^4 - T_{\text{sur}}^4) = 0.42 * 5.67 * 10^{-8} * (498^4 - 298^4) = \frac{230}{2 * 0.3 * 0.3} \text{ W}$$

**1.73** Consider the conditions of Problem 1.22, but the surroundings temperature is 25°C and radiation exchange with the surroundings is not negligible. If the convection coefficient is 6.4 W/m<sup>2</sup>·K and the emissivity of the plate is  $\varepsilon = 0.42$ , determine the time rate of change of the plate temperature,  $dT/dt$ , when the plate temperature is 225°C. Evaluate the heat loss by convection and the heat loss by radiation.

$$-2 \dot{q}_{\text{rad}} - 2 \dot{q}_{\text{conv}} = m c_p \frac{dT}{dt}$$

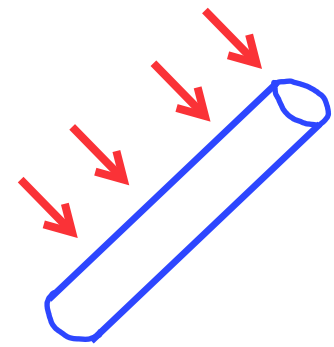
$$\frac{dT}{dt} = \frac{-2 \left[ 0.42 \times 5.67 \times 10^{-8} \times 0.3 \times 0.3 \times (498^4 - 298^4) + 6.4 \times 0.3 \times 0.3 (225 - 25) \right]}{3.75 \times 2770}$$

$$\frac{dT}{dt} = -0.044 \text{ K/s} \quad = 230.4 \text{ W} \neq$$

$$\dot{q}_{\text{rad}} = 230 \text{ W} \neq \dot{q}_{\text{conv}} = h A (T_s - T_\infty) = 6.4 \times 2 \times 0.3 \times 0.3 (225 - 25)$$

**1.25** A common procedure for measuring the velocity of an airstream involves the insertion of an electrically heated wire (called a *hot-wire anemometer*) into the airflow, with the axis of the wire oriented perpendicular to the flow direction. The electrical energy dissipated in the wire is assumed to be transferred to the air by forced convection. Hence, for a prescribed electrical power, the temperature of the wire depends on the convection coefficient, which, in turn, depends on the velocity of the air.

Consider a wire of length  $L = 20$  mm and diameter  $D = 0.5$  mm, for which a calibration of the form  $V = 6.25 \times 10^{-5} h^2$  has been determined. The velocity  $V$  and the convection coefficient  $h$  have units of m/s and  $\text{W/m}^2 \cdot \text{K}$ , respectively. In an application involving air at a temperature of  $T_\infty = 25^\circ\text{C}$ , the surface temperature of the anemometer is maintained at  $T_s = 75^\circ\text{C}$  with a voltage drop of 5 V and an electric current of 0.1 A. What is the velocity of the air?



$$A = \pi D L$$

$$\begin{aligned} A &= \pi \times 0.5 \times 10^{-3} \times \\ &\quad 20 \times 10^{-3} \\ &= 3.14 \times 10^{-5} \text{ m}^2 \end{aligned}$$

$$P_{elec} = \varnothing$$

$$E\Gamma = hA(T_s - T_\infty)$$

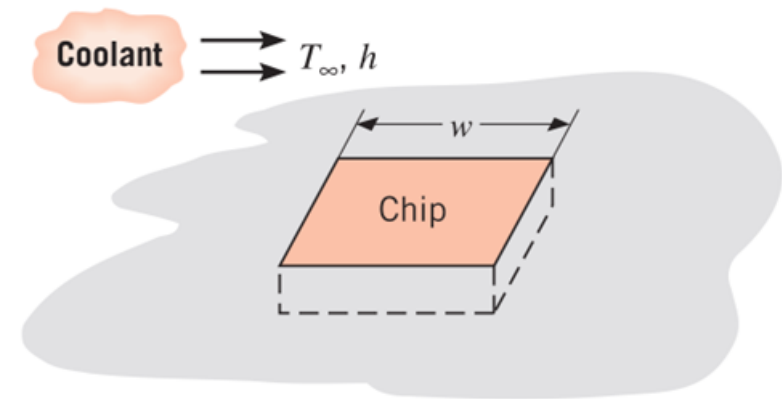
$$h = \frac{E\Gamma}{A(T_s - T_\infty)} = \frac{3 \times 0.1}{3.14 \times 10^{-5} \times (75 - 25)} = 318 \text{ W/m}^2 \cdot \text{K}$$

$$V = 6.25 \times 10^{-5} \text{ h}^2 = 6.25 \times 10^{-5} \times (318)^2 = 6.3 \text{ m/s}$$

#

**1.26** A square isothermal chip is of width  $w = 5 \text{ mm}$  on a side and is mounted in a substrate such that its side and back surfaces are well insulated; the front surface is exposed to the flow of a coolant at  $T_\infty = 15^\circ\text{C}$ . From reliability considerations, the chip temperature must not exceed  $T = 85^\circ\text{C}$ .

If the coolant is air and the corresponding convection coefficient is  $h = 200 \text{ W/m}^2 \cdot \text{K}$ , what is the maximum allowable chip power? If the coolant is a dielectric liquid for which  $h = 3000 \text{ W/m}^2 \cdot \text{K}$ , what is the maximum allowable power?



$$P = \dot{Q} = h A (T - T_\infty) = h w^2 (T - T_\infty)$$

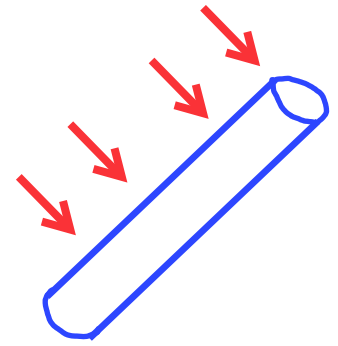
in air

$$P = 200 * (5 * 10^{-3})^2 * (85 - 15) = 0.35 \text{ W}$$

in dielectric liquid

$$P = 3000 * (5 * 10^{-3})^2 * (85 - 15) = 5.25 \text{ W}$$

**1.28** An overhead 25-m-long, uninsulated industrial steam pipe of 100-mm diameter is routed through a building whose walls and air are at 25°C. Pressurized steam maintains a pipe surface temperature of 150°C, and the coefficient associated with natural convection is  $h = 10 \text{ W/m}^2 \cdot \text{K}$ . The surface emissivity is  $\varepsilon = 0.8$ .



- (a) What is the rate of heat loss from the steam line?
- (b) If the steam is generated in a gas-fired boiler operating at an efficiency of  $\eta_f = 0.90$  and natural gas is priced at  $C_g = \$0.02$  per MJ, what is the annual cost of heat loss from the line?

$$T_{\text{sur}} = 25^\circ\text{C} \quad (T_\infty = 25^\circ\text{C})$$

a)

$$q_H = A(q_{\text{conv}}'' + q_{\text{rad}}'') = A[h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{\text{sur}}^4)]$$

$$A = \pi DL = \pi \times 100 \times 10^{-3} \times 25 \times 10^{-3} = 7.85 \text{ m}^2$$

$$q_H = 7.85 \times [10(150 - 25) + 0.8 \times 5.67 \times 10^{-8} \times (423^4 - 298^4)] = 18405 \text{ W} \quad \# \text{ J/s}$$

b) annual energy loss

$$E_p = q \cdot t = 18405 \times 365 \times 24 \times 60 \times 60 = 5.8 \times 10^{11} \text{ J}$$

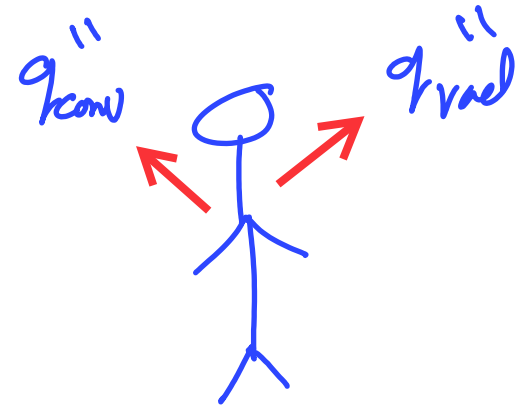
$$\eta = \frac{E_p}{E_c}$$

$$E_c = \frac{E_p}{\eta} = \frac{5.8 \times 10^{11}}{0.9} = 6.45 \times 10^{11} \text{ J}$$

$$C = C_g E_p = \frac{0.02}{106} \times 6.45 \times 10^{11} = \$12900$$



**1.29** Under conditions for which the same room temperature is maintained by a heating or cooling system, it is not uncommon for a person to feel chilled in the winter but comfortable in the summer. Provide a plausible explanation for this situation (with supporting calculations) by considering a room whose air temperature is maintained at  $20^\circ\text{C}$  throughout the year, while the walls of the room are nominally at  $27^\circ\text{C}$  and  $14^\circ\text{C}$  in the summer and winter, respectively. The exposed surface of a person in the room may be assumed to be at a temperature of  $32^\circ\text{C}$  throughout the year and to have an emissivity of 0.90. The coefficient associated with heat transfer by natural convection between the person and the room air is approximately  $2 \text{ W/m}^2 \cdot \text{K}$ .



$$q_{\text{conv}}'' = h(T - T_{\infty}) = 2 * (32 - 20) = 24 \text{ W/m}^2$$

$$\text{Summer: } q_{\text{rad}}'' = \epsilon \sigma (T_s^4 - T_{\text{sur}}^4) = 0.9 * 5.67 * 10^{-8} * (305^4 - 300^4) = 28.3 \text{ W/m}^2$$

$$\text{winter: } q_{\text{rad}}'' = \epsilon \sigma (T_s^4 - T_{\text{sur,w}}^4) = 0.9 * 5.67 * 10^{-8} * (305^4 - 287^4) = 95.4 \text{ W/m}^2$$



**1.30** A spherical interplanetary probe of 0.5-m diameter contains electronics that dissipate 150 W. If the probe surface has an emissivity of 0.8 and the probe does not receive radiation from other surfaces, as, for example, from the sun, what is its surface temperature?

$$\cancel{\dot{E}_{in}} - \dot{E}_{out} + \dot{E}_g = \cancel{\dot{E}_{st}}$$

$$T_{surv} = 0 \text{ K}$$

$$A = \pi D^2$$

$$\dot{E}_g = \dot{E}_{out}$$

$$150 = \epsilon \sigma A T_s^4$$

$$T_s = \left[ \frac{150}{0.8 \times 5.67 \times 10^{-8} \times \pi \times 0.5^2} \right]^{1/4} = 254.7 \text{ K} \quad \#$$