

dynamics

ديناميكا

هو علم دراسة الاجسام المتحركة ودراسة تأثير القوى الخارجية عليها

dynamics

Kinetics.

دراسة حركة الجسم
مع الاحتيا اعتبار
القوى المسببة للحركة

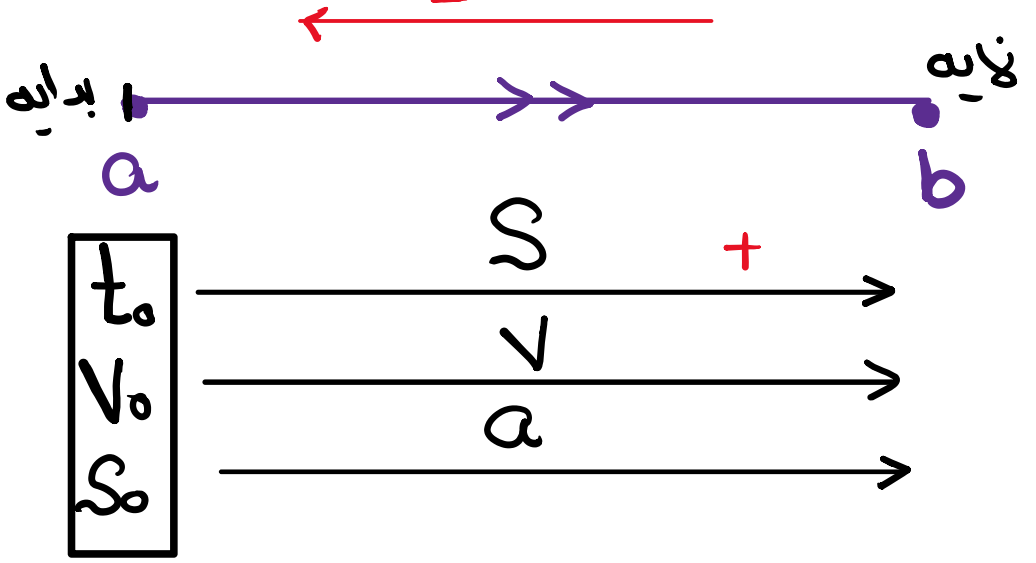
① kinematics

دراسة حركة الجسم
دون النظر للقوى المسببة للحركة

Ch1 :- Planer Motion:-

1.1 :- Rectilinear motion.

motion in straight line
حركة في خط مستقيم



initial Condition
الشروط بداية الحركة

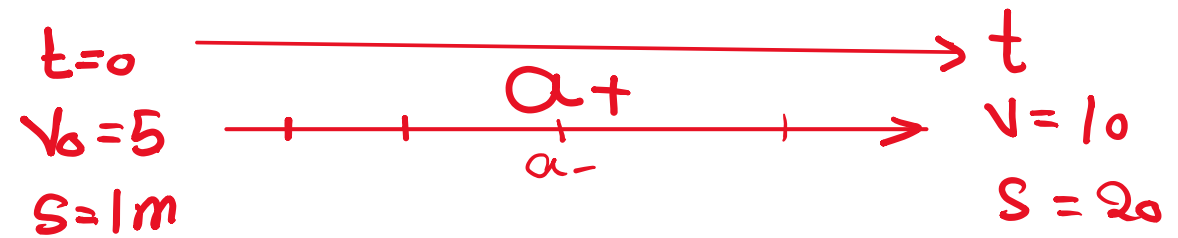
$t = \text{time}$ زمن
 $S = \text{displacement}$ إزاحة
 $v = \text{Velocity}$ سره
 $a = \text{acceleration}$ تسارع

الحركة الخطيه

$S = \text{displacement} = \pm m, km, ft$

$v = \text{Velocity} = \frac{ds}{dt} (\pm) \frac{m}{s}, \frac{km}{h}, \frac{ft}{s}$

$a = \text{acceleration} = \frac{dv}{dt} (\pm) \frac{m}{s^2}, \frac{k}{hr^2}, \frac{ft}{s^2}$



$a(+) \rightarrow \text{accelerate}$

$a(-) \rightarrow \text{decelerate}$

تسارع
تباطؤ

يوجد نوعين للحركة

Const acceleration $a = c$
عجلة ثابتة

← نقدم قوانين نيوتن

$$1- v = v_0 + at \rightarrow (v, t)$$

$$2- v^2 = v_0^2 + 2a(s - s_0) \quad (v, s)$$

$$3- s = s_0 + v_0 t + \frac{1}{2}at^2. \quad (s, t)$$

Variable acceleration عجلة متغيرة

$$a = f(t, v, s)$$

① لو $a = f(t)$ نستخدم التكامل

$$\Rightarrow a = \frac{dv}{dt}$$

$$\int_{t_0}^t a dt = \int_{v_0}^v dv$$

$$t^2 - 2t + \dots$$

$$\Rightarrow a = f(s)$$

$$a = v \frac{dv}{ds}$$

② لو a دالة في s

$$\Rightarrow \text{if } a = F(v)$$

یوحنا حالی

لوعاير علاف بين ۷ و ۱۰

$$a = F(v) = \frac{dv}{dt}$$

لوعايز عمل افه سن ۵۱۷ ②

$$a = F(v) = v \frac{dv}{ds}$$

$$\int_{t_0}^t f(t) dt = \int_{v_0}^v f(v) dv = \int_{s_0}^s f(s) ds$$

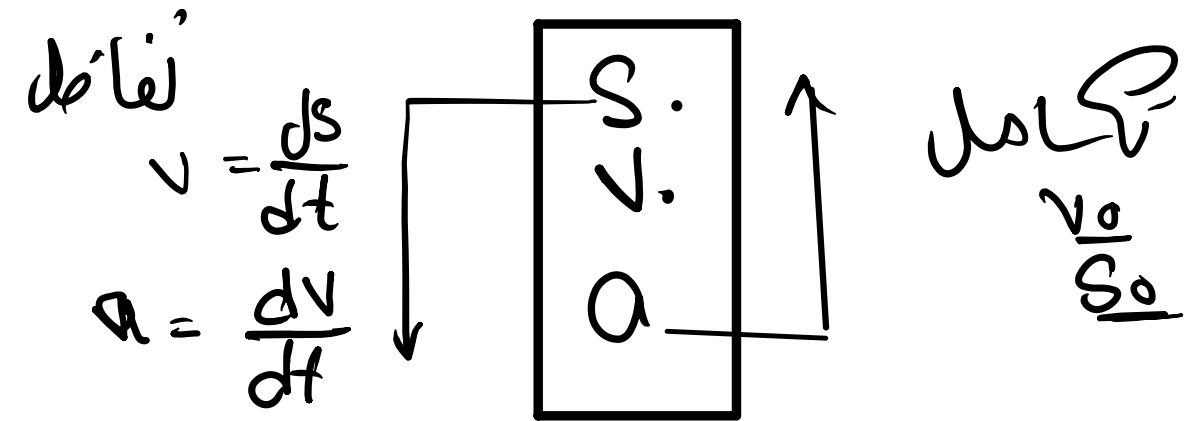
$$\int_0^t 2t + 2 \, dt = 2t^{\frac{3}{3}} + 2t \Big|_0^t$$
$$= \left[2t^{\frac{3}{3}} + 2t \right] - [2 \times 0 + 2 \times 0]$$
$$I = 2t^{\frac{3}{3}} + 2t.$$

⇒ Start from rest

$$\underline{V_0 = 0}$$

⇒ Start from origin

$$\underline{S_0 = 0}$$



خطوات الحل

① نقرأ السؤال

② given $a =$ $\begin{cases} \text{const} \\ \text{variable} \end{cases}$

$\underline{V_0 =}$ $\underline{S_0 =}$

③ Req:- مطلوب

④ نقدم الطريقة المناسبة في الحل

g) If $a = 4 \text{ m/s}^2$, determine s when $t = 3 \text{ s}$ if $v = 2 \text{ m/s}$ and $s = 2 \text{ m}$ when $t = 0$.

given:-

$$a = 4 \text{ m/s}^2$$

at $t_0 = 0$ $v_0 = 2 \text{ m/s}$

$$s_0 = 2 \text{ m}$$

Req :-

$$s = ?$$

$$t = 3 \text{ s}$$

الحل

قوانين

$$s = s_0 + v_0 t + \frac{1}{2} a t^2$$

$$at = 3 \text{ s}$$

$$s = 2 + 2 * 3 + \frac{1}{2} * 4 * 3^2$$

$$s = 26 \text{ m} *$$

$$v = v_0 + at$$

$$= 2 + 4 * 3 = 14 \text{ m/s}$$

لوطب

12-9.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1) \text{ m/s}^2$, where t is in seconds. If $s = 1 \text{ m}$ and $v = 2 \text{ m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6 \text{ s}$. Also,

given

$$a = (2t - 1)$$

$$s_0 = 1 \text{ m}$$

$$v_0 = 2 \text{ m/s}$$

Req

$v, s \rightarrow t=6$

$$a = (2t - 1) = \frac{dv}{dt}$$

$$\int_0^t (2t - 1) dt = \int_{v_0=2}^v dv$$

$$\left[2 \frac{t^2}{2} - t \right]_0^t = V - 2$$

$$[t^2 - t] - [0 - 0] = V - 2$$

المطلوب

$$v = (t^2 - t + 2) \Rightarrow \frac{ds}{dt}$$

$$\int_0^t (t^2 - t + 2) dt = \int_{s_0=2}^s ds$$

$$\left[\frac{t^3}{3} - \frac{t^2}{2} + 2t \right]_0^t = S - 2$$

$$\frac{t^3}{3} - \frac{t^2}{2} + 2t = S - 2$$

$$S = \frac{t^3}{3} - \frac{t^2}{2} + 2t + 2$$

$$V = t^2 - t + 2 \quad \#$$

$$\Rightarrow t = 6s$$

$$v = t^2 - t + 2 \Big|_{t=6} = (6^2) - 6 + 2 = \boxed{32 \text{ m/s}}$$

$$S = \frac{t^3}{3} - \frac{t^2}{2} + 2t + 2 \Big|_{t=6} = \frac{(6^3)}{3} - \frac{6^2}{2} + 2 \times 6 + 2 = \boxed{68 \text{ m}}$$

The car on the left in the photo and in Fig. 12-2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = \underline{0}$, $s = \underline{0}$.



given

$$v = (3t^2 + 2t)$$

$$s_0 = 0$$

Req:-

$$s \longrightarrow t = 3s$$

$$a \longrightarrow t = 3s$$

Solution

$$a = \frac{dv}{dt} = 6t + 2$$

$$\text{at } \boxed{t = 3s}$$

$$a = 3 \times 6 + 2 = \boxed{20 \text{ m/s}^2} \quad *$$

$$v = \frac{ds}{dt} = 3t^2 + 2t$$

$$\int_{s_0=0}^s ds = \int_0^t (3t^2 + 2t) dt$$

$$s \Big|_0^s = \frac{3t^3}{3} + \frac{2t^2}{2} \Big|_0^t$$

$$(s = t^3 + t^2) \longrightarrow \text{at } t = 3s$$

$$s = 3^3 + 3^2 = \boxed{36 \text{ m}}$$

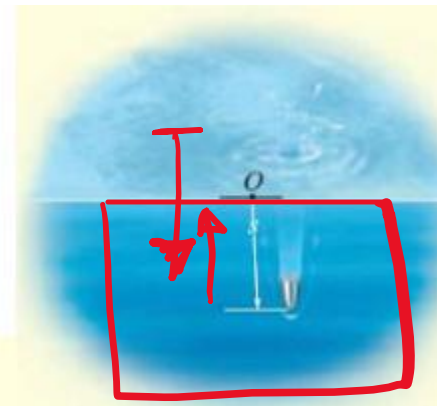
قذف رأسياً لأسفل

وسط

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3)$ m/s², where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.

مقاومة الماء

تباطؤ



SOLUTION

Given

$$v_0 = 60 \text{ m/s}$$

المطلوب

$$a = -0.4v^3$$

req:-

$$v = \text{---} \rightarrow t = 4 \text{ s}$$

$$s = \text{---} \rightarrow t = 4 \text{ s}$$

Solution

$$a = \frac{dv}{dt} = -0.4v^3$$

$$dv = -0.4v^3 dt$$

$$\int_{v_0}^v \frac{dv}{v^3} = \int_0^t -0.4 dt$$

$$\int_{60}^v v^{-3} dv = \int_0^t -0.4 dt$$

$$\left. \frac{v^{-2}}{-2} \right|_{60}^v = -0.4t \Big|_0^t$$

$$\frac{v^{-2}}{-2} + \frac{60^{-2}}{2} = 0.4t$$

$$\frac{1}{-2V^2} = -0.4t - \frac{60^{-2}}{2} \quad * (-2)$$

$$\frac{1}{V^2} = 0.8t + 60^{-2} = 0.8t + (3600)^{-1}$$

اقلب الكل

$$V^2 = \frac{1}{0.8t + \frac{1}{3600}}$$

نأخذ الجذر للعبارتين

$$V = \sqrt{\frac{1}{0.8t + \frac{1}{3600}}} = \frac{1}{\sqrt{0.8t + \frac{1}{3600}}}$$

at $t = 4s$

$$V = \frac{1}{\sqrt{0.8 \times 4 + \frac{1}{3600}}} = 0.556 \text{ m/s.}$$

$$V = \frac{ds}{dt} = \frac{1}{\sqrt{0.8t + \frac{1}{3600}}}$$

$$\int_0^s ds = \frac{1}{0.8} \int_0^t \frac{0.8 dt}{\sqrt{0.8t + \frac{1}{3600}}}$$

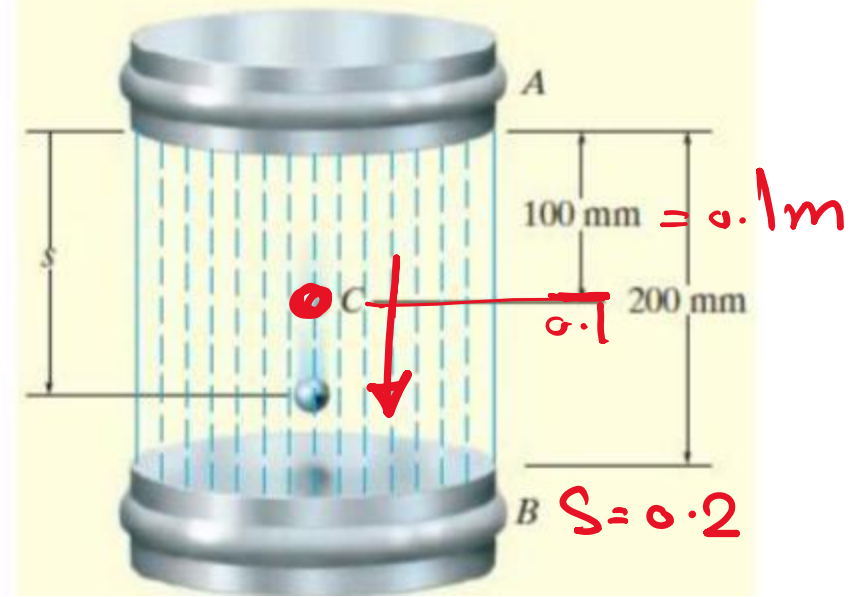
$$S = \frac{1}{0.8} 2 \sqrt{0.8t + \frac{1}{3600}} \quad \uparrow t=4$$

$$S = \frac{2}{0.8} \left[\sqrt{0.8 \times 4 + \frac{1}{3600}} - \sqrt{0 + \frac{1}{3600}} \right]$$

$$S = 4.43 \text{ m}$$

مسئله ۱۲-۵

A metallic particle is subjected to the influence of a magnetic field as it travels downward through a fluid that extends from plate A to plate B, Fig. 12-5. If the particle is released from rest at the midpoint C, $s = 100 \text{ mm}$, and the acceleration is $a = (4s) \text{ m/s}^2$, where s is in meters, determine the velocity of the particle when it reaches plate B, $s = 200 \text{ mm}$, and the time it takes to travel from C to B.



SOLUTION

Coordinate System. As shown in Fig. 12-5, s is positive downward, measured from plate A.

given $v_0 = 0$ $s_0 = 0.1 \text{ m}$

$a = 4s$

req.: $v \longrightarrow$ at $s = 0.2 \text{ m}$

$v(s) = v \frac{dv}{ds}$

Sol

$a = 4s = v \frac{dv}{ds}$

$$\int_0^v v dv = \int_{0.1}^s 4s ds$$

$$\frac{v^2}{2} \Big|_0^v = \frac{4s^2}{2} \Big|_{0.1}^s$$

$$\frac{v^2}{2} = 2s^2 - 2(0.1)^2 \quad \times 2$$

$$\boxed{v^2 = 4s^2 - 0.04}$$

$$V = \sqrt{4s^2 - 0.04} \quad \xrightarrow{\text{at } s=0.2}$$

$$V = \sqrt{4 \times 0.2^2 - 0.04} = 0.346 \text{ m/s} \quad \#$$

$$\Rightarrow t = ?$$

$$V = \frac{ds}{dt} = \sqrt{4s^2 - 0.04}$$

$$\int_0^t dt = \int_{0.1}^s \frac{ds}{\sqrt{4s^2 - 0.04}}$$

↖ 2s ↗ invers fn $\frac{u'}{\sqrt{u^2 - a^2}}$

$$\text{Cosh}^{-1}\left(\frac{u}{a}\right) \leftarrow \int \frac{u'}{\sqrt{u^2 - a^2}}$$

↖ 0.2 ↗

$$t = \text{Cosh}^{-1}\left(\frac{2s}{0.2}\right) \Big|_{0.1}^0.4 \quad \#$$

$$t = \text{Cosh}^{-1}\left(\frac{2 \times 0.4}{0.2}\right) - \text{Cosh}^{-1}(0.1)$$

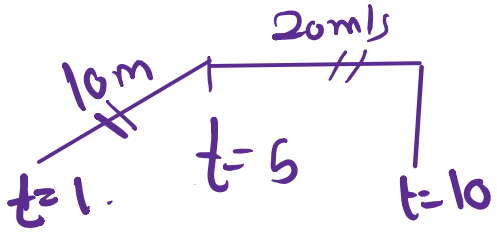
$$t = 0.658 \text{ s} \quad \#$$

12–18. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s) \text{ m/s}^2$, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100 \text{ m}$. Initially, $v = 0$ and $s = 0$ when $t = 0$.



eratic motion

الحركة الغير منتظمة



← حركة الجسم تتغير مع فترات زمنية مختلفة

ندرس حركة الجسم على فترات $[a, v, s]$

خطوات الحل

① نقسم الزمن الى فترات $t=0 \rightarrow t_1$ $0 \leq t \leq t_1$

$a =$

$v =$

$s =$

② $t_1 \leq t \leq t_2$

$a =$

$v =$

$s =$

② نفايه المرحلة الاولى هربداية المرحلة الثانية

لـ شروط ابتداء للناسه

$$v_{t_1} = v_0$$

$$s_{t_1} = s_0$$

③ المطلوب .

$$a = F(t) = F(v) = F(s)$$

$$v = F(t) = F(s)$$

$$s = F(t) = F(v)$$

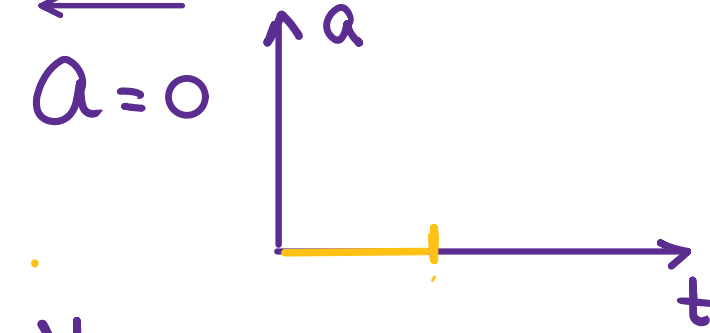
نرسم العلاقة بين متغيرين



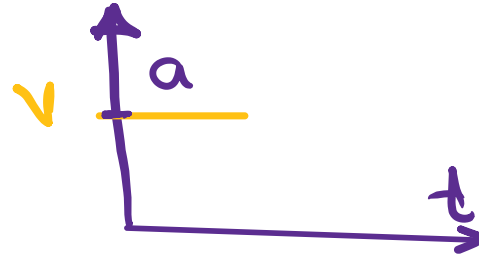
acceleration

Case ①

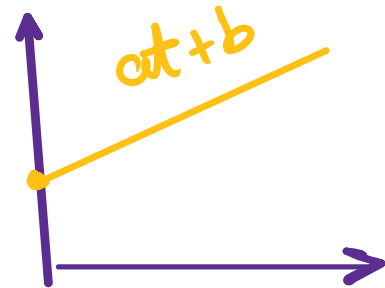
$$a = 0$$



$$v = \text{Const} = a$$

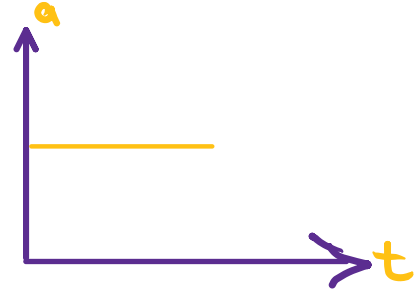


$$s = at + b$$

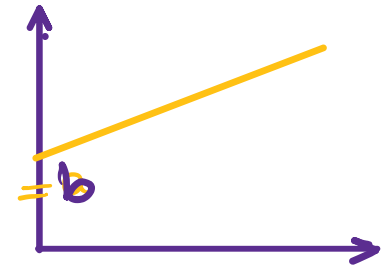


Case 2

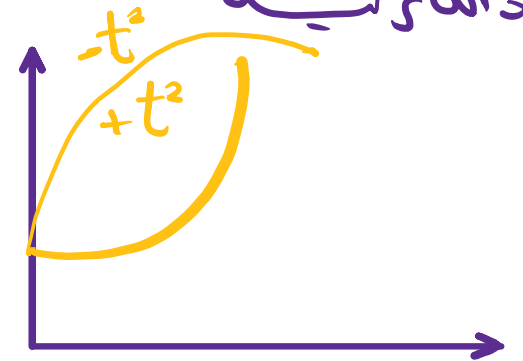
$$a = c$$



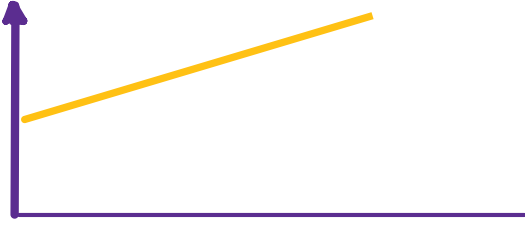
$$v = ct + b$$



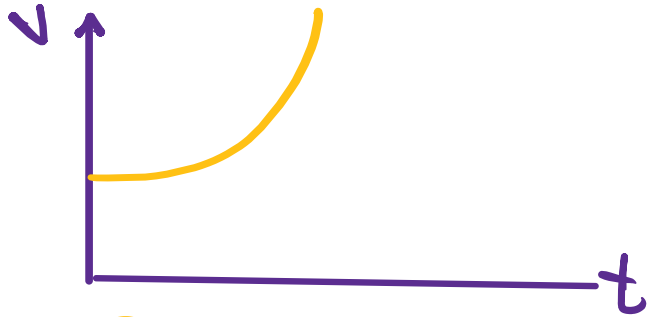
$$s = ct^2 + bt + d$$



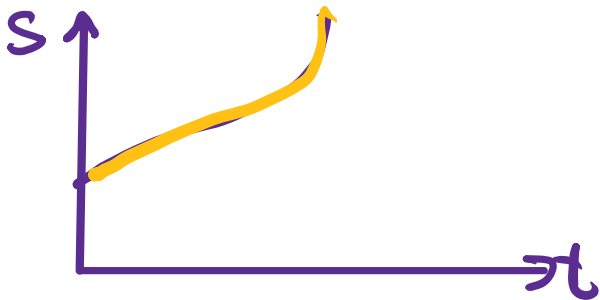
$$a = f(t) = ct + b$$



$$V = ct^2 + bt + d$$



$$S = ct^3 + bt^2 + dt$$



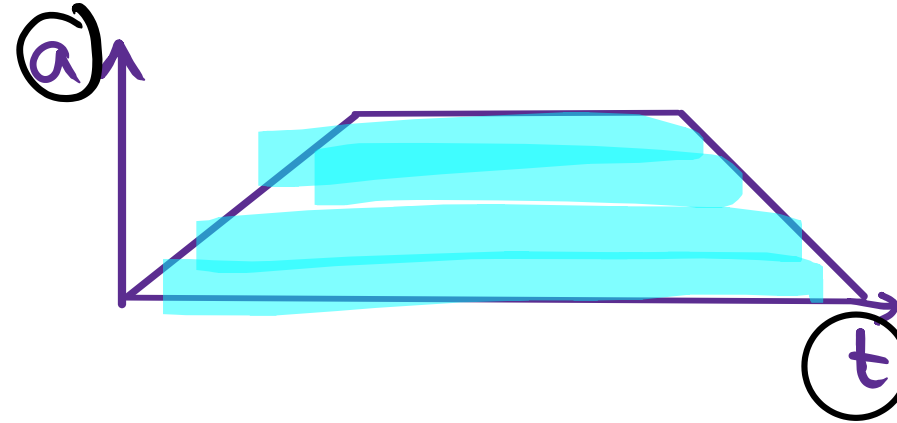
$$a = \frac{dV}{dt}$$

التكامل على مساحة
المنحنى

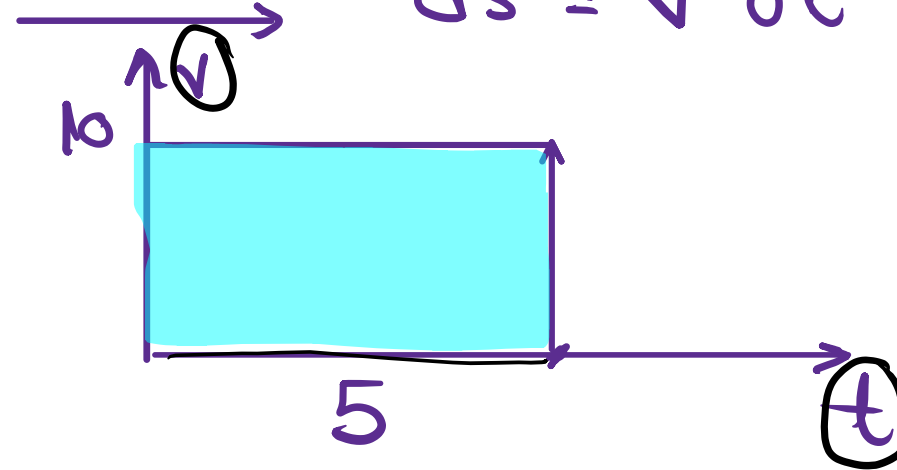
$$V = \frac{dS}{dt}$$

$$S = 5 \times 10 \\ = 50$$

$$dV = a dt$$

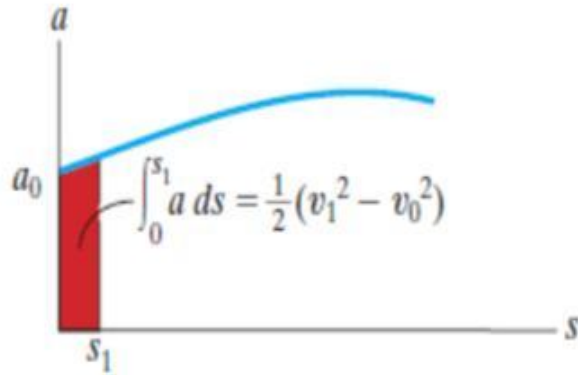


$$dS = V dt$$

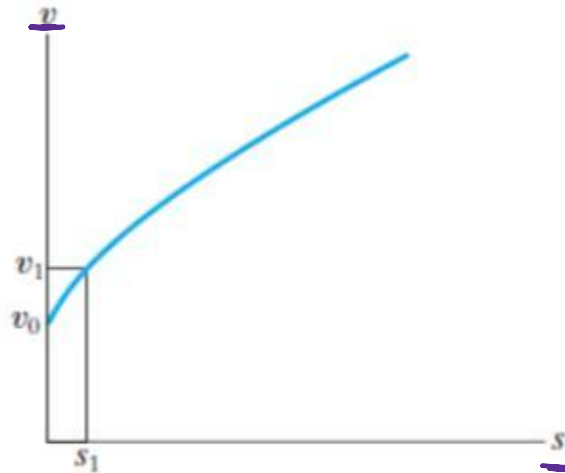


2. How to construct v-s and a-s graphs ?

- If the a-s graph can be constructed, than points on v-s graph can be determined by using:



(a)



(b)

$$\frac{1}{2}(v_1^2 - v_0^2) = \int_{s_0}^{s_1} a ds$$

area under
a-s graph

- If the v-s graph is known, the acceleration a at any position s can be determined using:

$$\Rightarrow a = v \left(\frac{dv}{ds} \right)$$

velocity times
acceleration = slope of
v-s graph

$$V = 2s + 1$$

$$\frac{dv}{ds} = 2$$

$$a = v \frac{dv}{ds}$$

$$= (2s + 1)(2)$$

$$a = v \frac{dv}{ds}$$

$$\int v dv = \int a ds$$

الحل

① $0 \leq t \leq 10$

given $s = t^2$

$$V = \frac{ds}{dt} = 2t$$

$$a = \frac{dV}{dt} = 2$$

② $10 \leq t \leq 30$

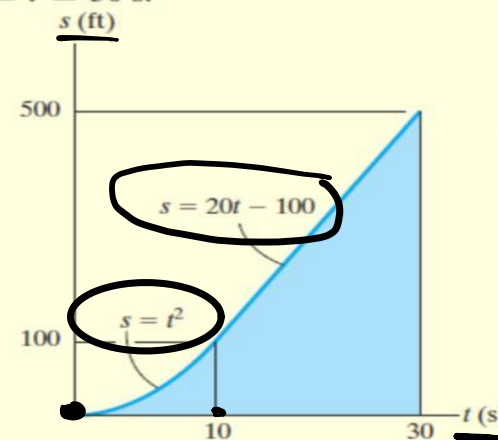
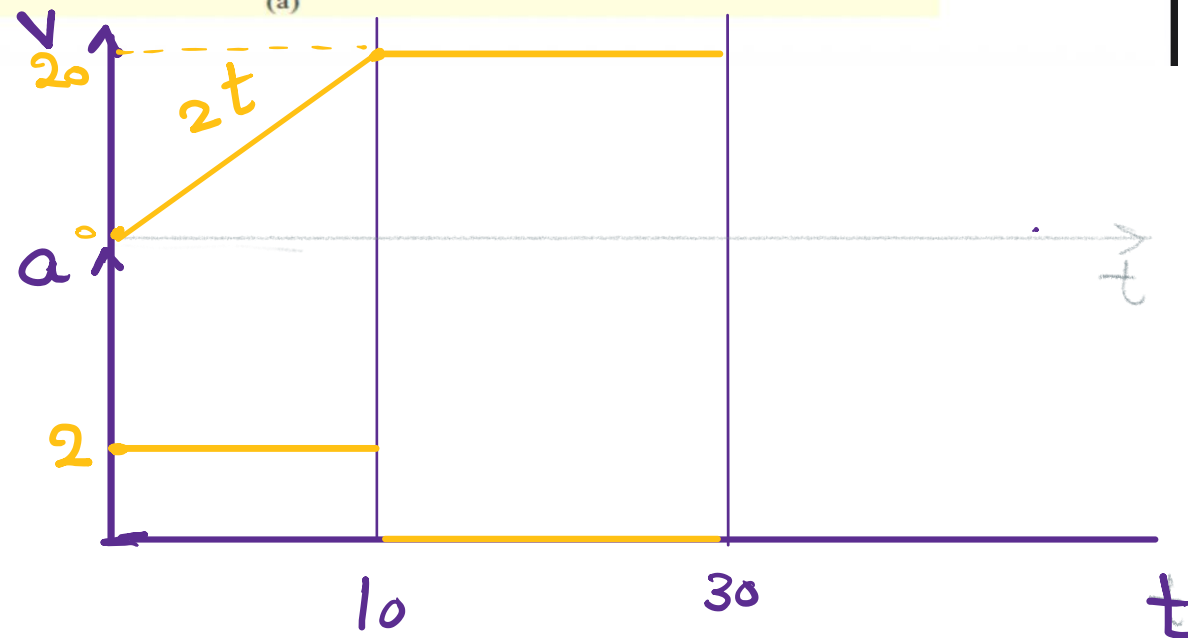
$$s = 20t - 100$$

$$V = \frac{ds}{dt} = 20$$

$$a = 0$$

Example 12.6

A bicycle moves along a straight road such that its position is described by the graph shown in Fig. 12-13a. Construct the $v-t$ and $a-t$ graphs for $0 \leq t \leq 30$ s.



Example 12.8

The $v-s$ graph describing the motion of a motorcycle is shown in Fig. 12-15a. Construct the $a-s$ graph of the motion and determine the time needed for the motorcycle to reach the position $s = 400$ ft.

SOLUTION



$$\Rightarrow 0 \leq s \leq 200$$

Req: $(a-s)$ graph

$$v = 0.2s + 10$$

$$a = v \frac{dv}{ds} = (0.2s + 10) \times 0.2$$

$$a = 0.04s + 2 \quad \text{خط مستقیم}$$

at $s=0$

$a=2$

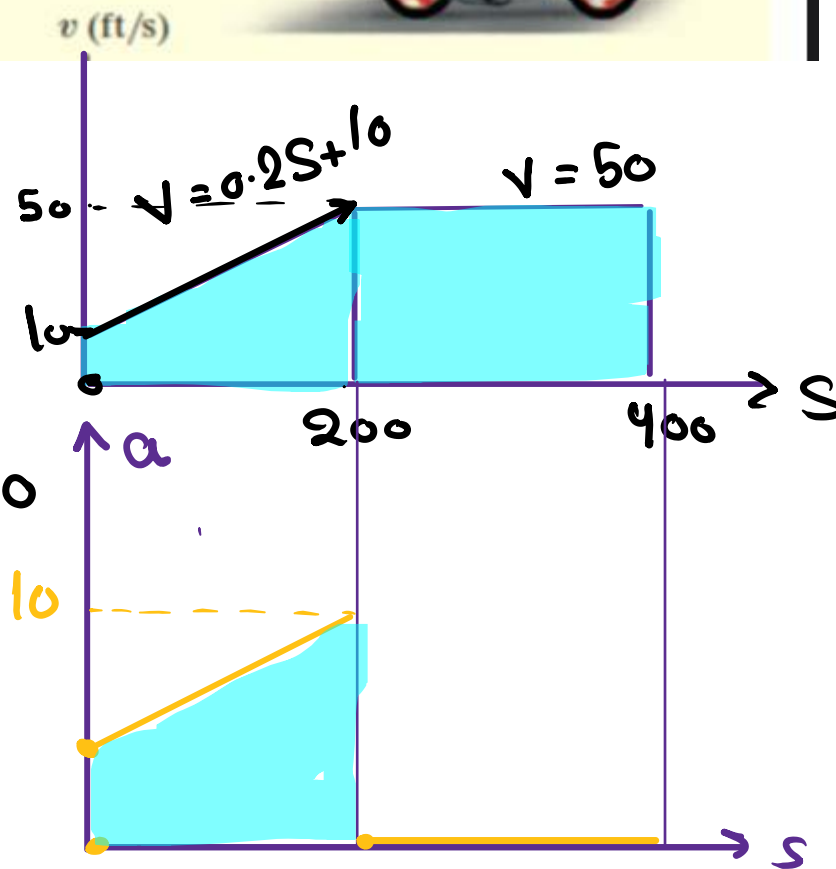
at $s=200$

$a=10$

$$\Rightarrow 200 \leq s \leq 400$$

$$v = 50$$

$$a = v \frac{dv}{ds} = \boxed{0}$$



$$0 \leq s \leq 200$$

$$V = (0.2s + 10) = \frac{ds}{dt}$$

$$\int_0^t dt = \frac{1}{0.2} \int_0^{200} \frac{0.2 ds}{0.2s + 10}$$

$$t_1 = \frac{1}{0.2} \ln|0.2s + 10| \Big|_0^{200}$$

$$t = 8.05 \text{ s}$$

$$200 \leq s \leq 400$$

$$V = 50 \quad \text{السرعة ثابتة}$$

$$S = 400 - 200 = 200$$

$$t_2 = \frac{S}{V} = \frac{200}{50} = 4$$

$$t = t_1 + t_2 = 8.05 + 4 = 12.05 \text{ s}$$

$$\textcircled{1} 0 \leq t \leq 5$$

$$V = 30t = \frac{ds}{dt}$$

$$ds = \int_0^t 30t dt$$

$$s = 15t$$

$$\text{at } t = 5 \quad s = 75 \text{ m}$$

$$\textcircled{2} 5 \leq t \leq 15$$

$$V = -15t + 225 = \frac{ds}{dt}$$

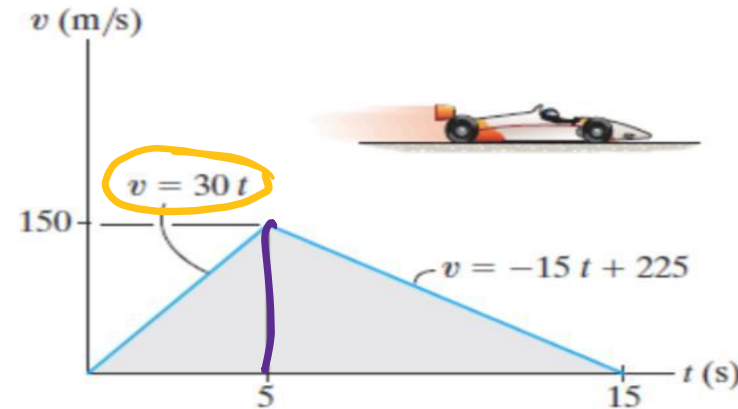
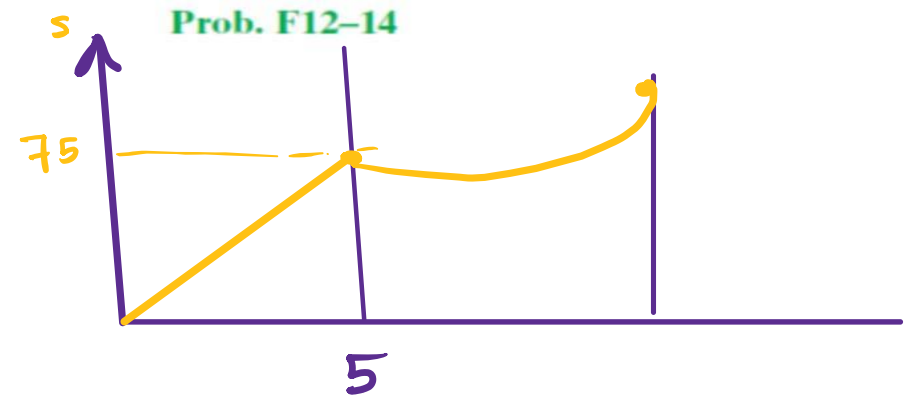
$$\int_{75}^s ds = \int_5^{15} -15t + 225$$

$$s - 75 = -15 \frac{t^2}{2} + 225t \Big|_5^{15}$$

Problem 12-14

$$v_0 = 0$$

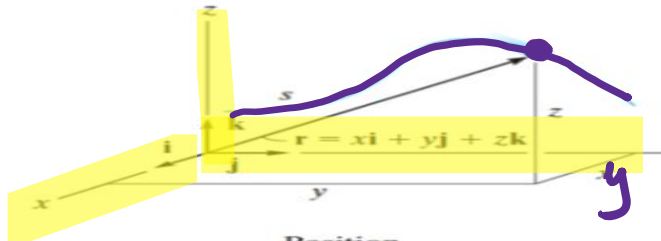
F12-14. The dragster starts from rest and has a velocity described by the graph. Construct the $s-t$ graph during the time interval $0 \leq t \leq 15$ s. Also, determine the total distance traveled during this time interval.



$$S_{\text{Tot}} = \text{الارتفاع} \times \frac{\text{القاعدة}}{2} = \frac{1}{2} \times 15 \times 150 = 1125 \text{ m.}$$

→ Rectangular Components

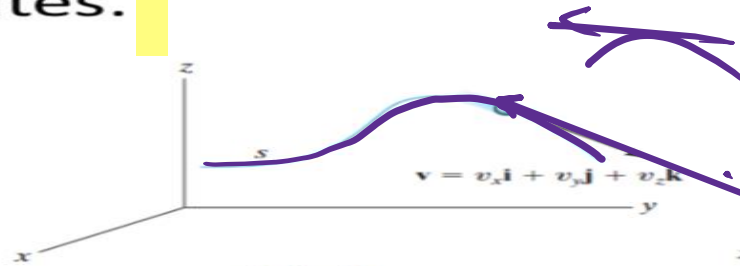
- The motion of a particle along a path that is expressed as x, y, z coordinates.



Position
(a)

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

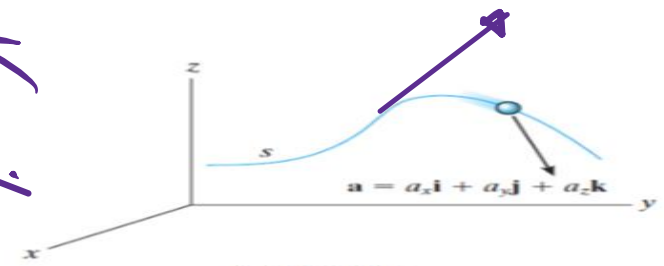
$$r = \sqrt{x^2 + y^2 + z^2}$$



Velocity
(b)

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$



Acceleration
(c)

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$X = F_1(t)$$

$$\mathbf{r} = X(t)\mathbf{i} + Y(t)\mathbf{j} + Z(t)\mathbf{k}$$

$$Y = F_2(t)$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dX}{dt}\mathbf{i} + \frac{dY}{dt}\mathbf{j} + \frac{dZ}{dt}\mathbf{k}$$

$$Z = F_3(t)$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2X}{dt^2}\mathbf{i} + \frac{d^2Y}{dt^2}\mathbf{j} + \frac{d^2Z}{dt^2}\mathbf{k}$$

motion in 2D

$$Z=0$$

$$\vec{r} = x\vec{i} + y\vec{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\vec{i} + v_y\vec{j}$$

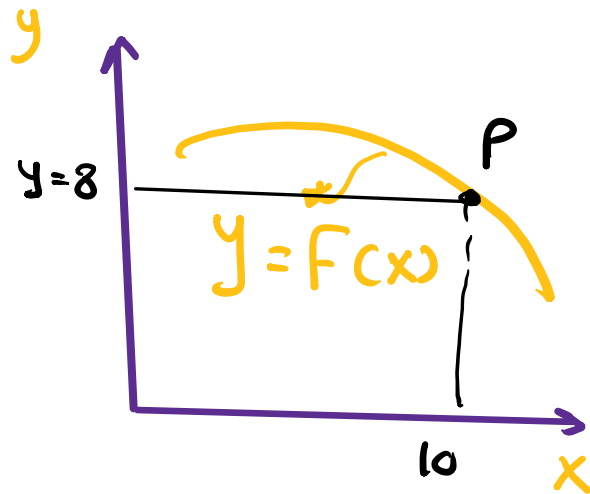
$$\vec{a} = \frac{d\vec{v}}{dt} = a_x\vec{i} + a_y\vec{j}$$

$$y = f(x)$$

Path eqn

معادلة المسار

$$y = x^2 +$$



$\Rightarrow$ given $y = x^2$

at $x = 10$

v_x, v_y
 $\searrow 2\text{ m/s}$

$y = 8$

a_x, a_y
 $\searrow 0$

$y = x^2$ $\xrightarrow[\text{بالنسبة لـ } t]{\text{نفاذ الحل}}$

$$y^{\circ} = 2x x^{\circ}$$

$$v_y = 2x v_x$$

$$2 = 20 v_x$$

محدد مرتبة دالة

$$a_y = 2x \cdot a_x + 2v_x v_x^{\circ}$$

$$a_y = 2x a_x + 2v_x^2$$

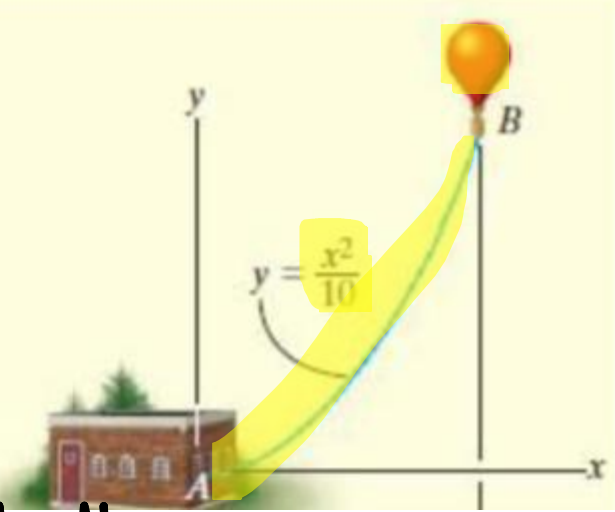
$$0 = 20 a_x + 2 \times 0.1$$

$$a_x = -\frac{92}{20}$$

At any instant the horizontal position of the weather balloon in Fig. 12-18a is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

SOLUTION

Velocity. The velocity component in the x direction is



Given:- $y = \frac{x^2}{10}$, $x = 8t$
 $v_x = \frac{dx}{dt} = 8$
 $a_x = \frac{dv}{dt} = 0$

at $t = 2$

$x = 16$ $v_x = 8$ $a_x = 0$

Req:- a_y , v_y

$V = \sqrt{v_x^2 + v_y^2}$ $a = \sqrt{a_x^2 + a_y^2}$

الحل
 $y = \frac{x^2}{10} \xrightarrow{\frac{d}{dt}}$
 $a_y = \frac{2x}{10} a_x + \frac{2v_x^2}{10}$
 $a_y = \frac{2 \times 16 \times 0}{10} + \frac{2 \times 8^2}{10}$
 $a_y = 12.8 \text{ ft/s}^2$

$v_y = \frac{2x v_x}{10}$
 at $x = 16$, $v_x = 8$
 $v_y = \frac{2 \times 16 \times 8}{10} = 25.6$

$$\vec{V} = v_x \hat{i} + v_y \hat{j} = \underline{8} \hat{i} + \underline{25.6} \hat{j}$$

$$|\vec{V}| = \sqrt{8^2 + 25.6^2} = \underline{26.8} \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{25.6}{8}\right) = \underline{72.6^\circ}$$

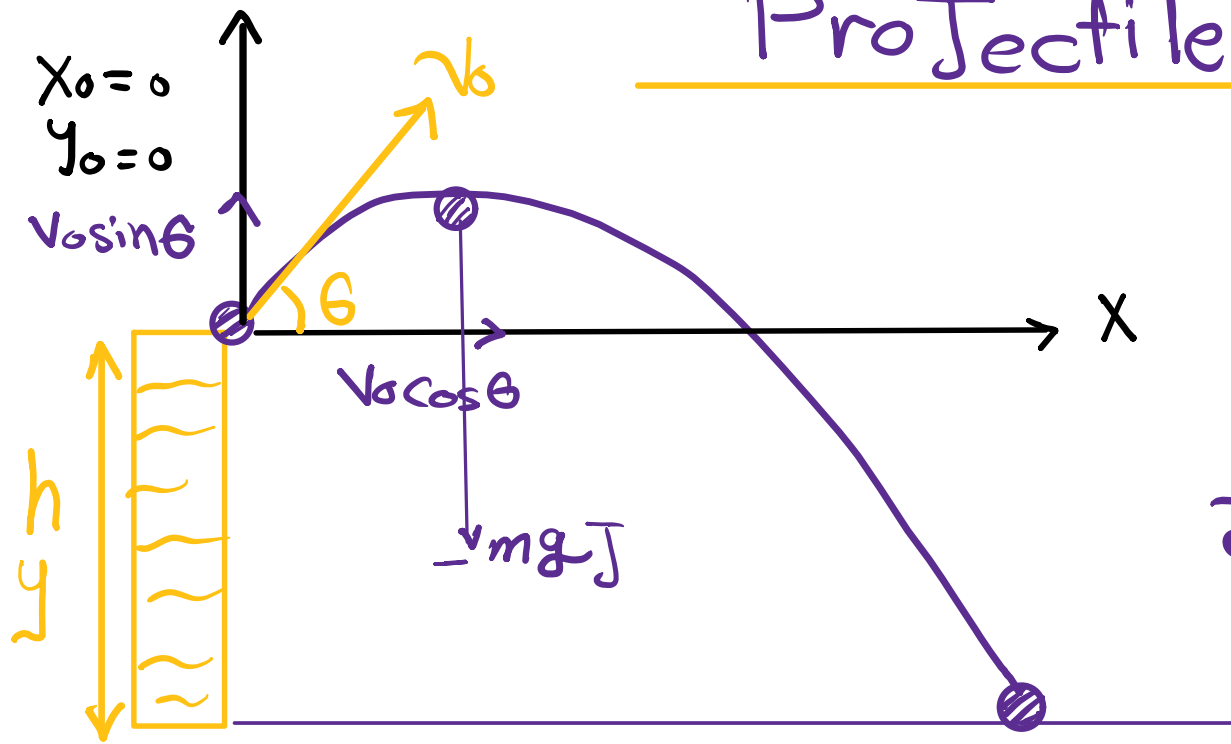
$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$
$$= 0 + 12.8 \hat{j}$$

$$|\vec{a}| = \underline{12.8} \text{ m/s}^2 \quad \uparrow$$

$$\theta = \underline{90^\circ}$$

Projectile

المقذوفات



حركة هم في المستوى xy تحت
تأثير وزن mg .

يتحرك بحجم ثابتة عجلة الجاذبية

$$g = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2$$

$$\vec{r} = x \vec{i} + y \vec{j}$$

$$\vec{v} = v_x \vec{i} + v_y \vec{j}$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j}$$

$\downarrow \quad \downarrow$
 $0 \quad -g_j$

$$\Sigma F = mg$$

قانون نيوتن الثاني

$$\Sigma F = ma$$

$$-mg_j = ma$$

$$a_y = -g_j$$

المقذوف يتحرك بحوله ثابتة
سرعة الابتدائية

$$a_x = 0 \quad a_y = -g$$

$$v_x = v_0 \cos \theta \quad \longrightarrow \quad x_0 = 0$$

$$v_{0y} = v_0 \sin \theta \quad y_0 = 0$$

in x direction

$$1 \quad v = v_0 \cos \theta + 0$$

$$2 \quad v^2 = (v_{0x})^2 + 2 a_x X = (v_0 \cos \theta)^2$$

$$3 \quad X = v_0 t + \frac{1}{2} a t^2$$

$$X = v_0 \cos \theta t$$

$$t = \frac{X}{v_0 \cos \theta}$$

→ نقدم قوانين نيوتن

⇒ in y direction

$$v = v_0 + at$$

$$v_y = v_0 \sin \alpha - g t \quad \longrightarrow \textcircled{1}$$

$$v_y^2 = (v_0 \sin \alpha)^2 - 2 g y \quad \longrightarrow \textcircled{2}$$

$$y = v_0 t + \frac{1}{2} a t^2$$

$$y = v_0 \sin \alpha t - \frac{g}{2} t^2 \quad \longrightarrow \textcircled{3}$$

A sack slides off the ramp, shown in Fig. 12-21, with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range R where sacks begin to pile up.

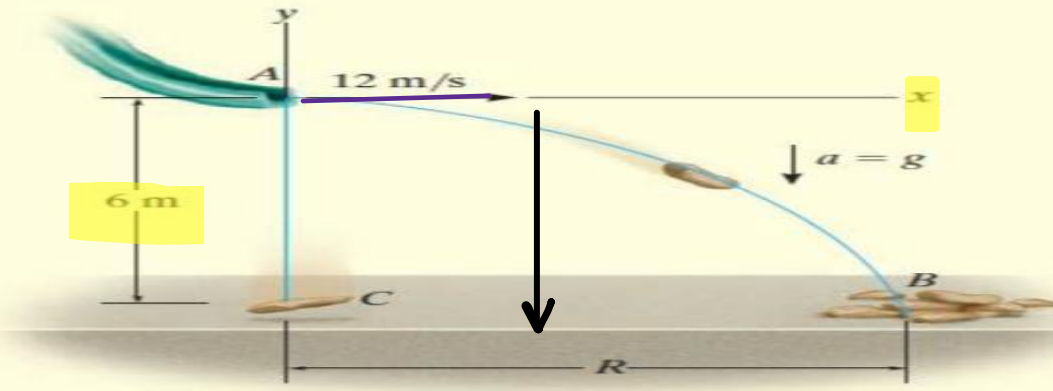


Fig. 12-21

$$V_0 = 12 \text{ m/s}$$

$$\text{horizontal}$$

$$\theta = 0$$

$$V_{0y} = V_0 \sin \theta = 0$$

$$V_{0x} = V_0 \cos \theta = V_0 = 12$$

$$\text{Req: } t_B$$

$$X_B \leftarrow R_B$$

$\Rightarrow$ in y direction

at B $y = -6 \text{ m}, V_{0y} = 0$

$$a_y = -g = -9.81$$

$$\Rightarrow y = V_{0y} t - \frac{1}{2} g t^2$$

$$+6 = \frac{1}{2} \times 9.8 t^2 \Rightarrow t = \sqrt{\frac{6 \times 2}{9.8}} = 1.1 \text{ s}$$

in x-direction

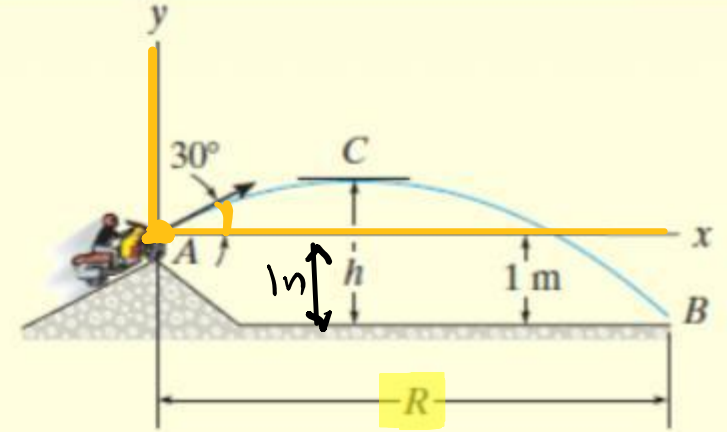
$$X = V_0 t \cos \theta = 12 t_B$$

$$X_B = 12 \times 1.1 = 13.3 \text{ m} = R_B$$

SOLUTION

Coordinate System. As shown in Fig. 12-23b, the origin of the coordinates is established at A. Between the end points of the path AB the three unknowns are the initial speed v_A , range R , and the vertical component of velocity $(v_B)_y$.

Vertical Motion. Since the time of flight and the vertical distance between the ends of the path are known, we can determine v_A .



given $t = 1.5s$

$\Rightarrow$ given $\theta = 30^\circ$
 $v_0 \leftarrow v_A = ?$

$R = ?$

$v_{By} = ?$
 $h_y = 1m$

$a_y = -9.8m/s^2$
 $y_B = -1m$

in y direction

$$y_B = v_{0y} t_B + \frac{1}{2} a t_B^2$$

$$-1 = v_0 \sin 30^\circ \times 1.5 - \frac{9.81}{2} \times 1.5^2$$

$v_0 = 13.4 m/s$

in x direction

$$X = R = v_0 \cos \theta \cdot t$$

$$R = 13.4 \cos 30^\circ \times 1.5 = 17.4 m$$

$$v_{By} = v_0 \sin \alpha - g t$$

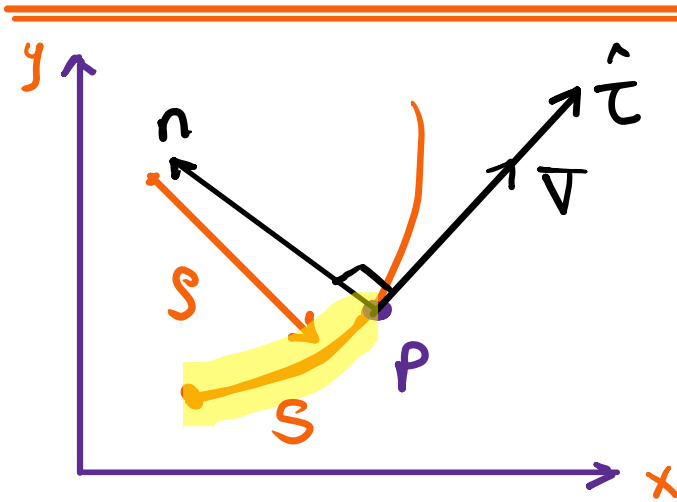
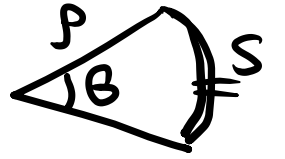
$$= 13.4 \times \sin 30^\circ - 9.8 \times 1.5$$

$v_{By} = 8.02 m/s$

~~✗~~

* Normal & Tangent Component.

المماس والعمودي



سرعة حركته

$\hat{T}$ = Tangent unit vector متجه وحدة مماس

$\hat{n}$ = Normal unit vector متجه وحدة عمودي

ρ = Radius of curvature نصف قطر الانحناء

s = Curved distance حول القوس .

$$① s = \int \theta = f(t)$$

$$② \vec{v} = \frac{ds}{dt} \hat{T} = |\vec{v}| \hat{T}$$

$$③ \vec{a} = a_t \hat{T} + a_n \hat{n}$$

$$- a_t = \frac{dv}{dt} = v \frac{dv}{ds} = \text{أي عجلته في السوال}$$

$$- a_n = \frac{v^2}{\rho}$$

$$④ \rho = \frac{[1 + y'^2]^{3/2}}{y''}$$

لومعة $y = f(x)$ عند $x=0$
 y''

given at A.

$$v_A = 6 \text{ m/s}$$

$$a_t = 2 \text{ m/s}^2$$

$$y = \frac{1}{20} x^2$$

$$\Rightarrow x_A = 10 \quad y = 5$$

Req:- θ_v , a , θ_a

الحل

$$y = \frac{x^2}{20}$$
$$y' = \frac{2x}{20} = \frac{x}{10}$$
$$y'' = \frac{1}{10}$$

at A

$$y = \frac{100}{20} = 5$$
$$y' = \frac{10}{10} = 1$$
$$y'' = \frac{1}{10} = 0.1$$

When the skier reaches point A along the parabolic path in Fig. 12-27a, he has a speed of 6 m/s which is increasing at 2 m/s². Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.

SOLUTION

Coordinate System. Although the path has been expressed in terms of its x and y coordinates, we can still establish the origin of the n, t axes at the fixed point A on the path and determine the components of v and a along these axes, Fig. 12-27a.

Velocity. By definition, the velocity is always directed tangent to the path. Since $y = \frac{1}{20}x^2$, $dy/dx = \frac{1}{10}x$, then at $x = 10 \text{ m}$, $dy/dx = 1$. Hence, at A, v makes an angle of $\theta = \tan^{-1}1 = 45^\circ$ with the x axis, Fig. 12-27b. Therefore,

$$v_A = 6 \text{ m/s} \quad 45^\circ \nearrow$$

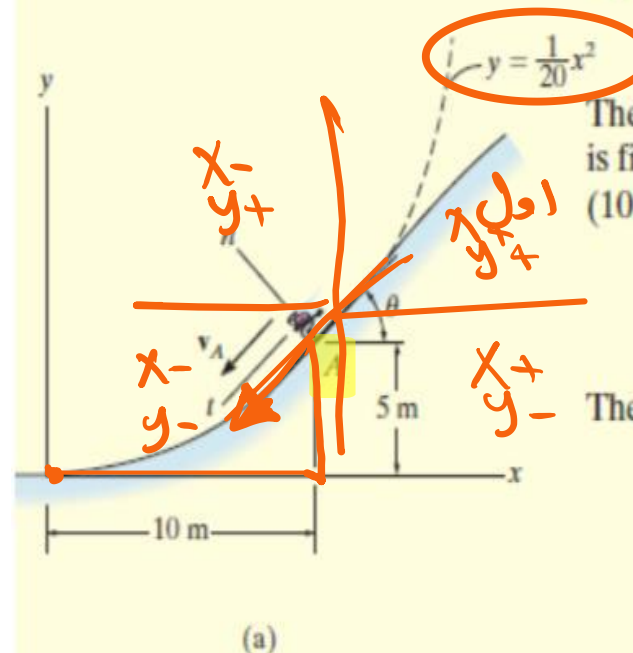
Ans.

The acceleration is determined from $a = \dot{v}u_t + (v^2/\rho)u_n$. However, it is first necessary to determine the radius of curvature of the path at A (10 m, 5 m). Since $d^2y/dx^2 = \frac{1}{10}$, then

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \frac{[1 + (\frac{1}{10}x)^2]^{3/2}}{|\frac{1}{10}|} \bigg|_{x=10 \text{ m}} = 28.28 \text{ m}$$

The acceleration becomes

$$\begin{aligned} a_A &= \dot{v}u_t + \frac{v^2}{\rho}u_n \\ &= 2u_t + \frac{(6 \text{ m/s})^2}{28.28 \text{ m}}u_n \\ &= \{2u_t + 1.273u_n\} \text{ m/s}^2 \end{aligned}$$



$$\rho = \frac{[1 + y'^2]^{3/2}}{y''} = \frac{[1 + 1^2]^{3/2}}{0.1}$$

$$\rho = 28.28 \text{ m}$$

$$\begin{aligned} \Rightarrow \vec{a} &= a_t \hat{t} + \frac{v^2}{\rho} \hat{n} \\ &= 2 \tau + \frac{6^2}{28.28} \hat{n} \end{aligned}$$

$$\vec{a} = 2 \hat{t} + 1.273 \hat{n}$$

$$|a| = \sqrt{2^2 + 1.27^2} = 2.37 \text{ m/s}^2$$

$$\theta_a = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{1.27}{2}\right) = 57.5^\circ$$

زاوية مع $\hat{t}$

$\Rightarrow$ direction of velocity

$$\text{slope of T.L} = \frac{dy}{dx} \Big|_{x_A} = \tan \theta$$

$$y' = \tan \theta \longrightarrow A$$

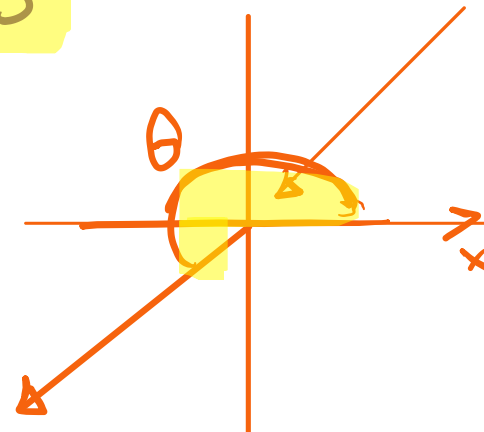
$$\frac{x}{10} = \tan \theta \longrightarrow x_A = 10$$

$$\tan \theta = \frac{10}{10} = 1$$

$$\theta = \tan^{-1}(1) = 45^\circ$$

$$\begin{aligned} \theta_v &= 180^\circ + \theta \\ &= 225^\circ \end{aligned}$$

مع $\hat{t}$



Given $v_0 = 0$ $s_A = 0$

$a_t = 0.2t$

Req $\vec{a} = 0.2t \tau + \frac{v^2}{\rho} \hat{n}$

الحل

$\rho = 2 \text{ m}$

$\theta = \frac{\pi}{2}$

$s_{AB} = s_{AC} + s_{CB}$
 $= 3 + 2 * \frac{\pi}{2} = \boxed{6.14 \text{ m}}$

$v = \frac{ds}{dt}$

$a_t = \frac{dv}{dt}$

من المبدأ

Example 12.16



(a)

The boxes in Fig. 12-29a travel along the industrial conveyor. If a box as in Fig. 12-29b starts from rest at A and increases its speed such that $a_t = (0.2t) \text{ m/s}^2$, where t is in seconds, determine the magnitude of its acceleration when it arrives at point B.

SOLUTION

Coordinate System. The position of the box at any instant is defined from the fixed point A using the position or path coordinate s , Fig. 12-29b. The acceleration is to be determined at B, so the origin of the n, t axes is at this point.

Acceleration. To determine the acceleration components $a_t = \dot{v}$ and $a_n = v^2/\rho$, it is first necessary to formulate v and $\dot{v}$ so that they may be evaluated at B. Since $v_A = 0$ when $t = 0$, then

$$a_t = \dot{v} = 0.2t \quad (1)$$

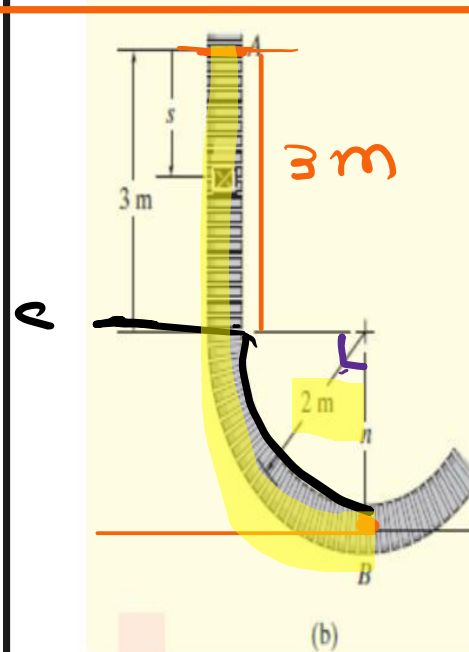
$$\int_0^v dv = \int_0^t 0.2t \, dt$$

$$v = 0.1t^2 \quad (2)$$

The time needed for the box to reach point B can be determined by realizing that the position of B is $s_B = 3 + 2\pi(2)/4 = 6.142 \text{ m}$, Fig. 12-29b, and since $s_A = 0$ when $t = 0$ we have

$$v = \frac{ds}{dt} = 0.1t^2$$

$$\int_0^{6.142 \text{ m}} ds = \int_0^{t_B} 0.1t^2 \, dt$$



(b)

$$a_t = \frac{dv}{dt} = 0.2t$$

$$\int_0^v dv = \int_0^t 0.2t dt$$

$$v = 0.1t^2$$

السرعة عند أي نقطة

$$v = \frac{ds}{dt} = 0.1t^2$$

$$\int_0^s ds = \int_0^t 0.1t^2 dt$$

$$s = 0.1 \frac{t^3}{3}$$

عند أي نقطة

$$\Rightarrow \underline{at_B} \quad J_B = \frac{0.1t_B^3}{3} \Rightarrow 6.14$$

$$\Rightarrow t^3 = \frac{3 \times 6.14}{0.1}$$

$$t_B = \sqrt[3]{\frac{3 \times 6.14}{0.1}} = 5.69 \text{ s}$$

$$v_B = 0.1t_B^2 = 0.1[5.69]^2 = 3.23 \frac{m}{s}$$

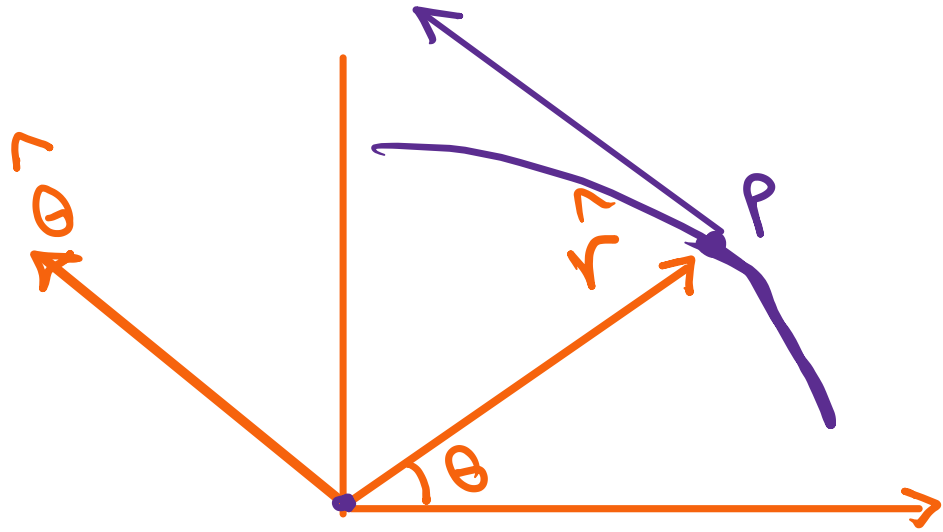
$$a_t = 0.2t = 0.2 \times 5.69 = 1.138$$

$$a_n = \frac{v^2}{r} = \frac{3.23^2}{2} = 5.24 \text{ m/s}^2$$

$$\vec{a} = 1.138 \hat{t} + 5.24 \hat{n}$$

$$|a| = \sqrt{(1.138)^2 + 5.24^2} = 5.36 \text{ m/s}^2$$

Cylindrical Component



⇒ Position $\vec{r}$ → radial distance
 $\hat{\theta}$ → Transverse $\vec{\theta}$ →

$$\vec{v} = \dot{r} \hat{r} + r \dot{\theta} \hat{\theta}$$

$$\vec{a} = (\ddot{r} - r \dot{\theta}^2) \hat{r} + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \hat{\theta}$$

→ حفظ

→ Velocity
 $\dot{r} = \frac{dr}{dt}$
 $\ddot{r} = \frac{d^2r}{dt^2}$

$\vec{r}$ = radial
 $r = f(t)$
 $\theta = g(t)$
 $\dot{\theta} = \frac{d\theta}{dt}$
 $\ddot{\theta} = \frac{d^2\theta}{dt^2}$

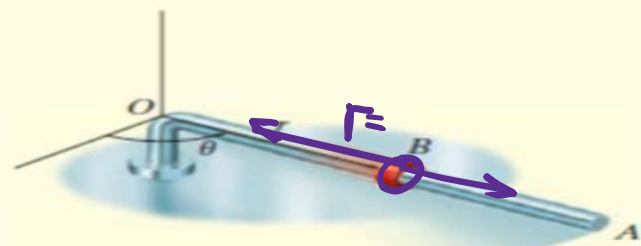
$\vec{v}_r = \dot{r}$
 $a_r = \ddot{r} - r \dot{\theta}^2$

$\vec{v}_\theta = r \dot{\theta}$
 $a_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta}$

$$\begin{matrix} r \\ \dot{r} \\ \ddot{r} \\ \dot{\theta} \\ \ddot{\theta} \end{matrix}$$

$t = 0$

$$\begin{matrix} r = 0 \\ \dot{r} = 0 \\ \ddot{r} = 0 \\ \dot{\theta} = 0 \\ \ddot{\theta} = 0 \end{matrix}$$



The rod OA in Fig. 12-33a rotates in the horizontal plane such that $\theta = (t^3)$ rad. At the same time, the collar B is sliding outward along OA so that $r = (100t^2)$ mm. If in both cases t is in seconds, determine the velocity and acceleration of the collar when $t = 1$ s.

SOLUTION

Coordinate System. Since time-parametric equations of the path are given, it is not necessary to relate r to θ .

Velocity and Acceleration. Determining the time derivatives and evaluating them when $t = 1$ s, we have

$$\Rightarrow r = 100t^2$$

$$\theta = t^3$$

$$\Rightarrow \text{req.}:- v, a \longrightarrow t = 1s$$

$$r = 100t^2$$

$$r' = 200t$$

$$r'' = 200$$

$$\theta = t^3$$

$$\theta' = 3t^2$$

$$\theta'' = 6t$$

$$t = 1s \longrightarrow$$

$$r = 100$$

$$r' = 200$$

$$r'' = 200$$

$$\theta' = 3$$

$$\theta'' = 6$$

$$\vec{v} = r' \hat{r} + r\theta' \hat{\theta}$$

$$= 200 \hat{r} + 100 \times 3 \hat{\theta} = 200 \hat{r} + 300 \hat{\theta}$$

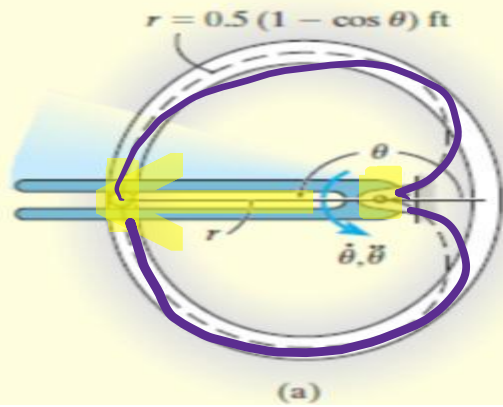
$$|\vec{v}| = \sqrt{200^2 + 300^2} = 361 \text{ mm/s}$$

$$\Rightarrow \vec{a} = (r'' - r\theta'^2) \hat{r} + (r\theta'' + 2r'\theta') \hat{\theta}$$

$$= (200 - 900) \hat{r} + (100 \times 6 + 2 \times 200 \times 3) \hat{\theta}$$

$$= -700 \hat{r} + 1800 \hat{\theta}$$

$$|\vec{a}| = \sqrt{(-700)^2 + 1800^2} = 1931 \text{ mm/s}^2$$



Due to the rotation of the forked rod, the ball in Fig. 12-35a travels around the slotted path, a portion of which is in the shape of a cardioid, $r = 0.5(1 - \cos \theta)$ ft, where θ is in radians. If the ball's velocity is $v = 4$ ft/s and its acceleration is $a = 30$ ft/s² at the instant $\theta = 180^\circ$, determine the angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$ of the fork.

SOLUTION

Coordinate System. This path is most unusual, and mathematically it is best expressed using polar coordinates, as done here, rather than rectangular coordinates. Also, since $\dot{\theta}$ and $\ddot{\theta}$ must be determined, then r, θ coordinates are an obvious choice.

$$\Rightarrow r = 0.5(1 - \cos \theta)$$

$$r' = 0.5(\sin \theta) \dot{\theta}$$

$$r'' = 0.5 \sin \theta \ddot{\theta} + 0.5 \cos \theta (\dot{\theta})^2$$

$$\text{at } \theta = 180^\circ \quad \begin{aligned} \sin 180^\circ &= 0 \\ \cos 180^\circ &= -1 \end{aligned}$$

$$r = 0.5(1 - \cos 180^\circ) = 0.5 \times 2 = 1$$

$$r' = 0$$

$$\begin{aligned} r'' &= 0.5(0) \ddot{\theta} + 0.5 \cos 180^\circ \dot{\theta}^2 \\ r'' &= -\frac{1}{2} \dot{\theta}^2 \end{aligned}$$

$$v = 4 = \sqrt{r'^2 + (r\dot{\theta})^2} = \sqrt{0 + (1 \times \dot{\theta})^2}$$

$$\boxed{4 = \dot{\theta}} \quad \#$$

$$\rightarrow r'' = -\frac{1}{2} \times 4^2 = \boxed{-8}$$

$$\begin{aligned}\bar{a} &= (r^{\circ\circ} - r\theta^{\circ 2})\hat{r} + (r\theta^{\circ\circ} + 2r^{\circ}\theta^{\circ})\hat{\theta} \\ &= (-8 - 1 \times 16)\hat{r} + (\theta^{\circ\circ} + 0)\hat{\theta} = -24\hat{r} + \theta^{\circ\circ}\hat{\theta}\end{aligned}$$

$$|a| = 30 = \sqrt{(-24)^2 + \theta^{\circ\circ 2}}$$

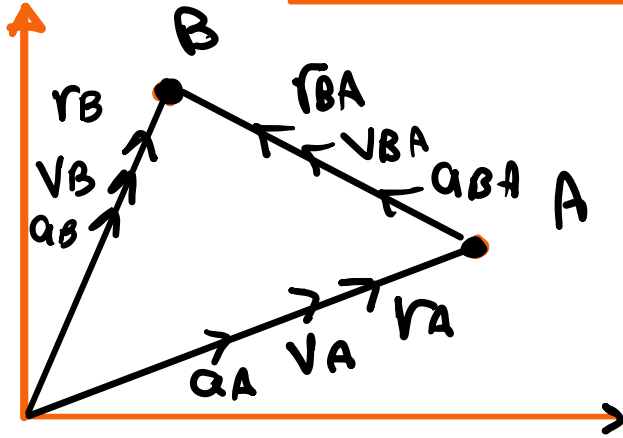
نُذِج العَرَضِين

$$30^2 = (-24)^2 + \theta^{\circ\circ 2}$$

$$\theta^{\circ\circ 2} = 30^2 - (-24)^2$$

$$\theta^{\circ\circ} = \sqrt{30^2 - (-24)^2} = \boxed{18 \text{ rad/s}^2} //$$

Relative Motion.



$$\Rightarrow \vec{r}_{BA} = \vec{r}_B - \vec{r}_A = -\vec{r}_{AB}$$

$$\Rightarrow \vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -\vec{v}_{AB}$$

$$\Rightarrow \vec{a}_{BA} = \vec{a}_B - \vec{a}_A = -\vec{a}_{AB}$$

حركة جسم بالنسبة لجسم آخر

عكس الاتجاهات

$$\Rightarrow (X, Y) \longrightarrow \begin{matrix} v_A = \dot{x} \\ a_A = \ddot{x} \end{matrix} \quad \begin{matrix} j \\ j \end{matrix}$$

نحلا

$$\Rightarrow (T, n) \longrightarrow \begin{matrix} \vec{v}_A = v \hat{T} \\ \vec{a} = a_t \hat{T} + a_n \hat{n} \end{matrix}$$

يتم التحرك في (X, Y) بشكل

لوائجهم يدور في مسار منحنى

(X, Y) نحو لها الى (T, n)

خطوات الحل

نذكر كل شيء على حدى

$$\begin{aligned} \overline{V_B} &= \dot{i} + J \\ \overline{Q_B} &= \dot{i} + J \end{aligned} \quad \uparrow \uparrow \quad \begin{aligned} \overline{V_A} &= \dot{i} + J \\ \overline{Q_A} &= \dot{i} + J \end{aligned} \quad (1)$$

$$\Rightarrow \overline{V_{BA}} = \overline{V_B} - \overline{V_A} = \square \dot{i} + \square J$$

$$|V_{BA}| = \sqrt{(\quad)^2 + (\quad)^2}$$

$$\theta_{V_{BA}} = \tan^{-1}\left(\frac{J}{\dot{i}}\right) =$$

$$\overrightarrow{Q_{BA}} = \overline{Q_B} - \overline{Q_A} = \square \dot{i} + \square J$$

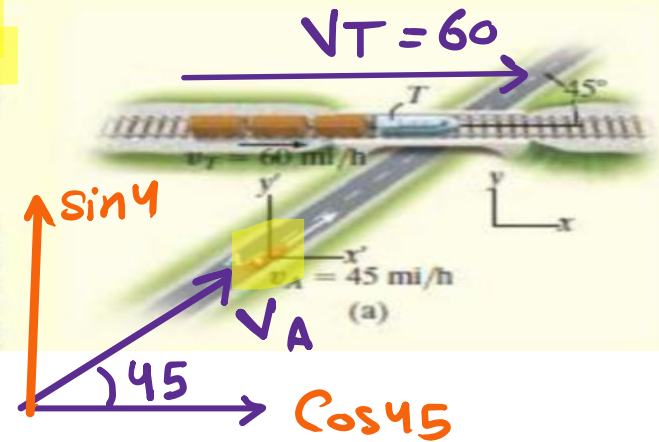
$$|Q_{BA}| = \sqrt{(\quad)^2 + (\quad)^2}$$

$$\theta = \tan^{-1}(\quad)$$

A train travels at a constant speed of 60 mi/h and crosses over a road as shown in Fig. 12-43a. If the automobile A is traveling at 45 mi/h along the road, determine the magnitude and direction of the velocity of the train relative to the automobile.

SOLUTION I

Vector Analysis. The relative velocity $\mathbf{v}_{T/A}$ is measured from the translating x' , y' axes attached to the automobile, Fig. 12-43a. It is determined from $\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$. Since $\mathbf{v}_T$ and $\mathbf{v}_A$ are known in both magnitude and direction, the unknowns become the x and y components



$$\Rightarrow V_T = 60 \text{ mi/h}$$

$$\overline{V}_T = 60 \hat{i}$$

$$\Rightarrow V_A = 45 \text{ mi/h}$$

$$\overline{V}_A = 45 \cos 45 \hat{i} + 45 \sin 45 \hat{j}$$

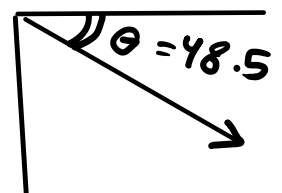
$$\Rightarrow = 31.8 \hat{i} + 31.8 \hat{j}$$

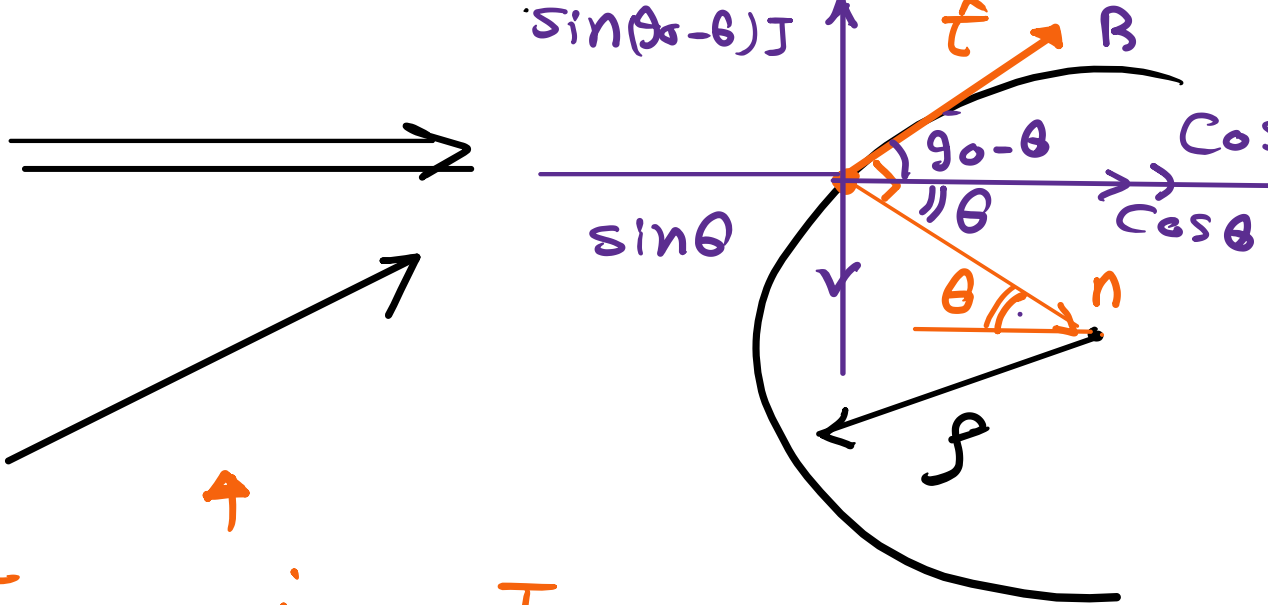
$$\begin{aligned} \overline{V}_{TA} &= \overline{V}_T - \overline{V}_A \\ &= 60 \hat{i} - 31.8 \hat{i} - 31.8 \hat{j} \end{aligned}$$

$$\overline{V}_{TA} = +28.2 \hat{i} - 31.8 \hat{j} \quad \text{or}$$

$$|V_{TA}| = \sqrt{(28.2)^2 + (31.8)^2} = 25.5 \text{ mi/h}$$

$$\theta = \tan^{-1} \left(\frac{31.8}{28.2} \right) = 48.5^\circ$$





$(i, j) \leftarrow (T, n)$ جَوَاز

$\vec{V} =$
 $a = \frac{i + j}{i + j}$

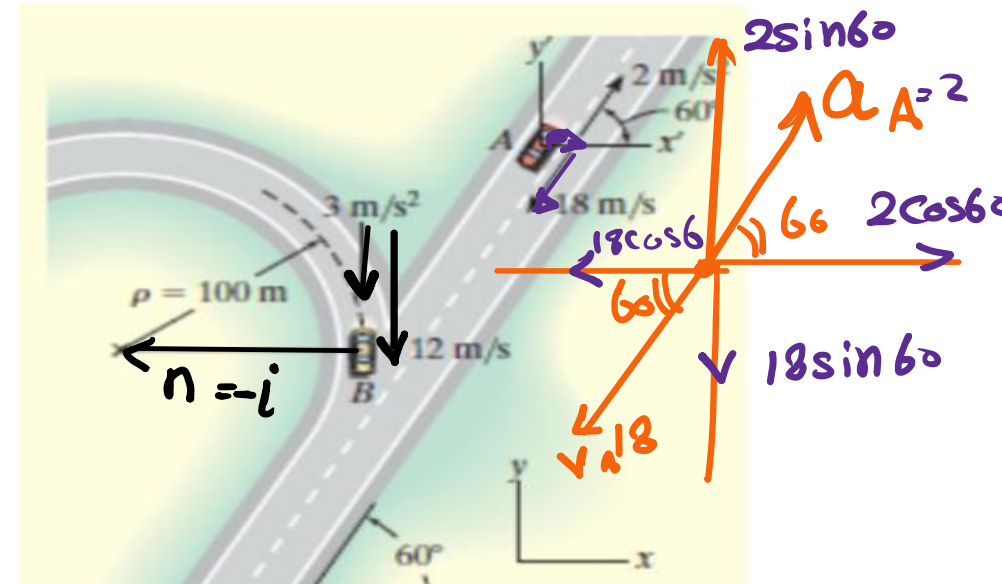
$$\hat{T} = \cos(90 - \theta) \hat{i} + \sin(90 - \theta) \hat{j}$$

$$\hat{N} = \cos \theta \hat{i} - \sin \theta \hat{j}$$

$$\Rightarrow V_B = V \hat{T}$$

$$\vec{a} = a_t \hat{T} + \frac{V^2}{\rho} \hat{N}$$

At the instant shown in Fig. 12-45a, cars A and B are traveling with speeds of 18 m/s and 12 m/s, respectively. Also at this instant, A has a decrease in speed of 2 m/s², and B has an increase in speed of 3 m/s². Determine the velocity and acceleration of B with respect to A.



$$\Rightarrow \vec{V}_A = -18 \cos 60 \hat{i} - 18 \sin 60 \hat{j}$$

$$\vec{V}_A = -9 \hat{i} - 9\sqrt{3} \hat{j}$$

$$\vec{a}_A = 2 \cos 60 \hat{i} + 2 \sin 60 \hat{j}$$

$$\vec{a}_A = \hat{i} + \sqrt{3} \hat{j}$$

$$\begin{aligned} V_B &= 12 \text{ m/s} \\ \vec{a}_B &= a_B \hat{\tau} + a_{Bn} \hat{n} \\ &= -3 \hat{j} + \frac{12^2}{100} (-\hat{i}) \end{aligned}$$

$$\vec{a}_B = -1.44 \hat{i} - 3 \hat{j}$$

$$\Rightarrow V_B = 12 \hat{\tau} \quad \text{سرعت}$$

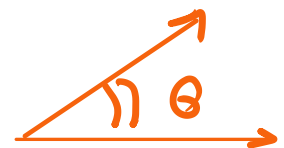
$$\tau = -\hat{j}$$

$$\therefore \vec{V}_B = -12 \hat{j}$$

$$\begin{aligned} \vec{V}_{BA} &= \vec{V}_B - \vec{V}_A = -12 \hat{j} - (-9 \hat{i} - 9\sqrt{3} \hat{j}) \\ &= 9 \hat{i} + 3.588 \hat{j} \end{aligned}$$

$$|V_{BA}| = \sqrt{9^2 + (3.588)^2} = 9.69 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{3.588}{9}\right) = 21.7^\circ$$

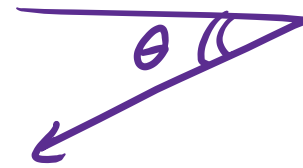


$$\overline{a_{BA}} = a_B - a_A$$

$$= -1.44 \hat{i} - 3 \hat{j} - \hat{i} - \sqrt{3} \hat{j} = -2.44 \hat{i} - 4.73 \hat{j}$$

$$|\overline{a_{BA}}| = \sqrt{(-2.44)^2 + (-4.73)^2} = 5.32 \text{ m/s}^2$$

$$\theta = \tan^{-1} \left(\frac{4.73}{2.44} \right) = 62.7^\circ$$

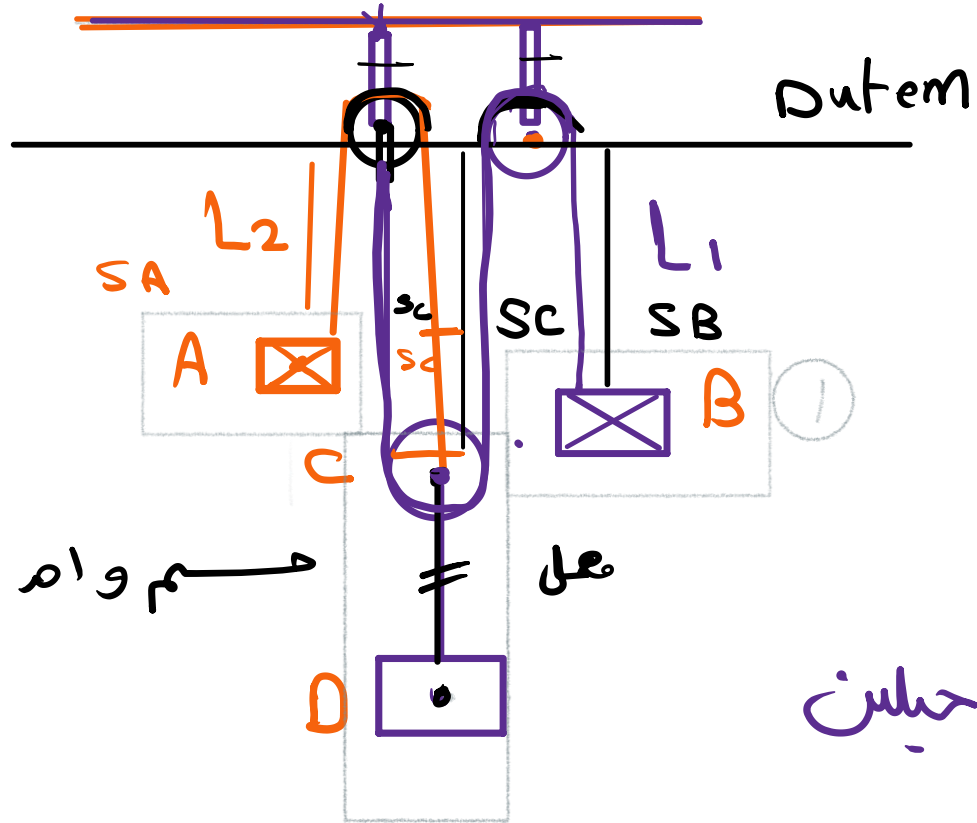


د.ع. ثالث

dependant motion.

حركة البكرات

Pully



$$L_1 = S_B + 2S_C$$

$$0 = v_B + 2v_C$$

$$0 = a_B + 2a_C$$

$$L_2 = S_A + S_C$$

$$0 = v_A + v_C$$

$$0 = a_A + a_C$$

- ① نأخذ Dutem عند مستوى ثابت
- ② نأخذ الأقسام المتحركة
إذا كان هناك حلول ثابت بين جسمين متحركين
نعتبرهم جسم واحد
- ③ نأخذ أطوال الأجزاء من النكات حول
- ④ نأخذ الأجزاء
لكل جلد يوحد معادله حركه
ثابت مجموع أطوال الأقسام = L
- ⑤ نفاضل للعلاوه عن ④

الحل

حبل واحد

$$L = S_A + 3S_B$$

تفاضل في الزمن

$$0 = V_A + 3V_B$$

$$0 = V_A + 3 \times 6$$

$$V_A = -18 \text{ m/s}$$

$V_A = 18 \text{ m/s}$ down word
↓ 18

Determine the speed of block A in Fig. 12-38 if block B has an upward speed of 6 ft/s.

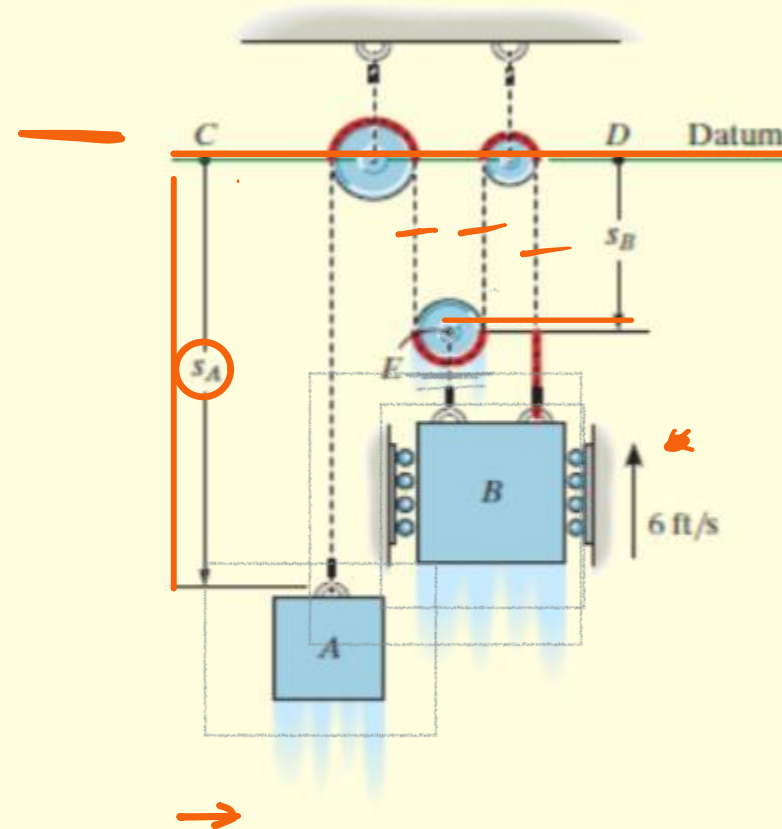


Fig. 12-38

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 $V_B = 6$

$$V_A = -2 \text{ m/s}$$

یو جرد حبلین

$$\Rightarrow L_1 = S_B + (S_B - S_C) + (S_A - S_C)$$

$$L_1 = 2S_B - 2S_C + S_A$$

$$0 = 2V_B - 2V_C + V_A$$

$$0 = 2V_B - 2V_C - 2$$

$$2 = 2V_B - 2V_C$$

$$\therefore \boxed{V_B - V_C = 1} \quad \leftarrow \textcircled{1}$$

$$L_2 = S_B + S_C$$

$$\boxed{0 = V_B + V_C} \quad \leftarrow \textcircled{2}$$

Determine the speed of block B in Fig. 12-40 if the end of the cord at A is pulled down with a speed of 2 m/s.

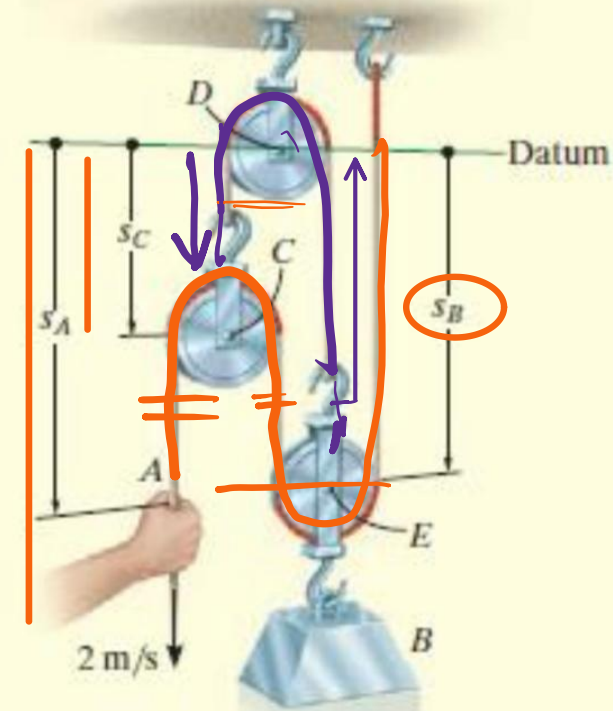


Fig. 12-40

$$2V_B = 1 \quad \textcircled{2} \text{ و } \textcircled{1} \text{ حل.$$

$$\boxed{V_B = \frac{1}{2} \text{ m/s}}$$

حل
✗