Links, Joints, and Kinematic Chains

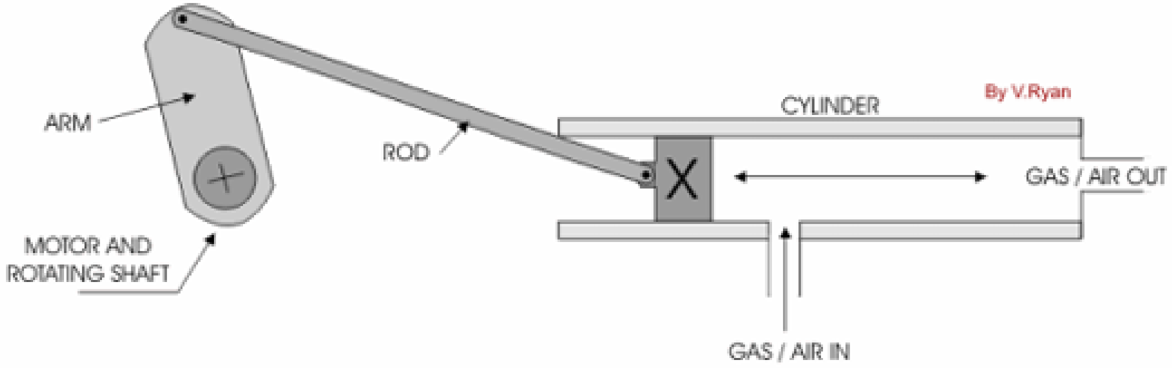
Linkage design:

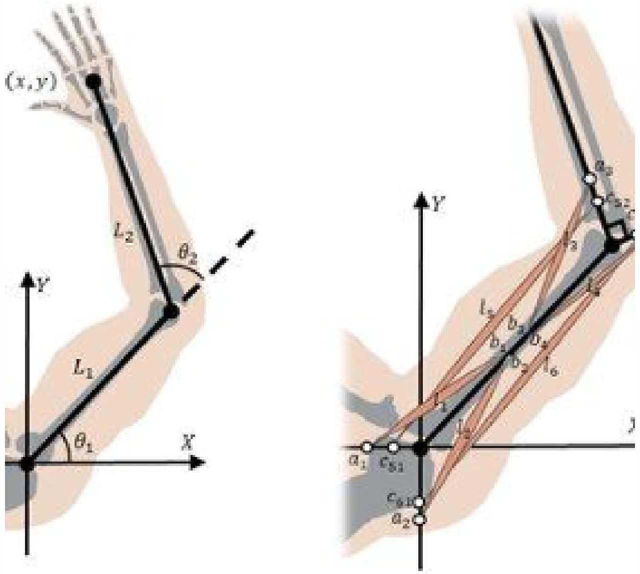
❖ Linkages are the basic building blocks of all mechanisms

❖ All common forms of mechanisms (cams, gears, belts, chains) are in fact variations of a common theme of linkages.

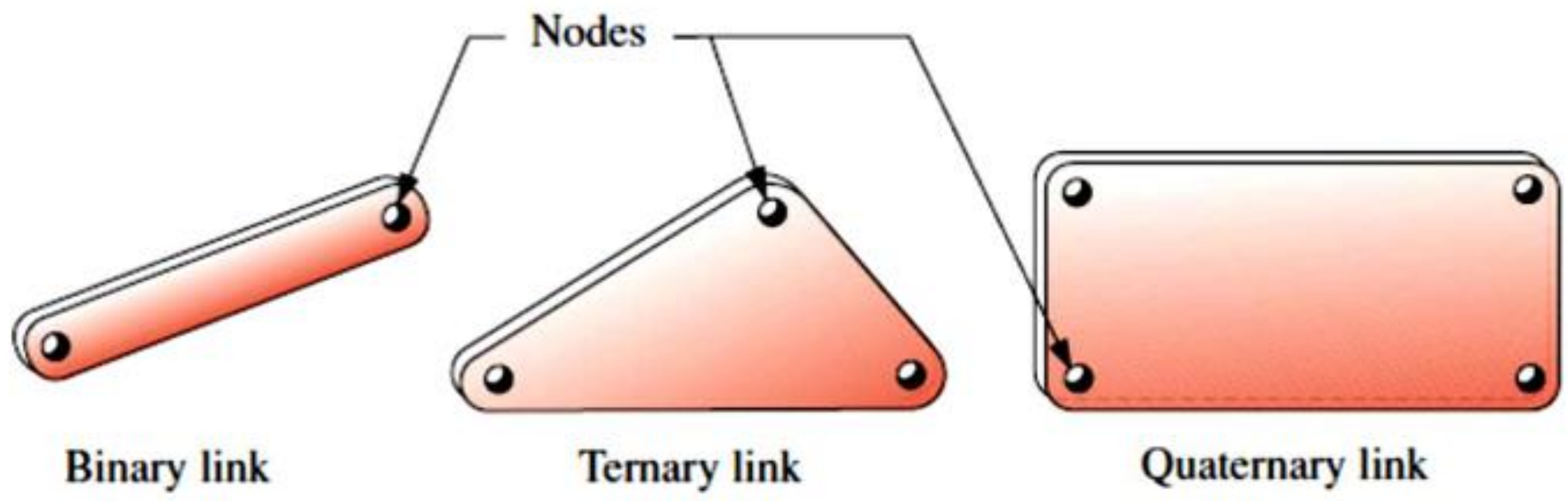
❖ Linkages are made up of links and joints.

- **Link:** rigid member having nodes
- **Node:** attachment points to other links



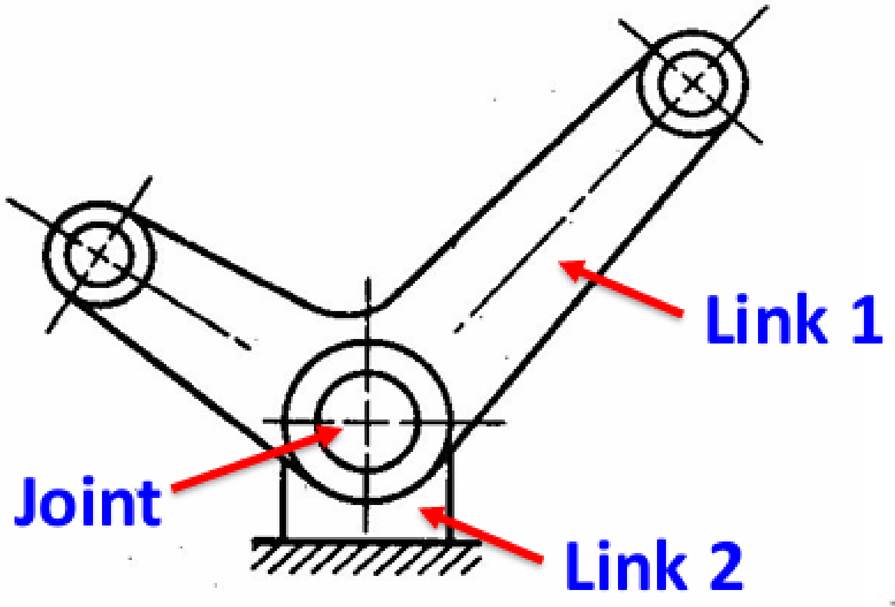


1. **Binary link: 2 nodes -** connected to other links at two points
2. **Ternary link: 3 nodes -** connected to other links at three points
3. **Quaternary link: 4 nodes -** connected to other links at four points
4. **Pentagonal Link: 5 nodes -** connected to other links at five points
5. **Hexagonal link: 6 nodes -** connected to other links at six points.

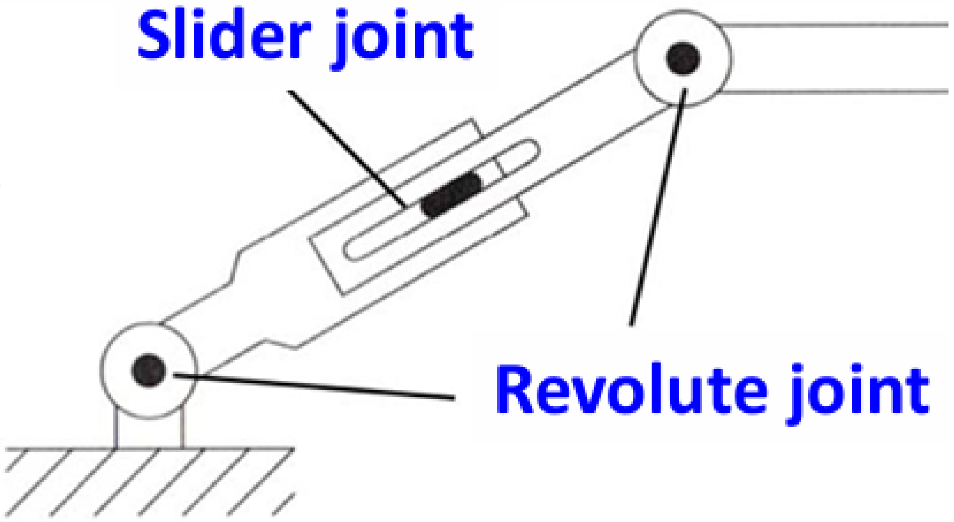


Links, Joints, and Kinematic Chains

Joint: connection between two or more links (at their nodes) which allows motion; **(joints also called kinematic pairs)**.



Kinematic pair



Links, Joints, and Kinematic Chains

Joints can be classified in several ways:

1. By the **type of contact** between the elements, line, point, or surface.

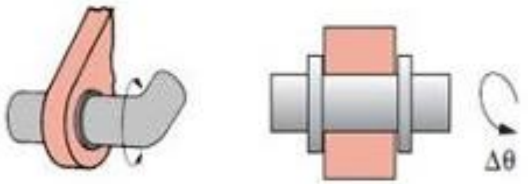
Lower Pair التلامس على مساحه

Higher Pair التلامس على خط أو نقطه

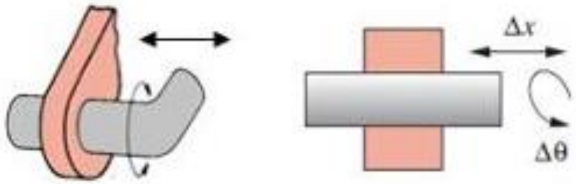
2. By the **number of DOF** allowed at the joint.
3. By the type of **physical closure** of the joint: either **force** or **form closed**.
4. By the **number of links joined (order of the joint)**.

1. Classification by the Type of Contact

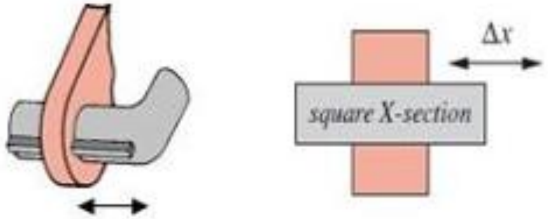
Six lower pair joints



Revolute (R) joint



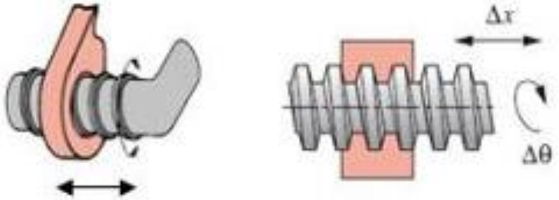
Cylindrical (C) joint



Prismatic (P) joint

Difference between helical and cylindrical joints?

Spherical (S) joint



Helical (H) joint

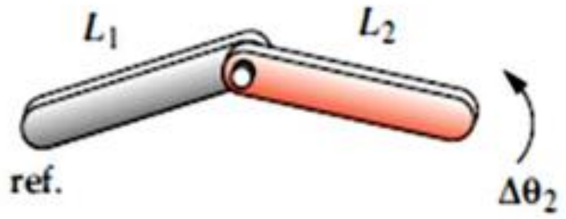
Flat (F) joint

2. Classification by the Number of DOF

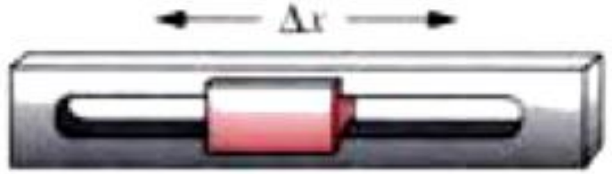
- **Full Joint = 1 DOF**
- **Half Joint = 2 DOF**
- **Three-freedom Joint = 3 DOF**

- **<u>1 DOF joints</u>:** a rotating pin joint (R) and a translating slider Joint (P).
- **<u>2 DOF joints</u>:** link against a plane and a pin in slot.
- **<u>3 DOF joints</u>:** a spherical, or ball-and-socket joint.

2. Classification by the Number of DOF

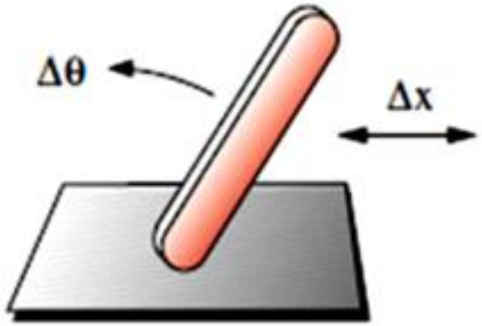


Rotating full pin (R) joint

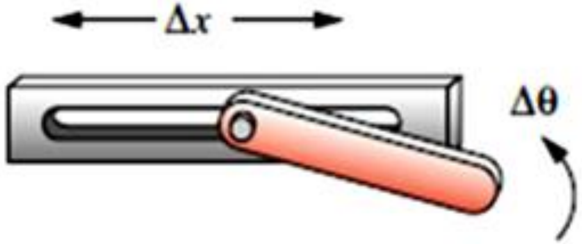


Translating full slider (P) joint

Full joint – 1 DOF



Link against plane

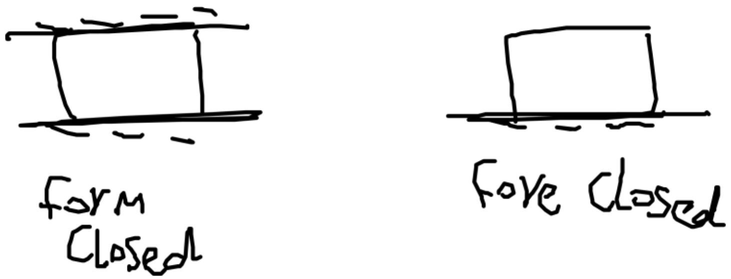


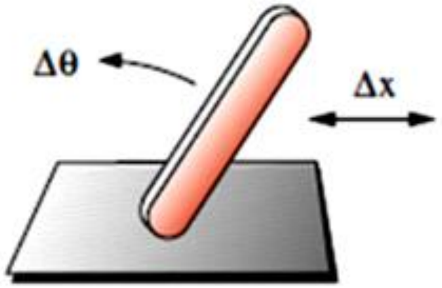
Pin in slot

half joint – 2 DOF

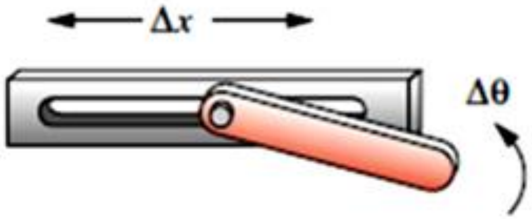
3. Classification by the Type of Physical Closure

- Physical closure joint → **Force** or **Form closed**
- A **form-closed joint** is kept together or closed by its geometry.
    - A pin in a hole or a slider in a two-sided slot are form closed.
- A **force-closed joint**, such as a pin in a half-bearing or a slider on a surface, requires some external force to keep it together or closed.
    - This force could be supplied by gravity, a spring, or any external means.





Link against plane (force closed)



Pin in slot (form closed)

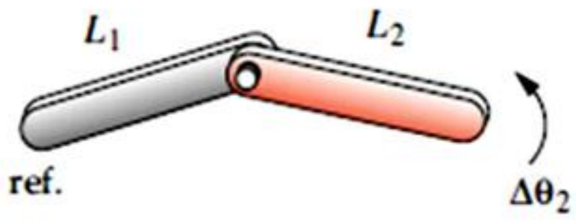
4. Classification by the order of Joints

1st order, 2nd order and so on.

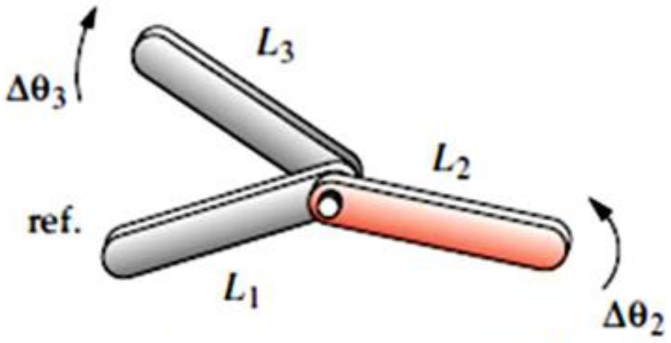
No. order = No. of links in Joint − 1

Joint Order: ***number of links -1.***

- It takes two links to make a single joint; thus the simplest joint combination of two links has ***order one***.
- Joint order has significance in the proper determination of overall degree of freedom for the assembly.



First order pin joint - one *DOF*
(two links joined)



Second order pin joint - two *DOF*
(three links joined)

Kinematic Chain

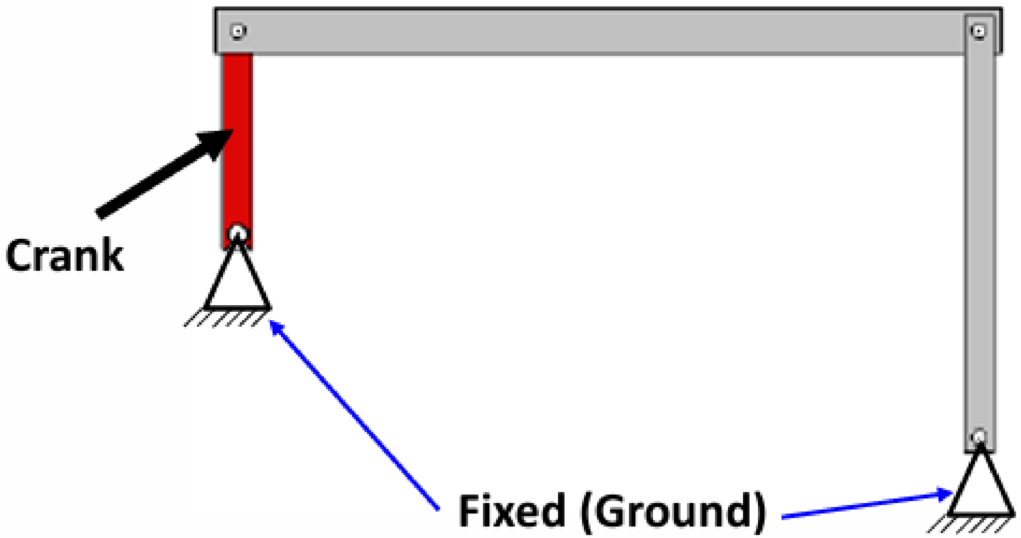
A **kinematic chain** is defined as:

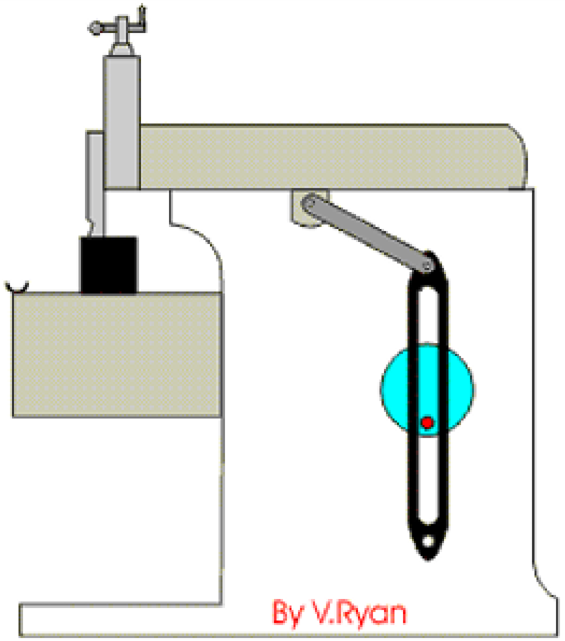
An assemblage of links and joints, interconnected in a way to provide a ***controlled output motion*** in response to a supplied input motion.

In short:

- Kinematic chain: links joined together for motion
- Mechanism: grounded kinematic chain
- Machine: mechanism designed to do work

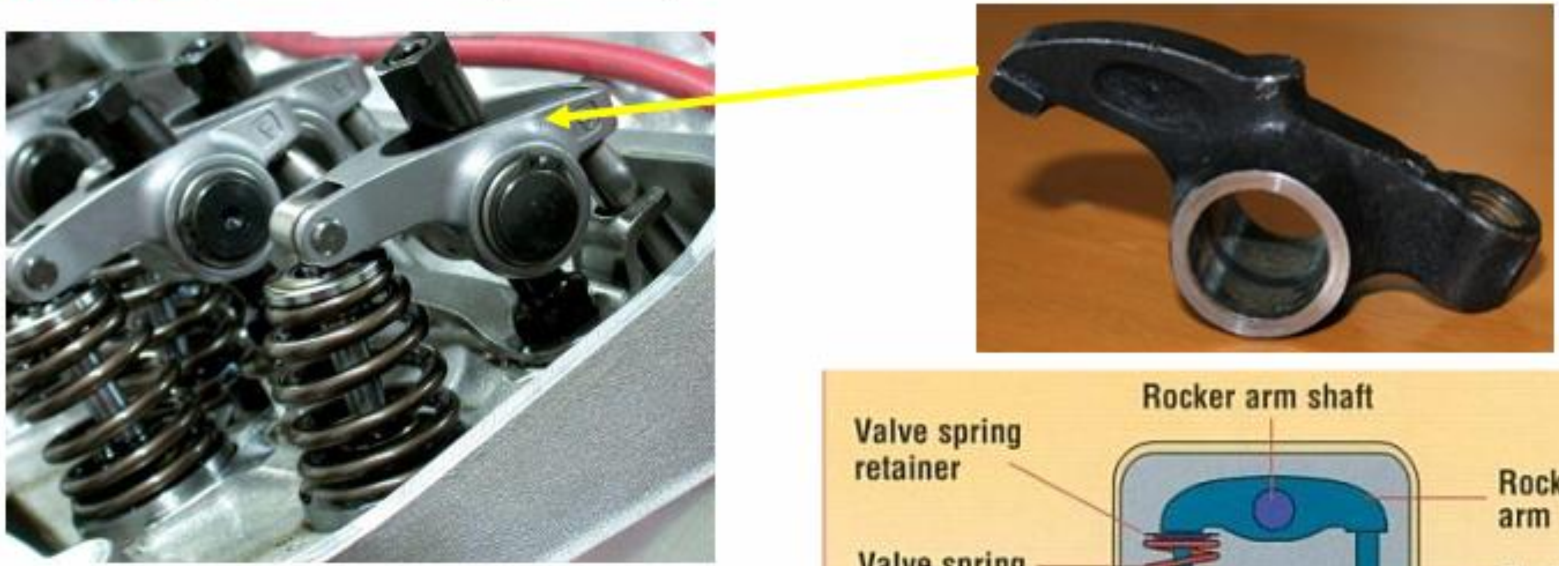
- ***Ground:*** any link or links that are fixed, nonmoving with respect to the frame of reference
- ***Crank:*** pivoted to ground, makes complete revolutions

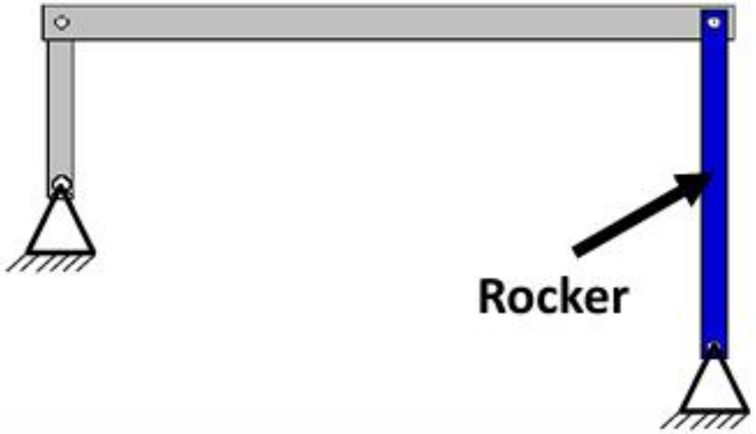


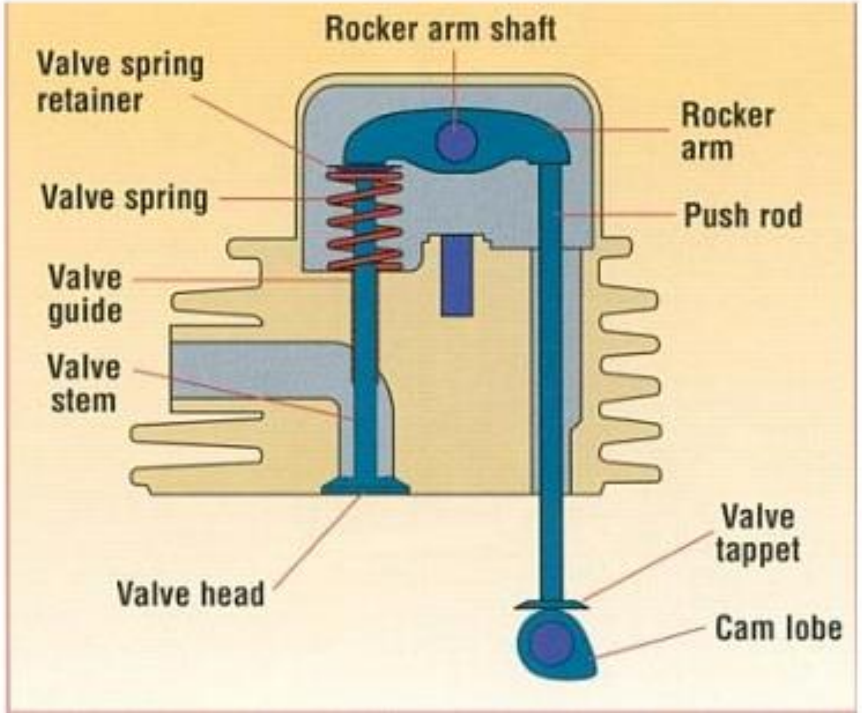


THE SHAPING MACHINE

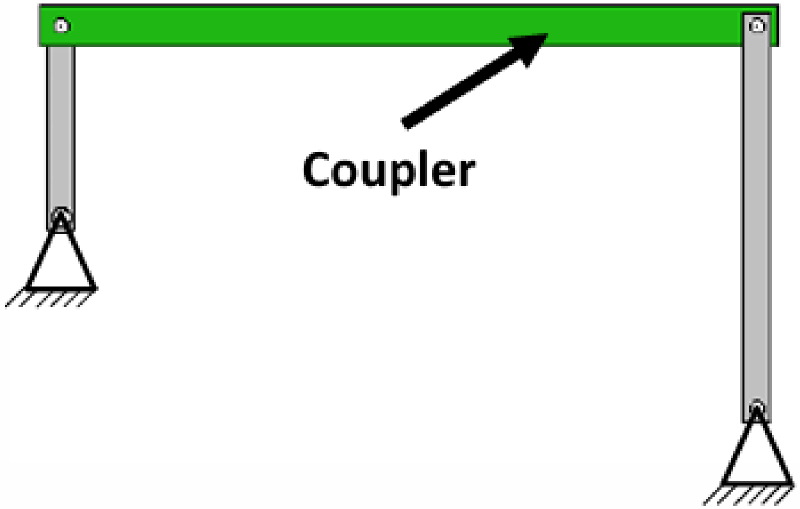
Rocker: pivoted to ground, has oscillatory motion

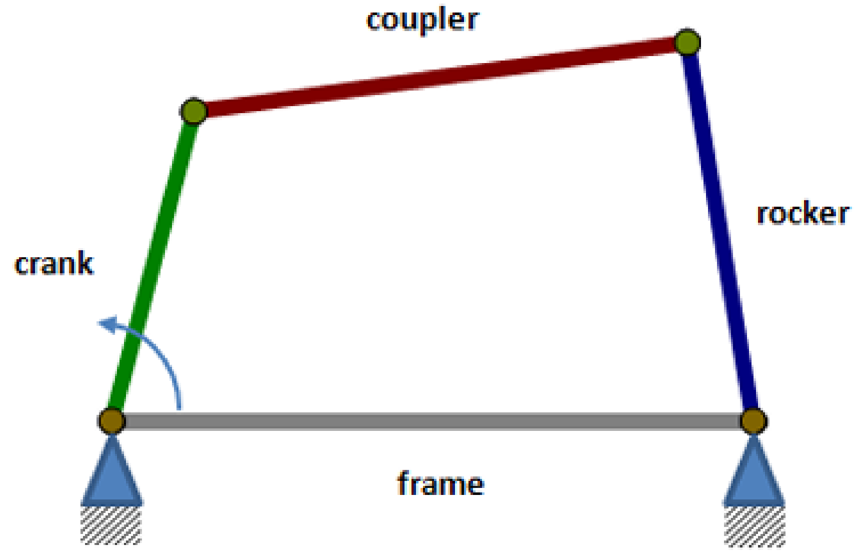




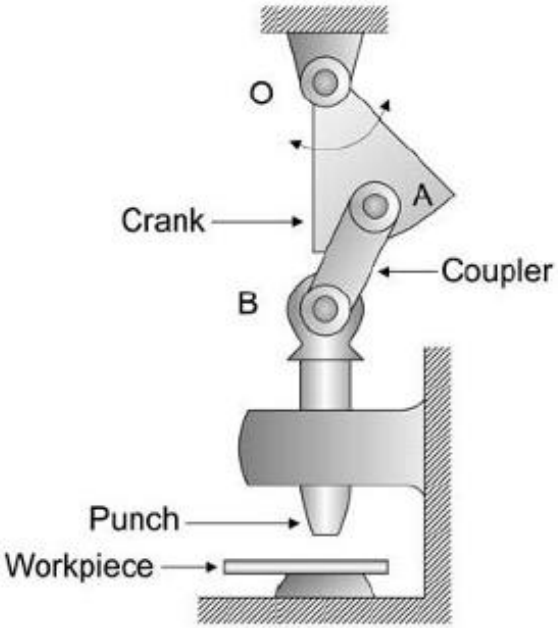


Coupler: link has complex motion, not attached to ground

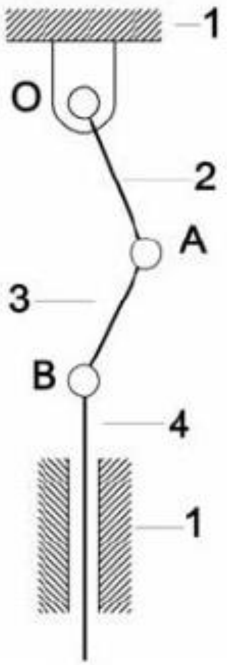




Kinematic diagram is used to express the complex parts of machines for easy design process.

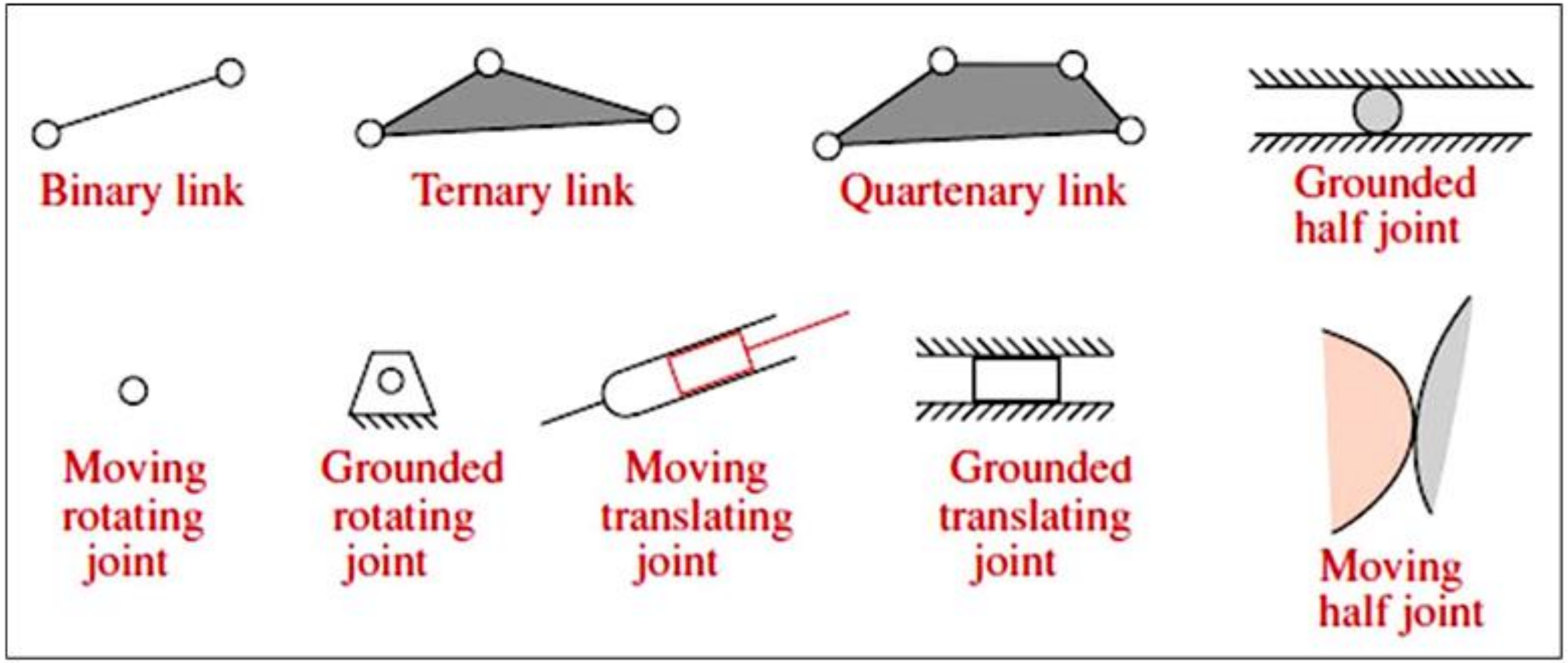


Punch mechanism



Punch Kinematic Diagram

Kinematic Diagrams Elements



Gruebler's Equation

Based on the above reasoning, ***Gruebler's Equation*** is

$$M = 3L-2J-3G \qquad (2.1a)$$

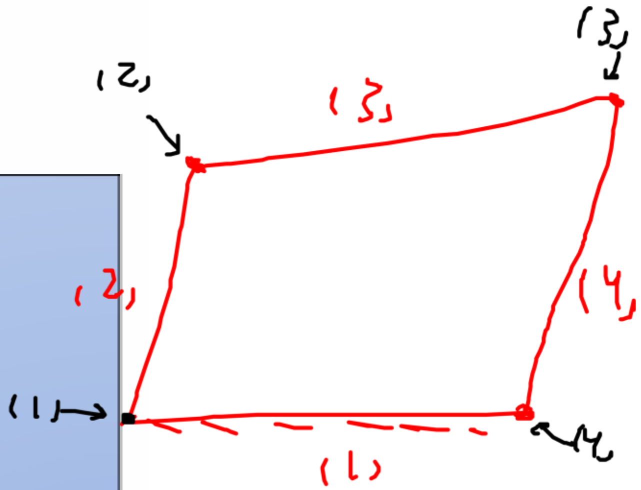
where: ***M = degree of freedom or mobility***

L = number of links

J = number of joints

G = number of grounded links

$$M = 3(L-1)-2J \qquad (2.1b)$$



$L = 4$

$G = 1$

$J = 4$

$M = 3*4 - 2*4 - 3*1$

$= \boxed{1}$

Kutzbach's Modification of Gruebler's Equation

- Kutzbach's Modification of Gruebler's Equation is:

$$M = 3(L-1)-2J_1-J_2 \qquad (2.1c)$$

where: *M = degree of freedom or mobility*

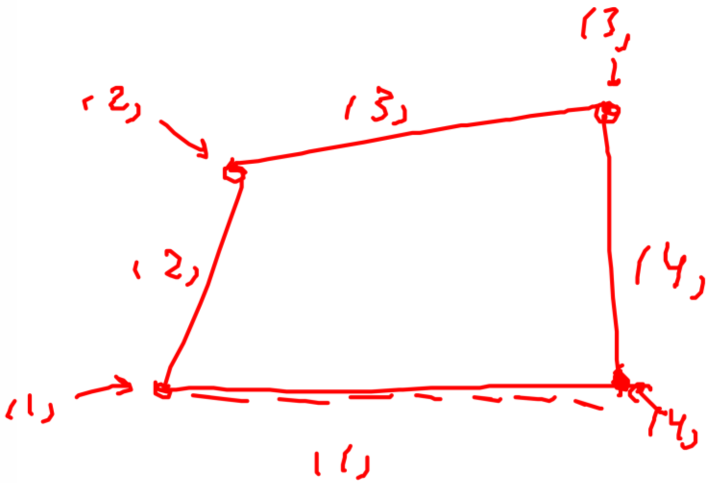
L = number of links

J_1 = number of 1 DOF (full) joints (lower Pair)

J_2 = number of 2 DOF (half) joints (Higher Pair)

- The value of **J_1** and **J_2** in these equations must still be carefully determined to account for all **full**, **half**, and **multiple joints** in any linkage.

- **Multiple joints:** count as one less than the number of links joined at that joint and add to the "full" (J_1) category.

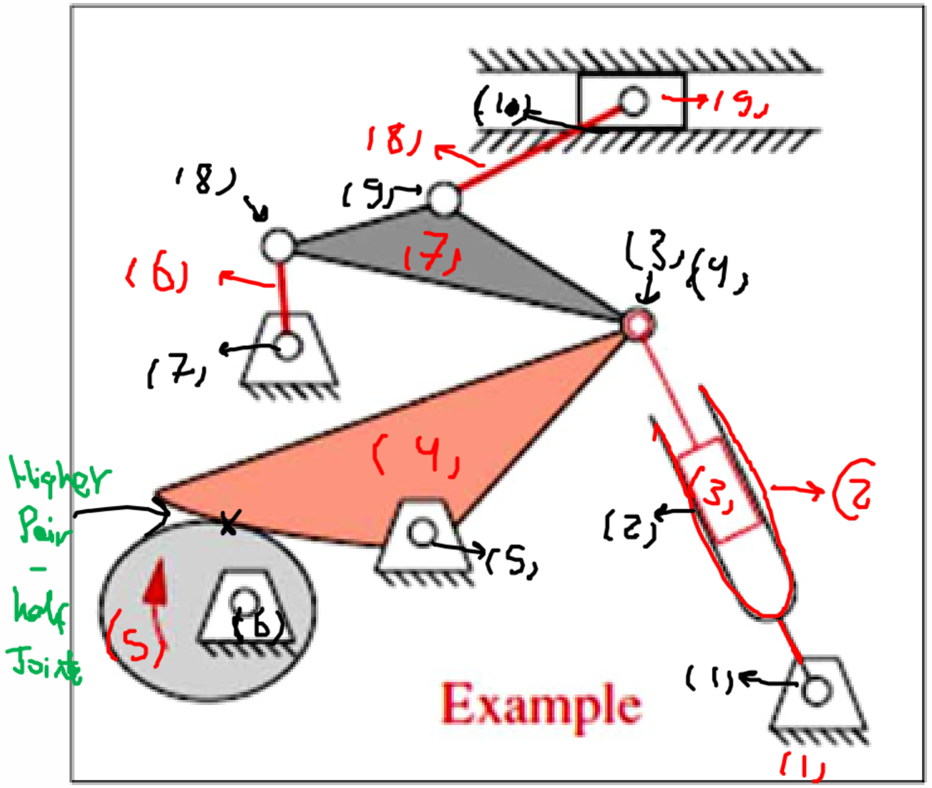


$L = 4$

$J_1 = 4$

$J_2 = 0$

$Dof = 3(4-1) - 2*4 = \boxed{1}$



Calculate the DOFs of the given mechanism

$L = 9$

$J_1 = 10$

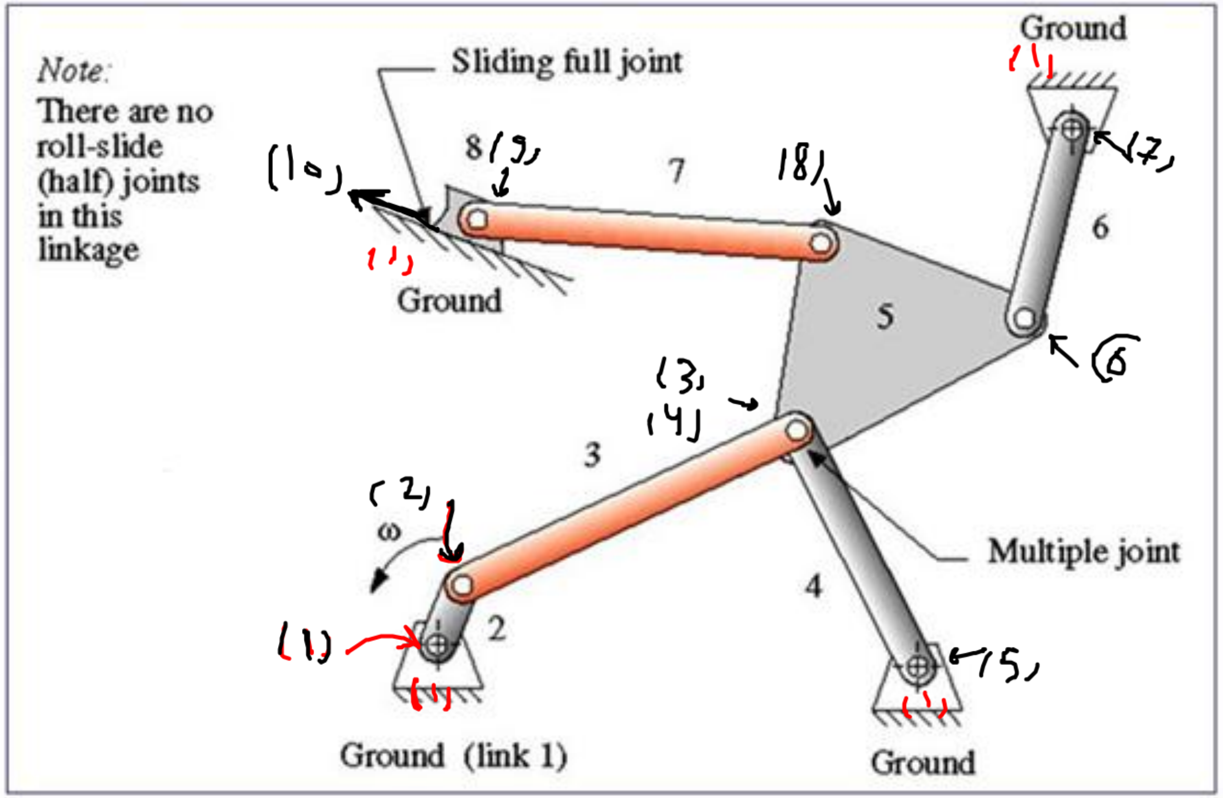
$J_2 = 1$

$$M = 3(L-1) - 2J_1 - J_2$$

$$= 3(9-1) - 2*10 - 1$$

$$= \boxed{3}$$

Compute the DOF of the following examples with **Kutzbach's and Gruebler's Equations.**



L = number of links = 8, J = number of full joints = 10, so **DOF = 1**

Kutzbach

$L = 8$

$J_1 = 10$

$J_2 = \text{zero}$

$M = 3(8-1) - 2*10 - 0 = \boxed{1}$

Grubler

$L = 8$

$J = 10$

$G = 1$

$M = 3L - 2J - 3G$

$= 3*8 - 2*10 - 3*1 = \boxed{1}$

Compute the DOF of the following examples with **Kutzbach's and Gruebler's Equations**.

Kutzbach

$L = 6$

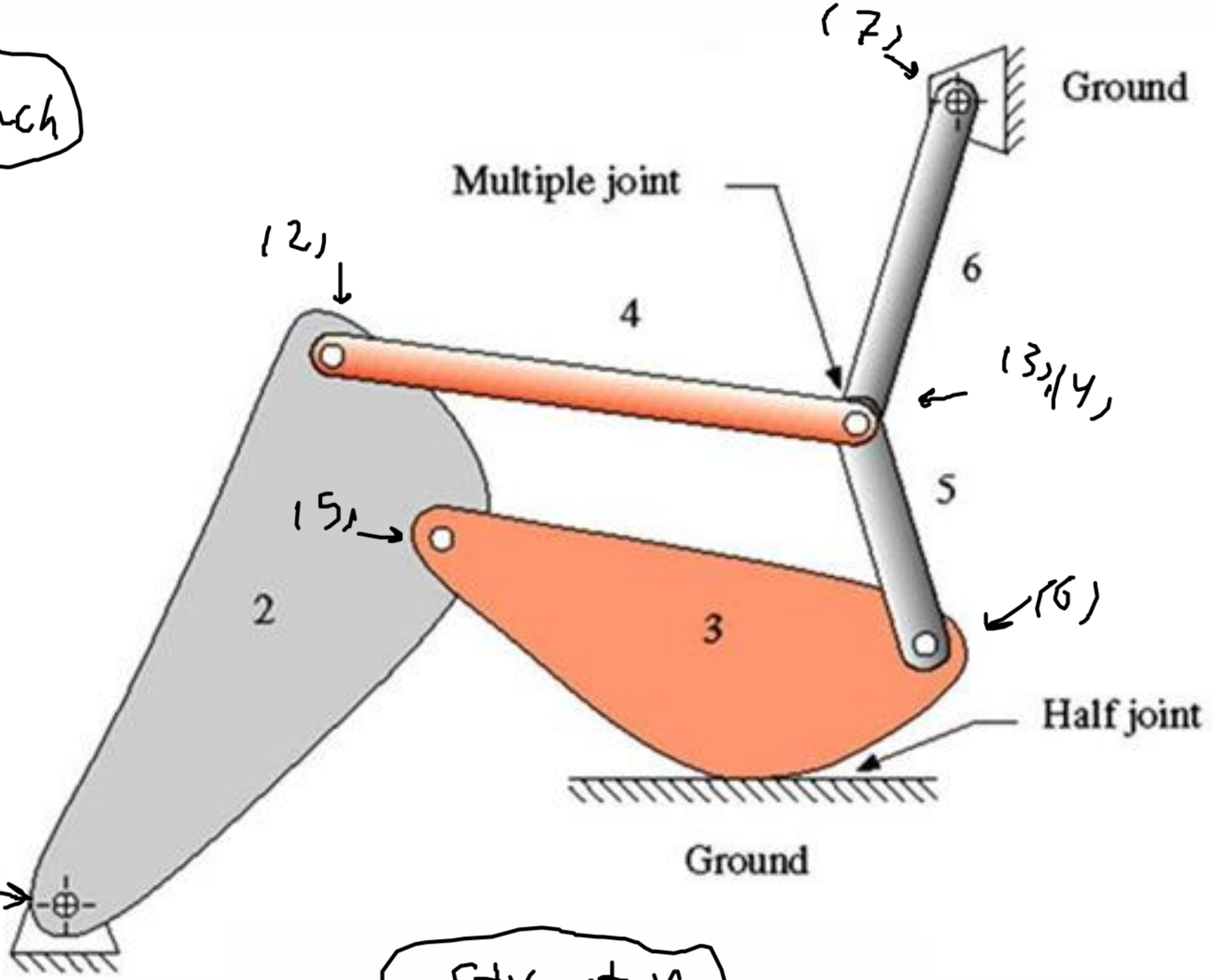
$J_1 = 7$

$J_2 = 1$

$M = 3(L-1) - 2J_1 - J_2$

$= 3(6-1) - 2*7 - 1$

$=$ Zero



Structure

$L = 6$

$J = 8$

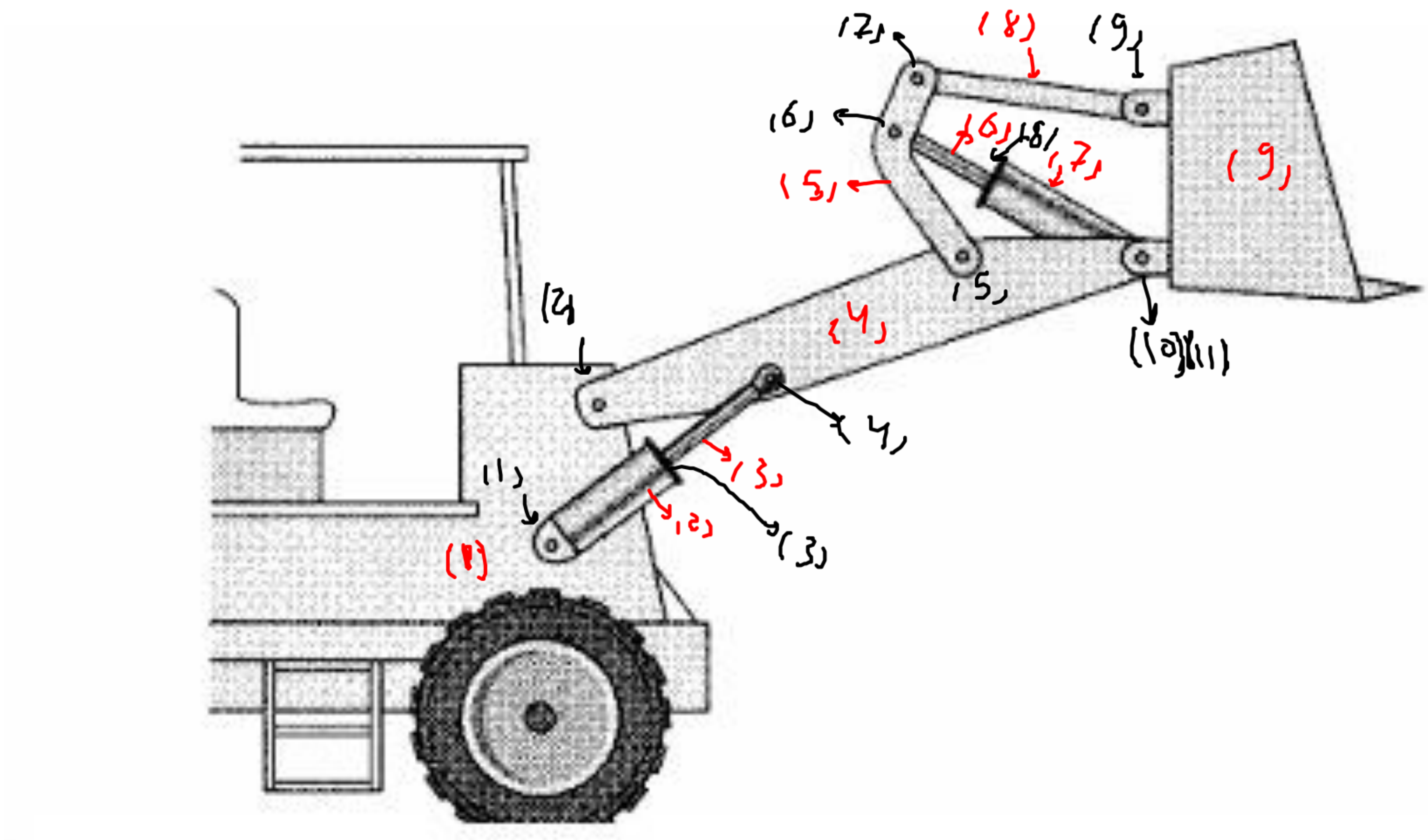
$G = 1$

Gruebler

$M = 3L - 2J - 3G$

$= 3*6 - 2*8 - 3*1$

$= \boxed{-1}$



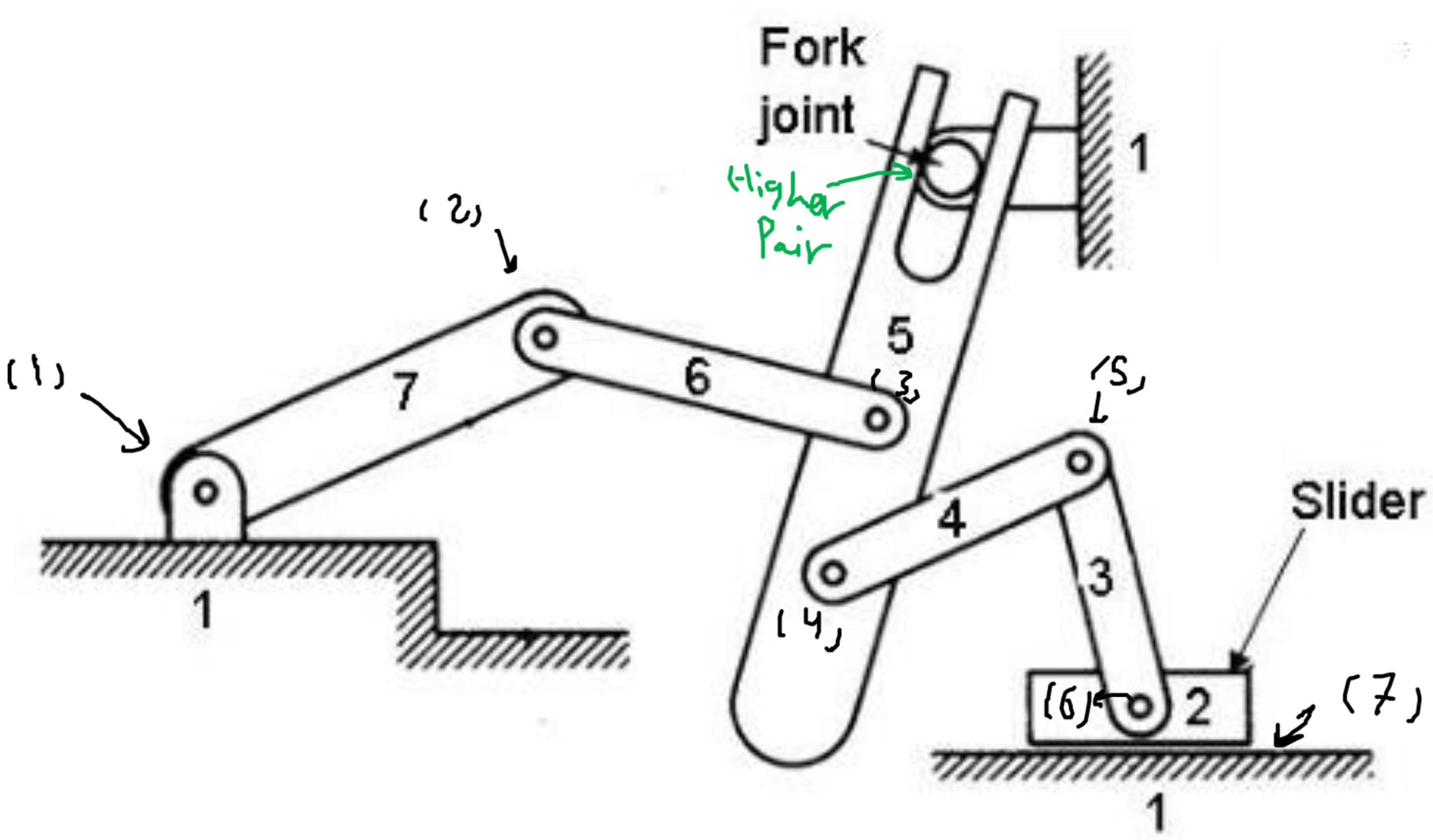
$L = 9$

$J_1 = 11$

$J_2 = \text{zero}$

$M = 3(9-1) - 2 * 11$

$= \boxed{21}$



$$l = 7$$

$$J_1 = 7$$

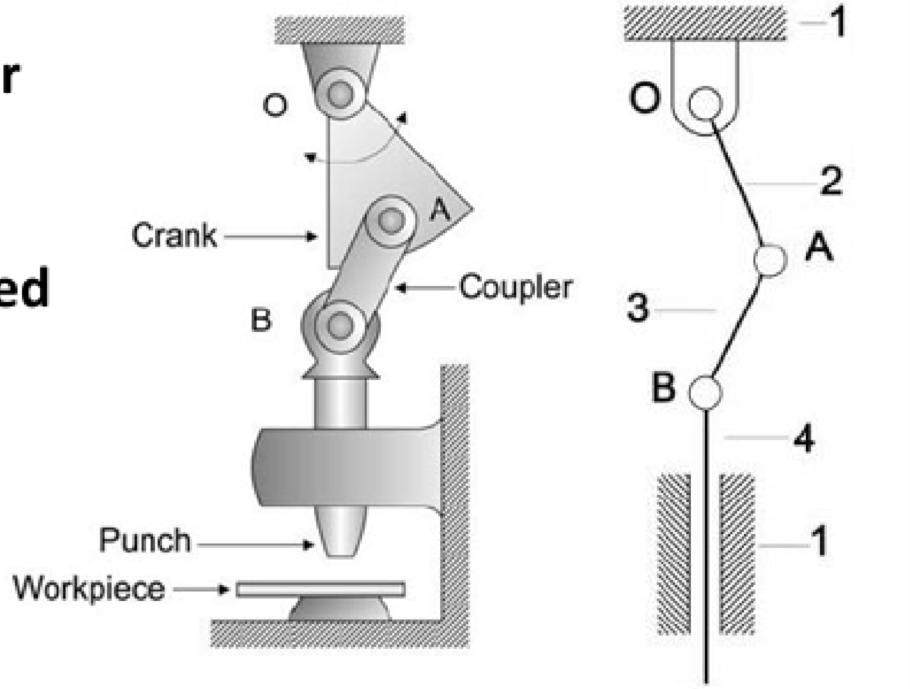
$$J_2 = 1$$

DoF or M $= 3(7-1) - 2*7 - 1$

$= \boxed{3}$

Example 4: Punch Slider

There are four links in punch slider mechanism. It has three revolute joints (full joints) and one grounded slider full joint.



Gruebler's Equation

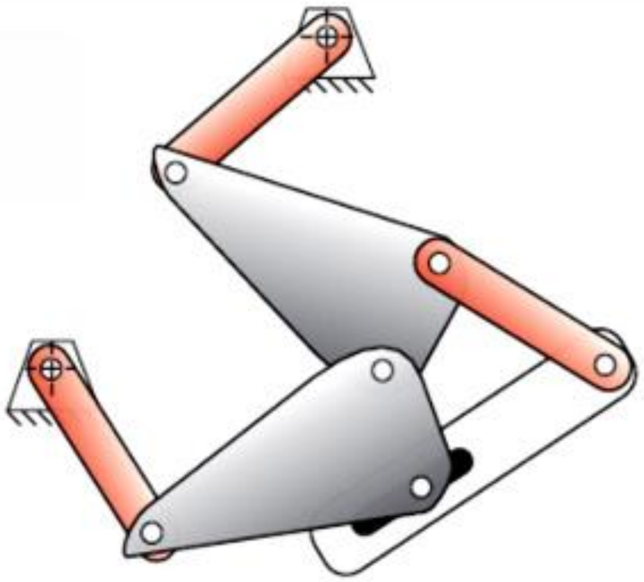
$$M = 3L - 2J - 3G$$
$$= 3(4) - 2(4) - 3(1) = 1 \text{ DOF}$$

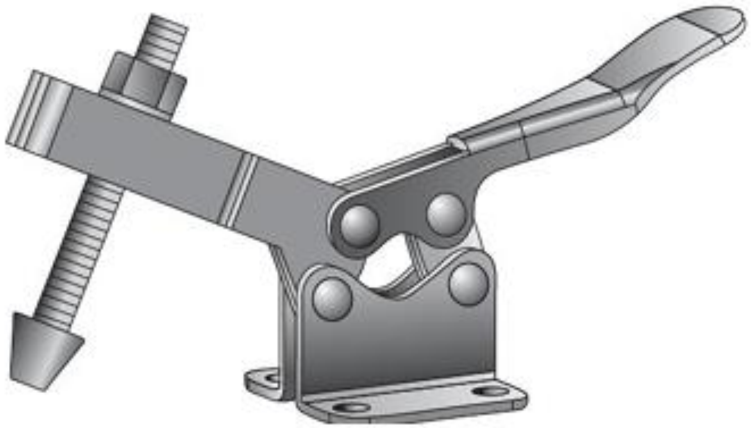
Kutzbach's Equation

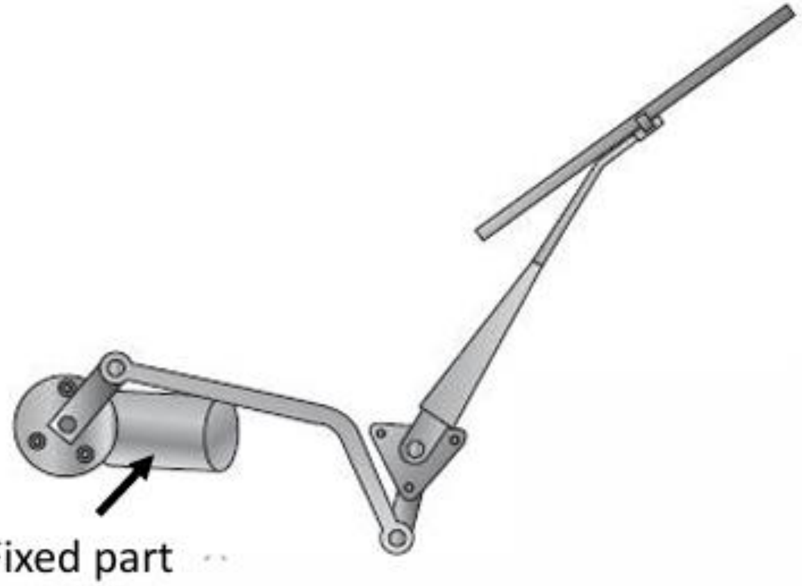
$$M = 3(L - 1) - 2J_1 - J_2$$
$$= 3(4-1) - 2(4) - 0 = 1 \text{ DOF}$$

Homework 1

Find mobility of given mechanisms through **Gruebler's and Kutzbach's Equations**







$\boxed{L=4}$

Kutzbach

$\boxed{J_1=4}$

$\boxed{J_2=Zero}$

$M = 3(4-1) - 2*4 - 0 = \boxed{1}$

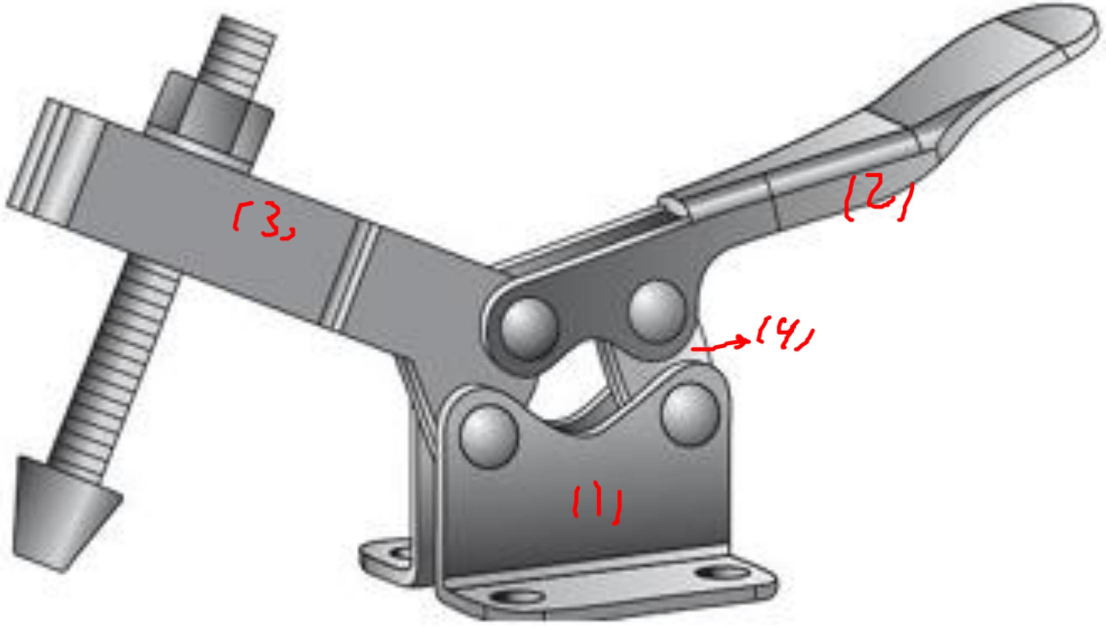
Grebbler

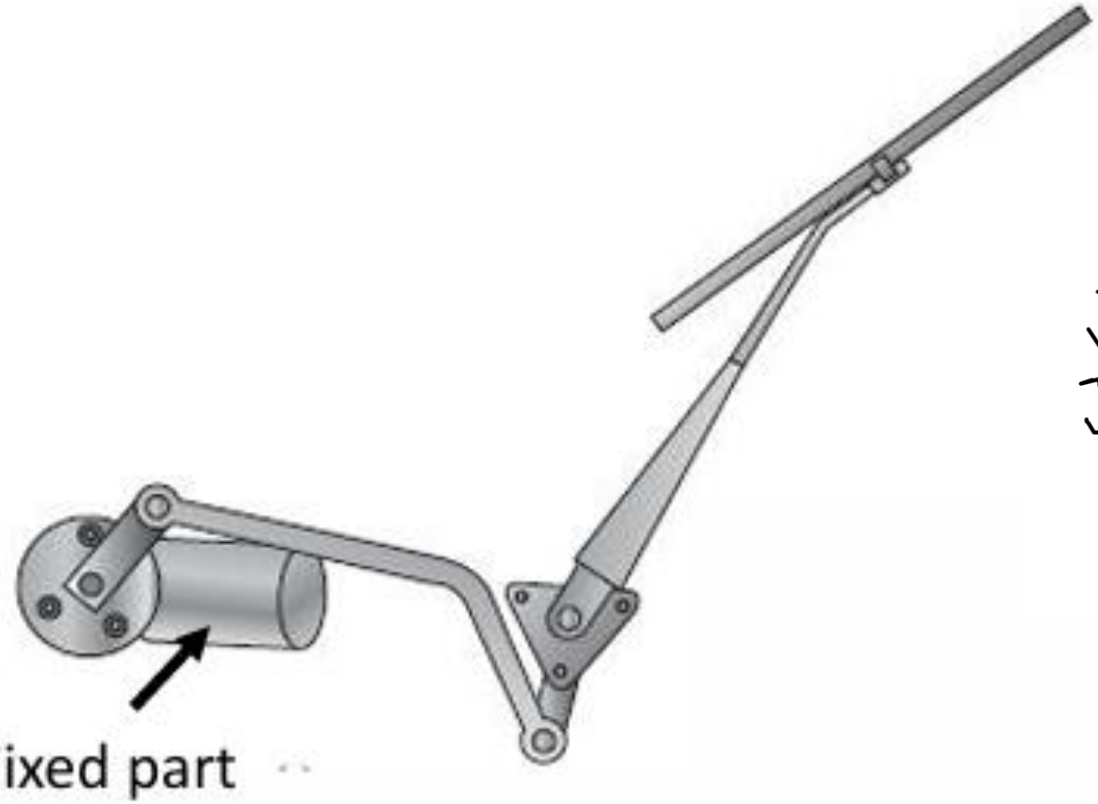
$L=4$

$J=4$

$G=1$

$M = 3*4 - 2*4 - 3*1 = \boxed{1}$

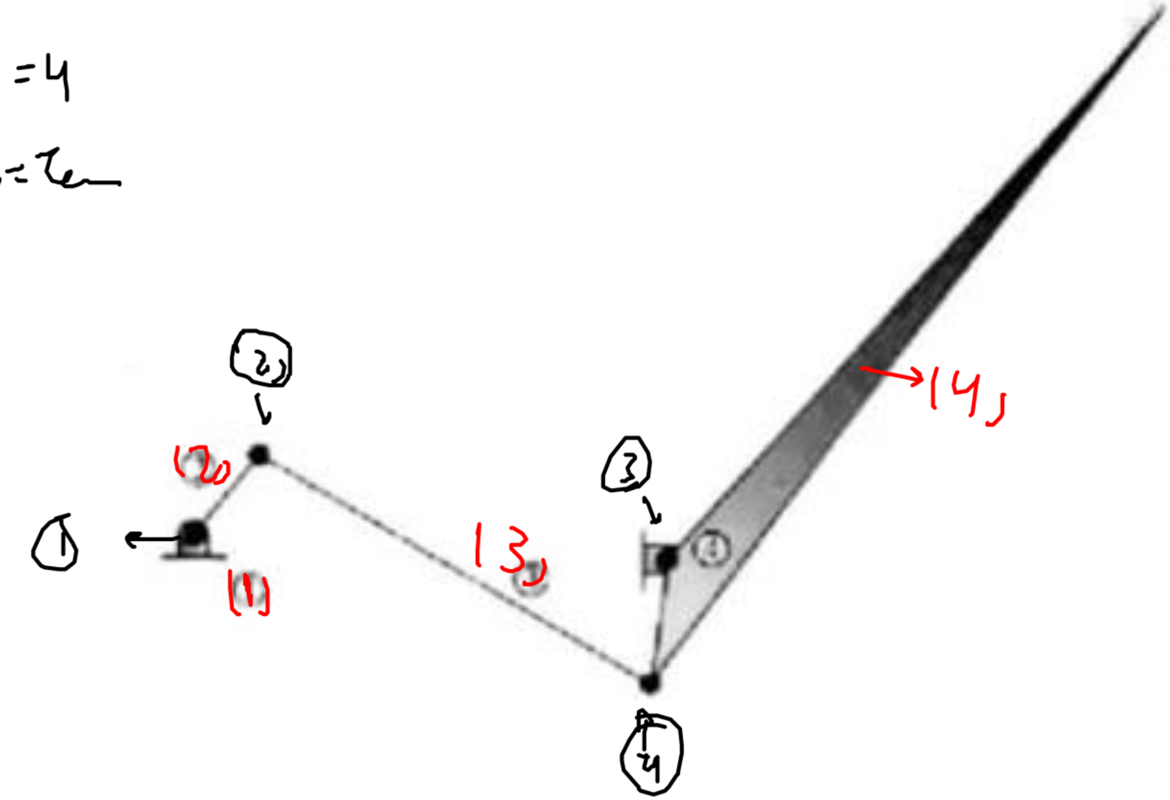




L = 4

$J_1 = 4$

$J_2 = \text{zero}$

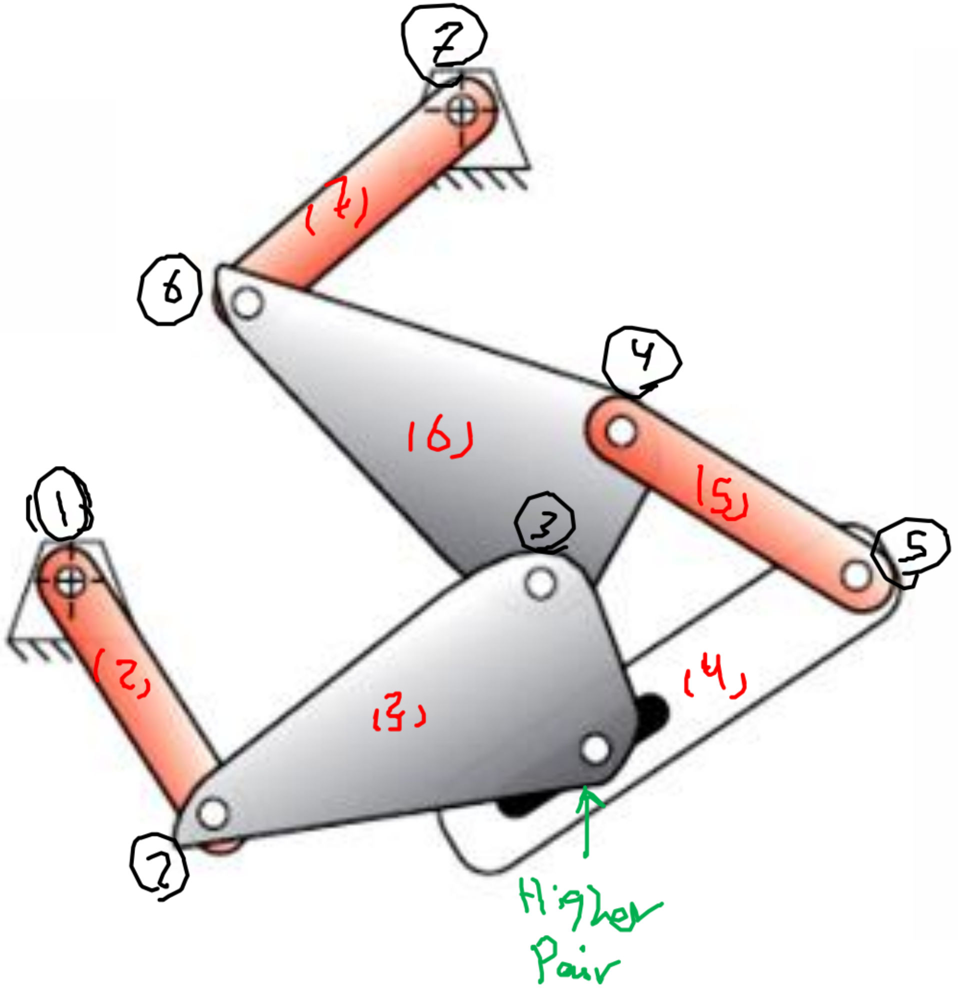


Kutzbach

$DOF = 3(4-1) - 2*4 = \boxed{1}$

Grubler

$DOF = 3*4 - 2*4 - 3*1 = \boxed{1}$



$\boxed{L = 7}$

$\boxed{J_1 = 7}$

$\boxed{J_2 = 1}$

Kutzbach

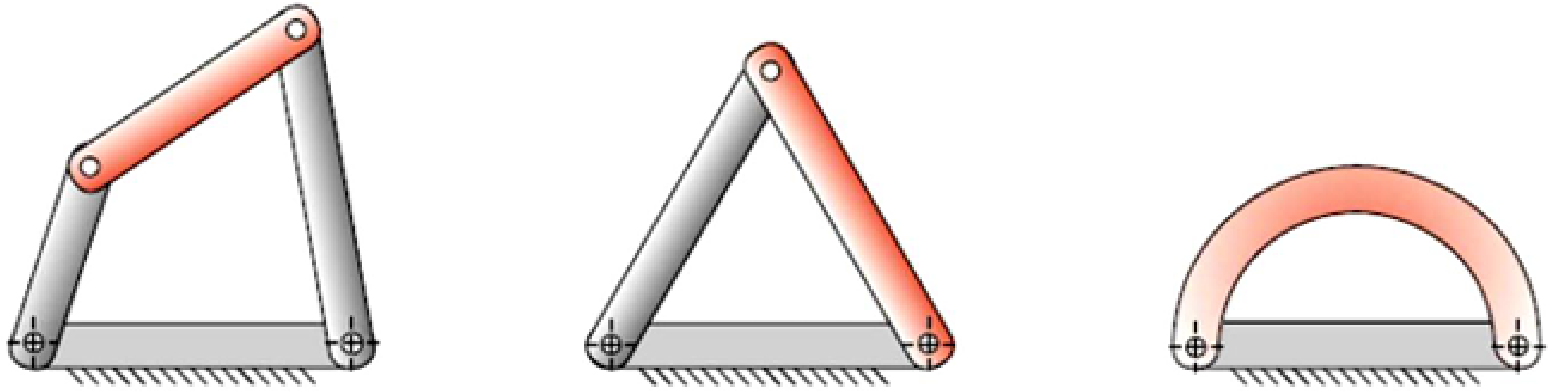
$$DOF = 3(7-1) - 2*7 - 1 = \boxed{3}$$

GReublen

$$DOF = 3*7 - 2*8 - 3*1 = \boxed{2}$$

Mechanisms and Structures

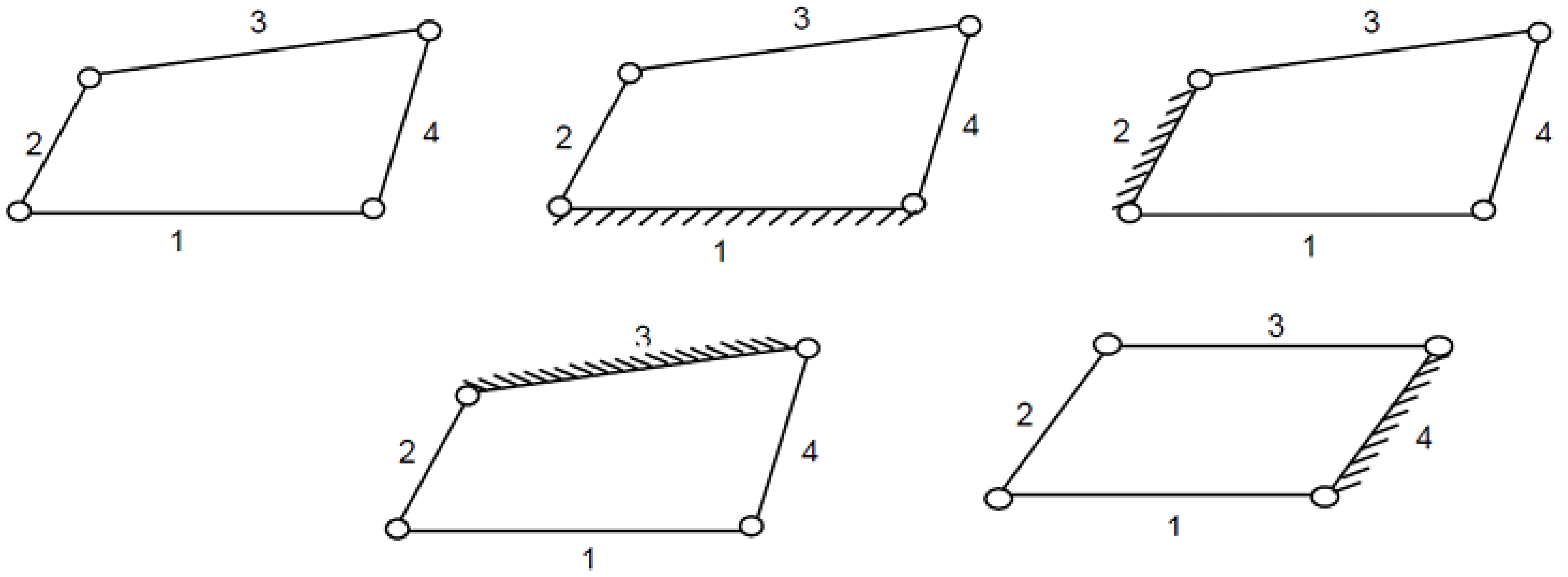
- If **DOF > 0**, the assembly of links is a **mechanism** and will exhibit relative motion
- If **DOF = 0**, the assembly of links is a **structure,** and no motion is possible.
- If **DOF < 0**, then the assembly is a **preloaded structure**, no motion is possible, and in general stresses are present.



(a) Mechanism—*DOF* = +1 (b) Structure—*DOF* = 0 (c) Preloaded structure—*DOF* = −1

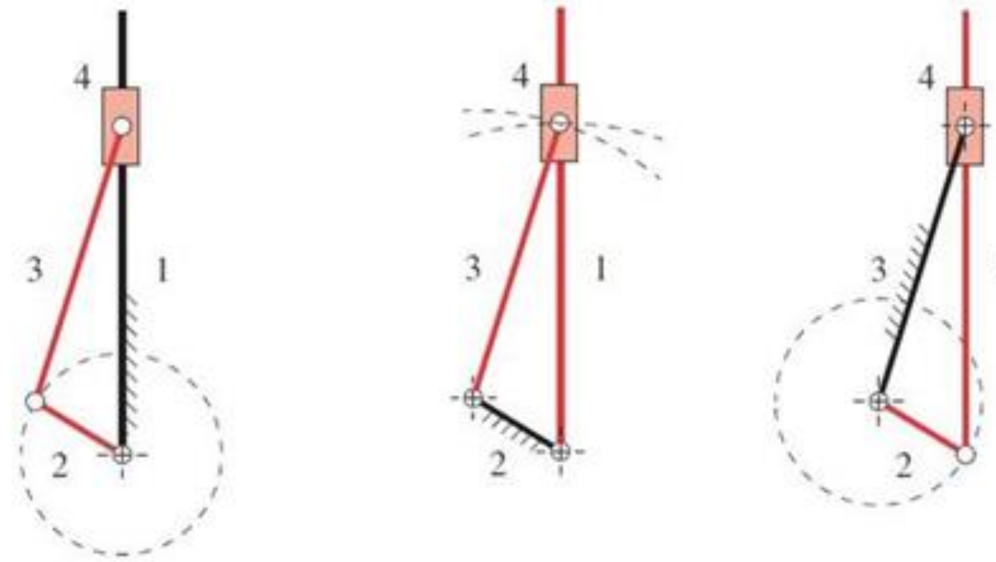
Inversions of Kinematic Chain

- All the mechanisms shown are called **inversions of the parent kinematic chain**.
- By this **principle of inversions** of a four-link chain, **several useful mechanisms** can be **obtained**.



Inversions of slider-crank linkage

- Figure shows the four inversions of the fourbar slider-crank linkage, all of which have distinct motions.



(a) Inversion # 1 slider block translates

(b) Inversion # 2 slider block has complex motion

(c) Inversion # 3 slider block rotates

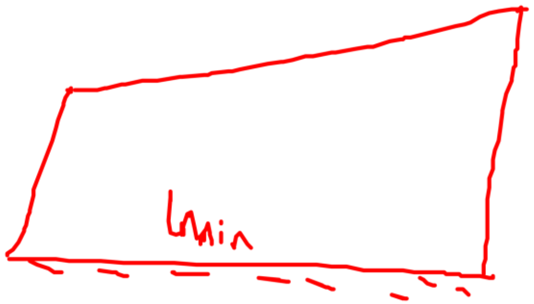
(d) Inversion # 4 slider block is stationary

* 4-bar Mechanism

→ Grashof s' law

$$\underbrace{L_{Max}}_{\text{link أطول}} + \underbrace{L_{Min}}_{\text{link أقصر}} \le \underbrace{L_a + L_b}_{\text{other links}}$$

$\boxed{L_{Max} + L_{Min} = L_a + L_b}$



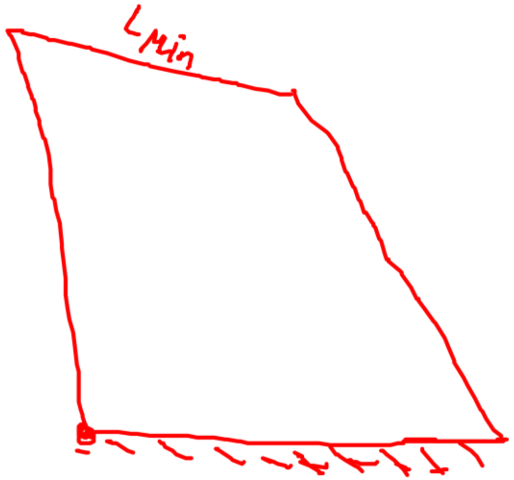
Crank rocker Mechanism

أقصر link على يمين أو يسار ال link الثابت

drag-link

Double-Crank Mechanis

أقصر link هو ال link الثابت



Double-rocker

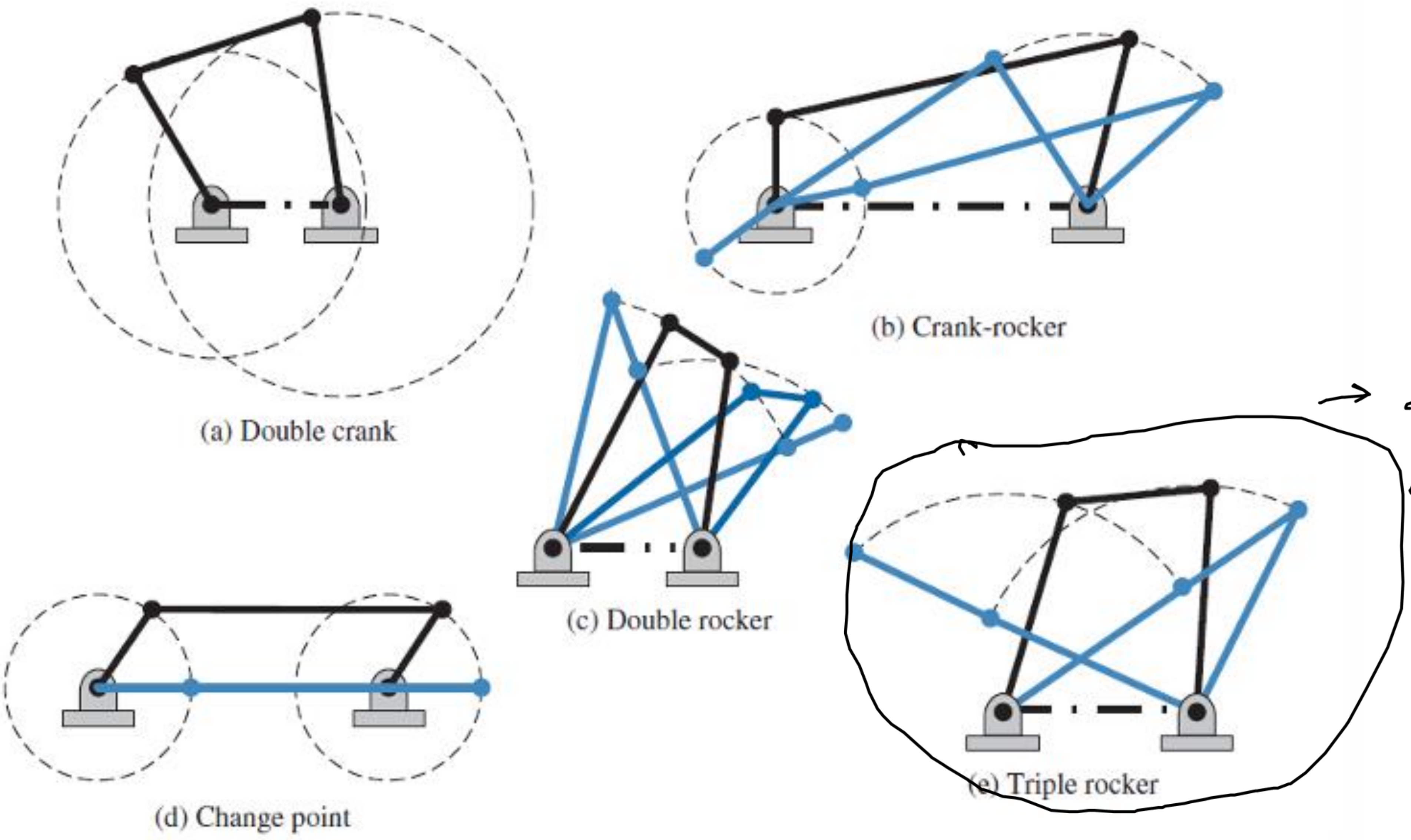
أقصر link في مواجهة ال link الثابت

Change-Point Mechanism

شرط تحقق ال 4-bar Mechanism

$$L_{Max} < L_{Min} + L_a + L_b$$

Schematics of Grashof Mechanisms

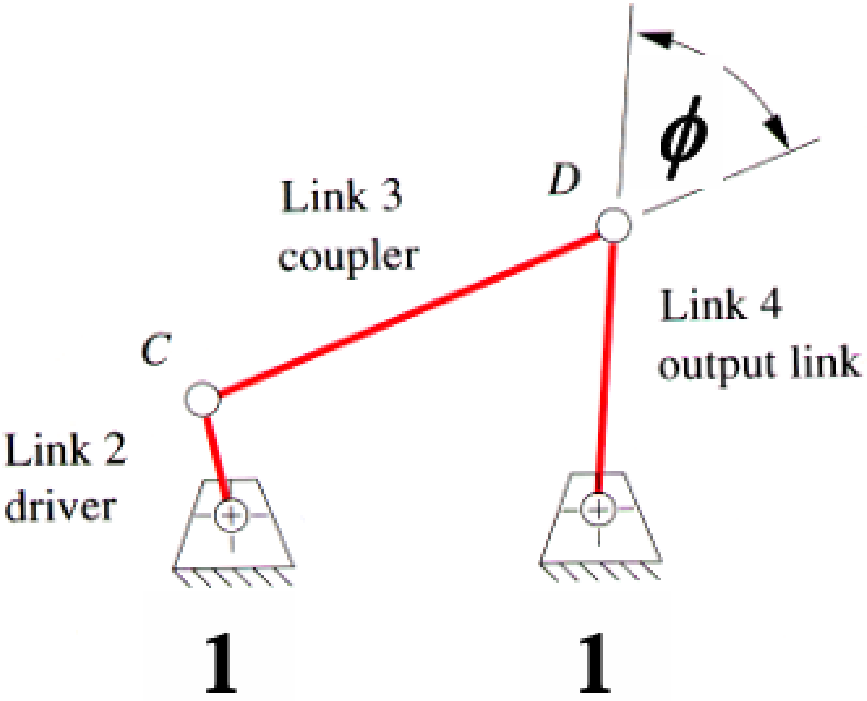


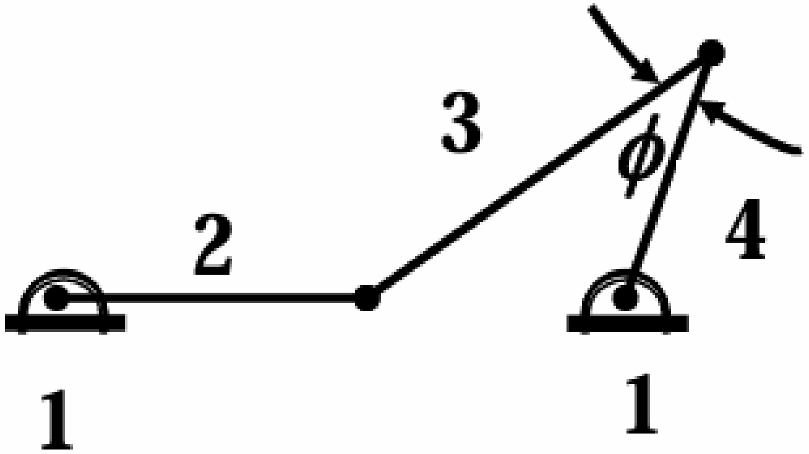
Grashof law غير محقق

$L_{Max} + L_{min} > L_a + L_b$

Transmission Angle [ϕ]

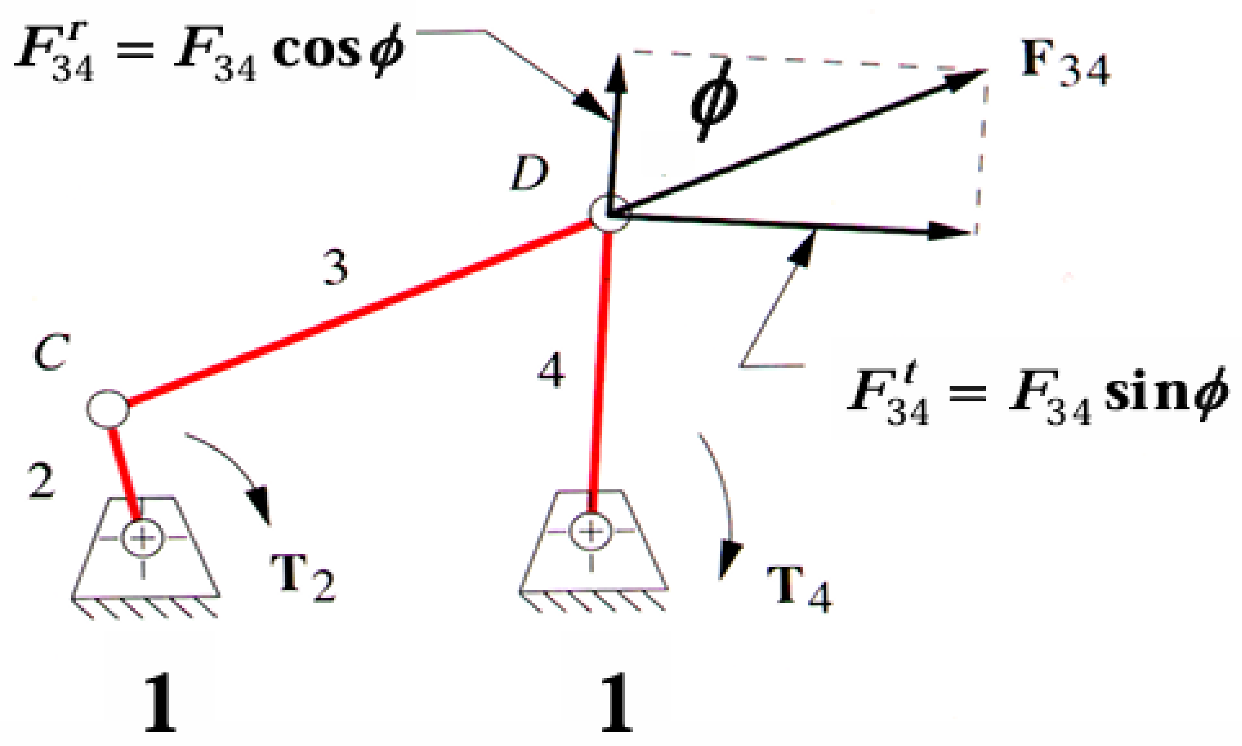
The angle between the coupler (link 3) and the driven (Rocker or Follower) (link 4)





Transmission Angle [ϕ]

The coupler (link 3) transmits a force along its centerline to the driven (rocker) link



$$T_4 = F_{34}^{t} * L_4$$

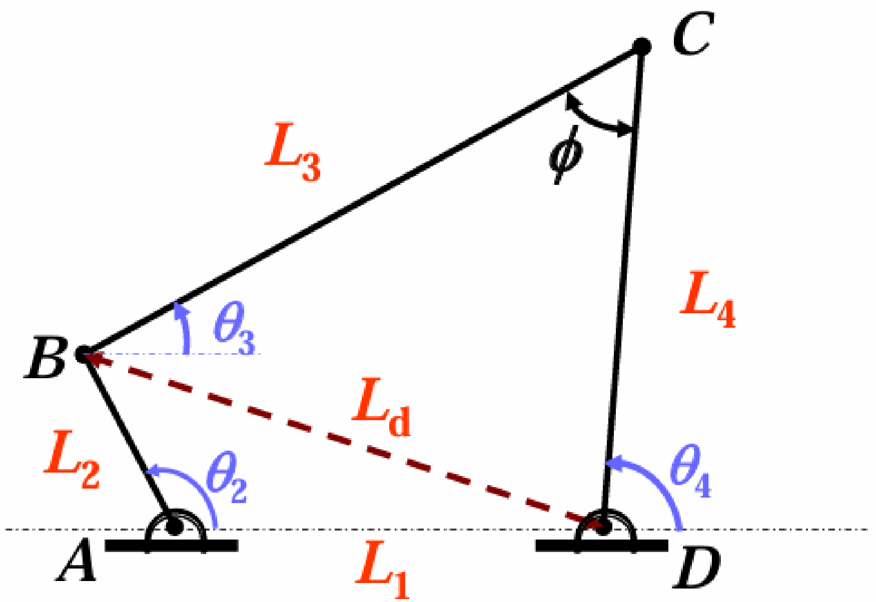
Transmission Angle [ϕ]

To get best performance of the mechanism ➔ $\varphi \approx 90^{\circ}$

In General

$$\frac{\text{To prevent problems in the mechanism}}{(40^{\circ} \leftrightarrow 45^{\circ}) \le \phi \le (135^{\circ} \leftrightarrow 140^{\circ})}$$

Transmission Angle [ϕ]



θ_2 : input angle (crank angle), θ_4 : Output angle (rocker angle)

θ_3 : Coupler angle, ϕ : Transmision angle, L_d : Diagonal

Transmission Angle [ϕ]

* L_d can be determined using the cosines law as follows :
  * For the triangle (ABD) :

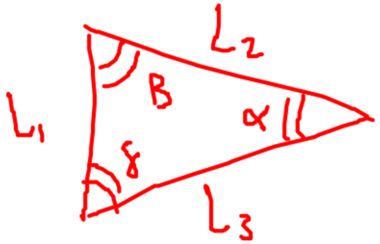
$$L_d^2 = L_1^2 + L_2^2 - 2\,L_1 L_2 \cos\theta_2 \qquad (1)$$

* For the triangle (BCD) :

$$L_d^2 = L_3^2 + L_4^2 - 2\,L_3 L_4 \cos\phi \qquad (2)$$

* From Eq. (2) :

$$\cos\phi = \frac{L_3^2 + L_4^2 - L_d^2}{2\,L_3 L_4} \qquad (3)$$



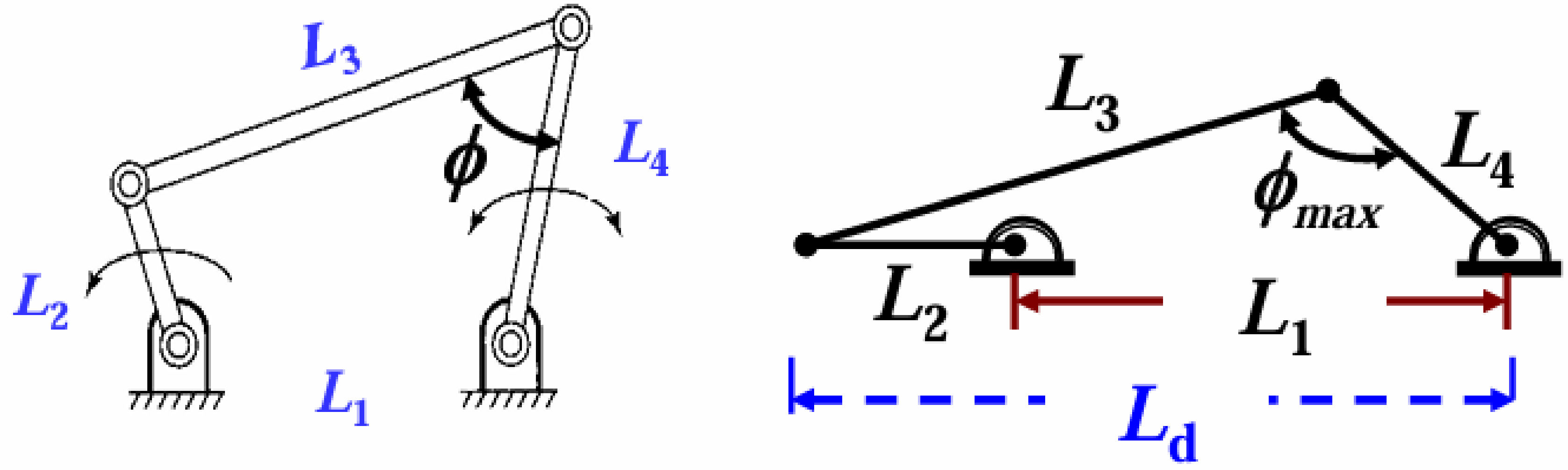
$$L_2^2 = L_1^2 + L_3^2 - 2L_1L_3\cos\gamma$$

$$L_1^2 = L_2^2 + L_3^2 - 2L_2L_3\cos\alpha$$

$$L_3^2 = L_1^2 + L_2^2 - 2L_1L_2\cos\beta$$

Transmission Angle [ϕ]

The maximum and minimum transmission angle occurs when *L_2 and L_1 are collinear*

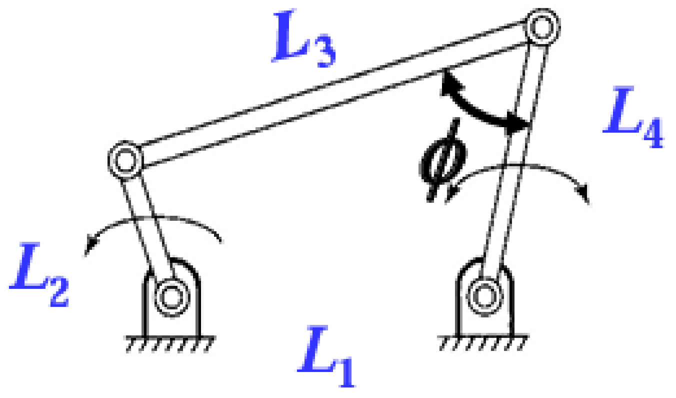


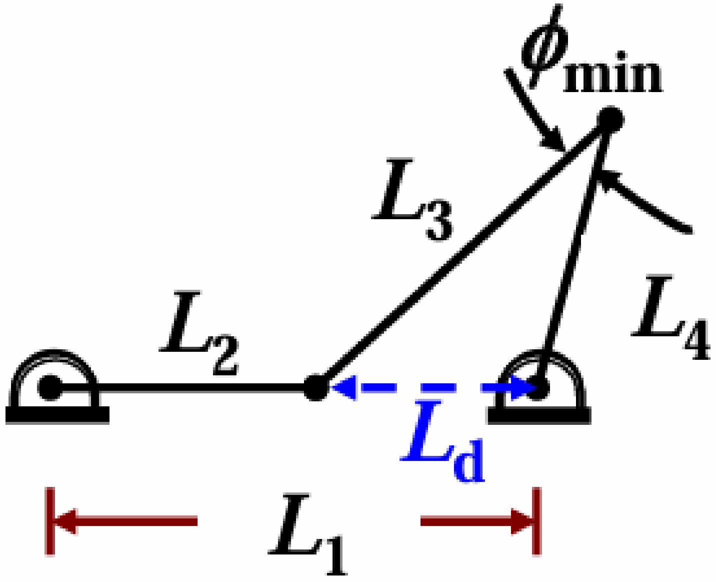
When; $L_d = (L_d)_{max} = L_1 + L_2$

$$\Rightarrow \quad \phi = \phi_{max}$$

Transmission Angle [ϕ]

The maximum and minimum transmission angle occurs when *L_2 and L_1 are collinear*





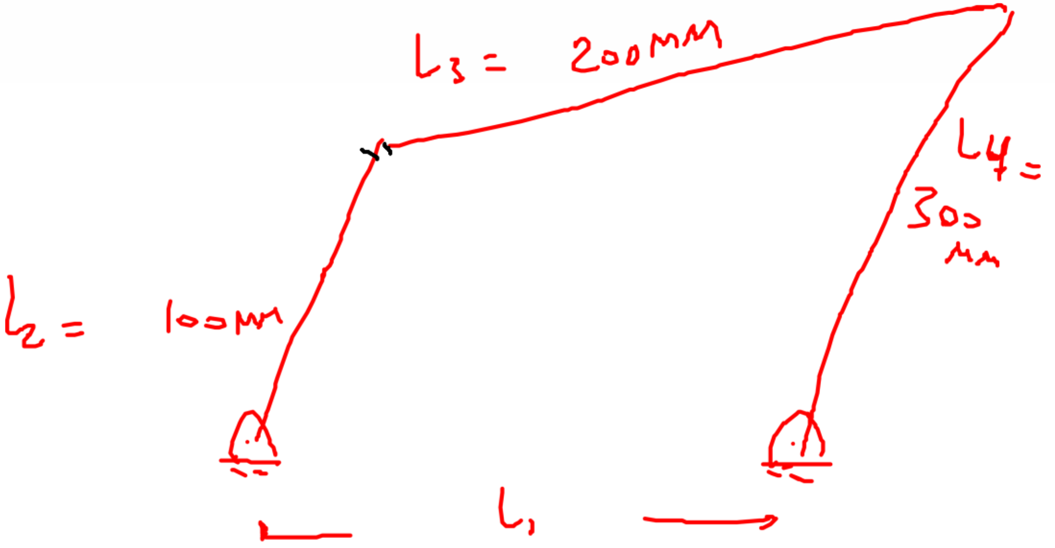
When; $L_d = (L_d)_{\min} = L_1 - L_2$

$\Rightarrow \quad \phi = \phi_{\min}$

Example: 3

$$\text{Given}\quad L_2 = 100\,\text{mm},\ L_3 = 200\,\text{mm, and}\ L_4 = 300\,\text{mm}$$

$$\text{Find}\ L_1 = ?\ \text{to be crank - rocker mechanism?}$$



$L_2 \rightarrow$ Shortest link

* L_1 is the longest link

$L_1 + 100 \leq 200 + 300$

$L_1 \leq 400$ → الشرط الأول

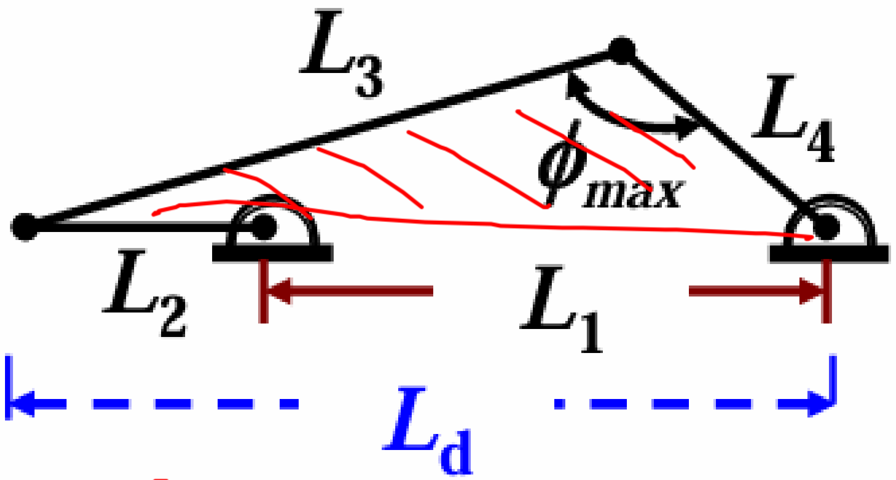
* L_1 is intermediate

$300 + 100 \leq L_1 + 200$

$L_1 \geq 200$ → الشرط الثاني

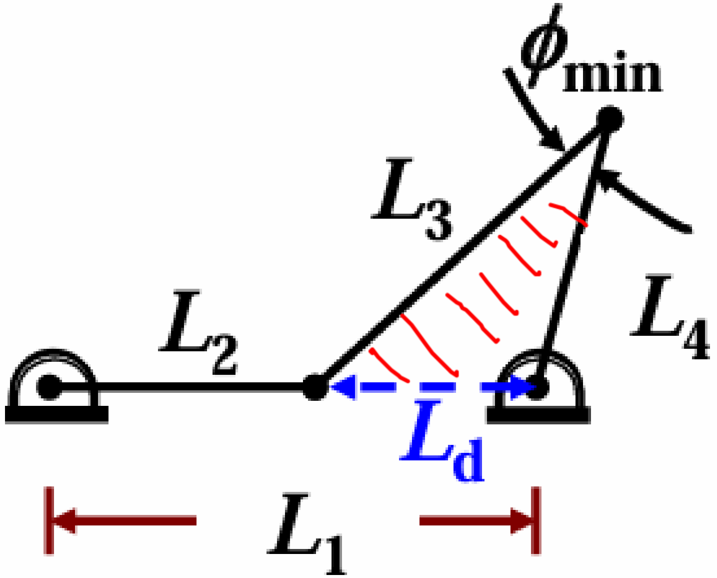
الشرطين متحققين

$200 < L_1 < 400$



$45° < \phi < 135°$

For $\phi_{min} = 45°$



$$L_d^2 = (L_1 + L_2)^2 = L_3^2 + L_4^2 - 2L_3L_4\cos\phi_{max}$$

$$(312{,}48 + 100)^2 = 200^2 + 300^2 - 2*200*300\cos\phi_{max}$$

$$\boxed{\phi_{max} = 109{,}54°}$$

↳ Accepted because it is within range

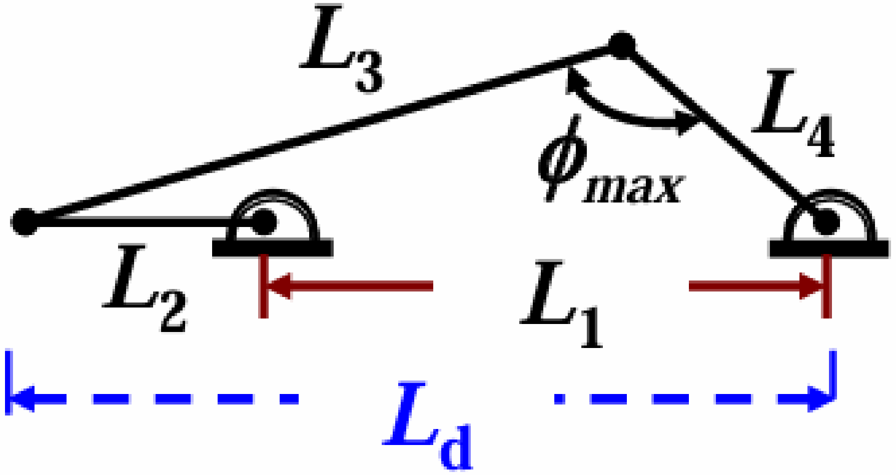
$$\phi_{min} = 45°$$

$$L_d^2 = (L_1 - L_2)^2 = L_3^2 + L_4^2 - 2L_3L_4\cos\phi_{min}$$

$$(L_1 - 100)^2 = 200^2 + 300^2 - 2*200*300\cos 45$$

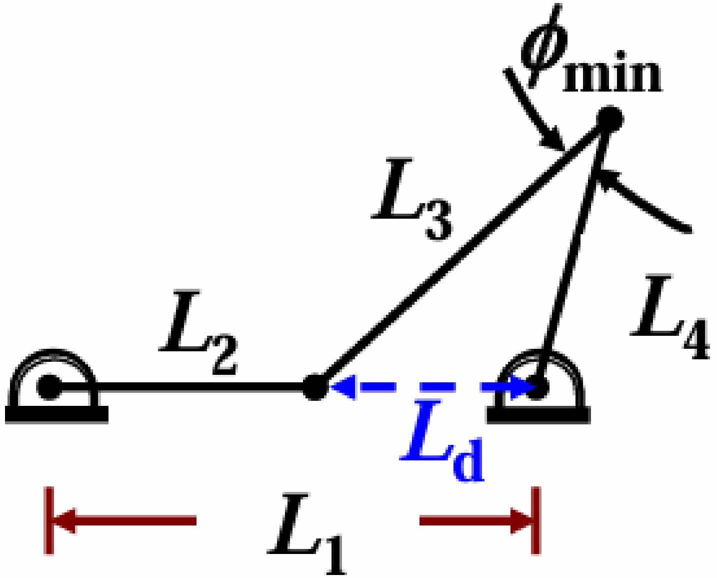
$$\boxed{L_1 = 312{,}48\,\text{mm}}$$

Check for ϕ_{max}



$45 < \phi < 135$

$\phi_{Max} = 135°$



$\phi_{Max} = 135°$

$$L_d^2 = (L_1 + L_2)^2 = L_3^2 + L_4^2 - 2L_3L_4\cos\phi_{Max}$$

$$(L_1 + 100)^2 = 200^2 + 300^2 - 2*200*300*\cos 135$$

$$L_1 = 363{,}52\text{ mm}$$

$$L_d^2 = (L_1 - L_2)^2 = L_3^2 + L_4^2 - 2L_3L_4\cos\phi_{min}$$

$$(363{,}52 - 100)^2 = 200^2 + 300^2 - 2*200*300\cos\phi_{min}$$

$$\phi_{min} = 59{,}69°$$

accepted because it's within range

check

$$312{,}48_{mm} < L_1 < 363{,}52_{mm}$$

Example: 4

A four-bar mechanism with L_1 = 3.0 in, L_2 = 2.0 in, L_3 = 5.0 in, and L_4 = 5.0 is connected in an open configuration. When θ_2 =30° , what is the value of θ_4 (output angle)?

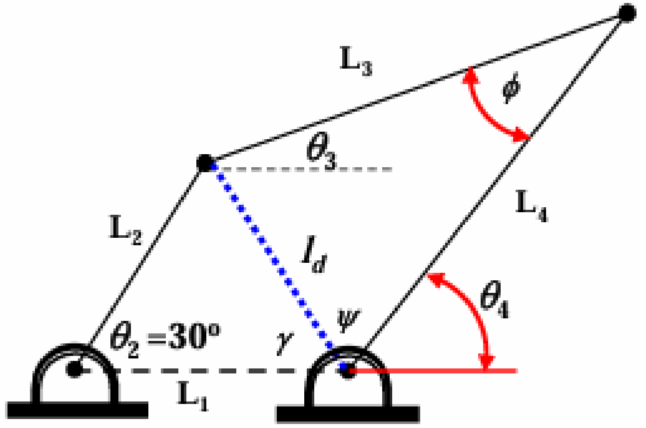
$$l_d = \sqrt{l_1^2 + l_2^2 - 2 l_1\, l_2 \cos\theta_2}$$
$$= \sqrt{3^2 + 2^2 - (2\times 3\times \cos 30^\circ)}$$
$$= 1.61\,\text{in}$$

$$\varphi = \cos^{-1}\left[\frac{l_3^2 + l_4^2 - l_d^2}{2 l_3 l_4}\right]$$
$$= \cos^{-1}\left[\frac{25+25-1.61^2}{2\times 5\times 5}\right] = 18.53^\circ$$

$$\psi = \cos^{-1}\left[\frac{l_d^2 + l_4^2 - l_3^2}{2 l_d l_4}\right] = \cos^{-1}\left[\frac{1.61^2 + 25 - 25}{2\times 1.61\times 5}\right] = 80.11^\circ$$

$$\gamma = \cos^{-1}\left[\frac{l_1^2 + l_d^2 - l_2^2}{2 l_d l_1}\right] = \cos^{-1}\left[\frac{9 + 1.61^2 - 4}{2\times 1.61\times 3}\right] = 38.19^\circ$$

$$\theta_4 = 180 - (\psi + \gamma) = 180 - (80.11 + 38.19) = 61.7^\circ$$



Example: 5

A four-bar mechanism with L_1 = 3.0 in, L_2 = 2.0 in L_3 = 5.0 in and L_4 = 5.0 is connected in open configuration. When θ_4 =120° , θ_2 (input angle) is:

$$d = \sqrt{l_1^2 + l_4^2 - 2 l_1 l_4 \cos 60^\circ}$$
$$= \sqrt{9 + 25 - (2 \times 5 \times \cos 60^\circ)}$$
$$= 4.35 \text{ mm}$$

$$\alpha = \tan^{-1}\left[\frac{l_4 \sin 60}{l_1 - l_4 \cos 60}\right]$$
$$= \tan^{-1}\left[\frac{5 \sin 60}{3 - 5\cos 60}\right] = 83.41^\circ$$

$$\beta = \cos^{-1}\left[\frac{l_2^2 + d^2 - l_3^2}{2 l_2 d}\right] = \cos^{-1}\left[\frac{4 + 4.35^2 - 25}{2 \times 2 \times 4.35}\right] = 96.85^\circ$$

$$\theta_2 = \alpha + \beta = 180.26^\circ$$