Mechanical Engineering Department
ME 384 Fluid Mechanics. Term 462.
Quiz 1. 23-01-2025. Please solve in a neat, organized and detailed manner and show all your work.

Name:____________________ **ID:**____________ **Section: 90350 Dr Yousuf**

Q1: If $f(x) = 3x^3 - 2x^2 + 4x - 1$,

a) use the Taylor series to estimate $f(4)$ based on knowing the value of $f(2)$ and neglect forth derivative and higher-order derivatives, and **(3 marks)**

b) dose this estimation approach the exact value? Why? **(1 marks)**

Solution

* Taylor series

$$f(x) = f(a) + f'(a)(x-a) + \tfrac{1}{2}f''(a)(x-a) + \underbrace{\cdots}_{\text{neglect}}$$

$f(x) = 3x^3 - 2x^2 + 4x - 1$	$f(2) = 23$
$f'(x) = 9x^2 - 4x + 4$	$f'(2) = 32$
$f''(x) = 18x - 4$	$f''(2) = 32$
$f'''(x) = 18$	$f'''(2) = 18$; $f^{(n)} = 0, n > 4$

$$f(4) = 23 + 32(4-2) + \frac{1}{2}(32)(4-2)^2 + \frac{1}{6}(18)(4-2)^3$$

$$f(4) = 151 + \frac{1}{6}(18)(4-2)^3 = 175 \text{ ; Real value} = 175$$

$$\text{for } f(4) = 151 \Rightarrow \text{Error\%} = \frac{175 - 151}{175} \times 100 = 13.7\%$$

Q2: $\vec{A}$ and $\vec{B}$ are two vectors in cartesian coordinates and you are given that $|\vec{A}| = 12$, $|\vec{B}| = 8$ and the angle between $\vec{A}$ and $\vec{B}$ is 30°. Find $\vec{A} \times \vec{B}$ if a vector $\vec{C} = -\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ has the same direction of $\vec{A} \times \vec{B}$? **(2 marks)**

Solution

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \, \hat{C} = 12 * 8 \sin 30 \, \hat{C} = 48 \hat{C}$$

$$\hat{C} = \frac{-\hat{i} + 2\hat{j} - 2\hat{k}}{\sqrt{1+4+4}} = -\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$$

$$\therefore \vec{A} \times \vec{B} = 48\left(-\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = -16\hat{i} + 32\hat{j} - 32\hat{k}$$

#

Q3: Use the definition of derivative to find

$$\frac{d(ae^{bx})}{dx}$$

where e is Euler's number and a and b are constant values and $b \neq 0$.

Solution

$$f(x) = ae^{bx} \longrightarrow f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{ae^{b[x+\Delta x]} - ae^{bx}}{\Delta x} = \lim_{\Delta x \to 0} ae^{bx}\left[\frac{e^{b\Delta x} - 1}{\Delta x}\right]$$

$$f'(x) = ae^{bx} \lim_{\Delta x \to 0} \frac{e^{b\Delta x} - 1}{\Delta x} \; ; \; \lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x} = 1$$

$$f'(x) = ae^{bx} \lim_{\Delta x \to 0} \frac{be^{b\Delta x} - 1}{b\Delta x} = abe^{bx}$$

$$\therefore f'(x) = abe^{bx}$$

#

Mechanical Engineering Department

ME 384 Fluid Mechanics. Term 462.

Quiz 1. 20-01-2025. Please solve in a neat, organized and detailed manner and show all your work.

Name:____________________ **ID:**__________ **Section: 66942 Dr Khaled**

Q1: If $f(x) = e^{1-10x}$, (**Use at least 6 significant figures in your calculations**.)

a) use the Taylor series to estimate $f(0.15)$ based on knowing the value of $f(0.10)$ and neglect third derivative and higher-order derivatives, and **(3 marks)**

b) what is the percentage of error in this estimation? **(1 mark)**

Solution

* Taylor series

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a) + \underbrace{\cdots}_{\text{neglect}}$$

$f(x) = e^{1-10x}$	$f(0.1) = 1$
$f'(x) = -10\,e^{1-10x}$	$f'(0.1) = -10$
$f''(x) = 100\,e^{1-10x}$	$f''(0.1) = 100$

$$f(0.15) = 1 - 10(0.15-0.1) + \frac{100}{2}(0.15-0.1)^2 = 0.625$$

Real value: $f(x) = e^{1-10x} \longrightarrow f(0.15) = 0.60653\,1$

$$\text{Error\%} = \frac{0.625 - 0.606531}{0.606531} \times 100 = 3.045021\%$$

Q2: Let two vectors $\vec{A} = 3\hat{\imath} - 5\hat{\jmath} + \hat{k}$ and $\vec{B} = 6\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$, are they parallel or opposite?

<u>Solution</u>

$$|\vec{A}| = \sqrt{3^2+5^2+1^2} = \sqrt{35} \quad \& \quad |\vec{B}| = \sqrt{6^2+2^2+2^2} = 2\sqrt{11}$$

$$\vec{A}\cdot\vec{B} = 3*6+5*2+2 = 30$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{30}{\sqrt{35}*2\sqrt{11}} \Rightarrow \theta = 40.14^\circ$$

$\because$ $\theta \neq 0$ or 180, then $\vec{A}$ & $\vec{B}$ are neither parallel nor opposite.

<u>Another Solution</u>

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & 1 \\ 6 & -2 & 2 \end{vmatrix} = \hat{i}\left[-10+2\right] - \hat{j}\left[\cancelto{0}{6-6}\right] + \hat{k}\left[-6+30\right]$$

$$= -8\hat{i} + 24\hat{k}$$

$\because$ $\vec{A}\times\vec{B} \neq 0 \Rightarrow$ then $\vec{A}$ & $\vec{B}$ are neither parallel nor opposite.

Q3: Given that $v = v(x, y) = 3xy^2$ and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

where x and y in m, u and v in m/s and $u = u(x, y)$. Find $u = u(2,3)$ if $u(1, y) = 2y$.

Solution

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} = -6xy$$

$$u = -6\int xy\, dx + f(y)$$

$$u = -\frac{6x^2}{2}y + f(y) \Rightarrow \boxed{u(x,y) = -3x^2y + f(y)}$$

but

$$u(1,y) = 2y \Rightarrow u(1,y) = -3y + f(y) = 2y$$

$$\boxed{f(y) = 5y}$$

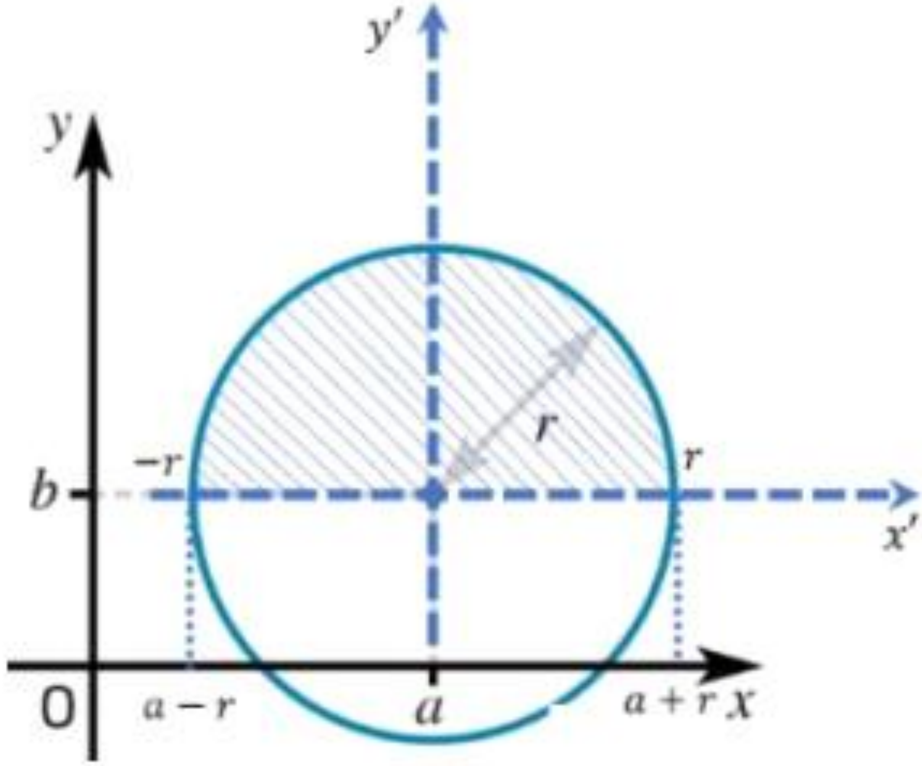
$$\therefore u(x,y) = -3x^2y + 5y$$

$$u(2,3) = -3(2)^2(3) + 5(3)$$

$$u(2,3) = -21 \text{ m/s}$$

#

Q3: The integral can be described as the area under the curve bounded by $f(x)$. Using this concept with the help of the figure, find the area of a circle with a center at the point (a,b) and a radius of r. **Hint**: *The general form of the equation of the circle is* $(x-a)^2+(y-b)^2=r^2$ *where the center is at the point (a,b), the radius is* r *and* a *and* b *are real positive numbers.* **(4 marks)**



$(x-a)^2+(y-b)^2=r^2$

$(y-b)^2 = r^2-(x-a)^2$

$y-b = \pm\sqrt{r^2-(x-a)^2}$

$y = b \pm\sqrt{r^2-(x-a)^2}$

for upper circle

$y = b+\sqrt{r^2-(x-a)^2}$

where, $a-r \le x \le a+r$

& $b \le y \le b+r$

Area $= 2\,A_{upper}$

$A_{upper} = \int_{a-r}^{a+r} \cancel{b} + \sqrt{r^2-(x-a)^2} - \cancel{b}\,dx$

$= \int_{a-r}^{a+r} \sqrt{r^2-(x-a)^2}\,dx$

$$\text{Area} = 2A_{\text{upper}} = 2\int_{a-r}^{a+r} \sqrt{r^2-(x-a)^2}\,dx$$

$\underline{\text{let:}}$ $x - a = r\sin\theta$ & $\theta = \sin^{-1}\left(\frac{x-a}{r}\right)$

$dx = r\cos\theta\, d\theta$

$\underline{\text{limits:}}$ at $x = a - r$

$$\theta = \sin^{-1}\left(\frac{\cancel{a}-r-\cancel{a}}{r}\right) = \sin^{-1}\left(\frac{-\cancel{r}}{\cancel{r}}\right) = \sin^{-1}(-1) = -\frac{\pi}{2}$$

at $x = a + r$

$$\theta = \sin^{-1}\left(\frac{\cancel{a}+r-\cancel{a}}{r}\right) = \sin^{-1}\left(\frac{\cancel{r}}{\cancel{r}}\right) = \sin^{-1}(1) = \frac{\pi}{2}$$

$$\therefore \text{Area} = 2\int_{-\pi/2}^{\pi/2} \sqrt{r^2 - r^2\sin^2\theta}\cdot r\cos\theta\, d\theta$$

$$= 2\int_{-\pi/2}^{\pi/2} r\cos\theta \cdot r\cos\theta\, d\theta = 2r^2\int_{-\pi/2}^{\pi/2} \cos^2\theta\, d\theta$$

$$\therefore \text{Area} = \cancel{2}r^2 \int_{-\pi/2}^{\pi/2} \frac{1}{\cancel{2}}(1+\cos 2\theta)\, d\theta$$

$$\text{Area} = r^2\left[\theta + \frac{\sin 2\theta}{2}\right]_{-\pi/2}^{\pi/2} = r^2\left[\frac{\pi}{2} + \frac{1}{2}\cancelto{0}{\sin\pi} + \frac{\pi}{2} - \frac{1}{2}\cancelto{0}{\sin(-\pi)}\right]$$

$$\text{Area} = r^2\left(\frac{\pi}{2} + \frac{\pi}{2}\right) \Rightarrow \boxed{\text{Area} = \pi r^2} \#$$

$$\text{Area} = r^2\left(\frac{\pi}{2} + \frac{\pi}{2}\right) \Rightarrow \boxed{\text{Area} = \pi r^2}$$